You can find a max flow in a flow network using the Edmonds-Karp algorithm in time $O(|V||E|^2)$.

**Problem 1.**
Find a max flow and a min cut in the following network. You must find the cut using your max flow, as we did in the proof of the max-flow min-cut theorem.

**Problem 2.**
We are given a flow network $(V, E, s, t, c)$ and a maximum flow $f$ for this network. The flow $f$ is given by specifying how much flow passes through each edge of $E$. Then, the capacity of one specific edge $e$ in $E$ is decreased by one. Describe an $O(|V| + |E|)$ algorithm for finding a maximum flow in the new network.

**Problem 3.**
Consider a directed $G = (V, E)$ and two sets of vertices $V_1$ and $V_2$. Both sets contain only vertices from $V$, and no vertex appears in both sets. We wish to disconnect every path that starts in a vertex of $V_1$ and ends in a vertex of $V_2$. Describe an algorithm that finds a minimum set of edges whose removal disconnects these paths. The running time should be $O(|V||E|^2)$.

**Problem 4.**
Our company transports products to the U.S. by ships and stores those products in warehouses. We have $n$ warehouses around the coasts, and the $i$'th warehouse can store up to $n_i$ products. There are $m$ ships coming, and the $j$'th ship brings $m_j$ products. Each ship has a list of specific warehouses it can reach (for example, a ship coming from Spain can reach the NY warehouse, but not the one in Alaska). A ship can visit any number of warehouses that it can reach.

We wish to find warehouses for all the products, according to the above restrictions. Describe this as a flow network problem.