Phase transitions in coalescing ballistic annihilation

Ballistic annihilation is a one-dimensional interacting particle system that caricatures chemical reactions. Particles are placed throughout the real line and assigned velocities at which they move from the onset. When two particles collide, they mutually annihilate and are removed from the system. While a variety of velocity distributions have been studied [6, 1, 9], what has become the canonical way to assign velocities is independently from $\{-1, 0, 1\}$ where velocity 0 is assigned with probability $p \in [0, 1)$, and velocities ± 1 symmetrically with probability $(1-p)/2$. We will refer to this system as *symmetric three-velocity ballistic annihilation* (BA).

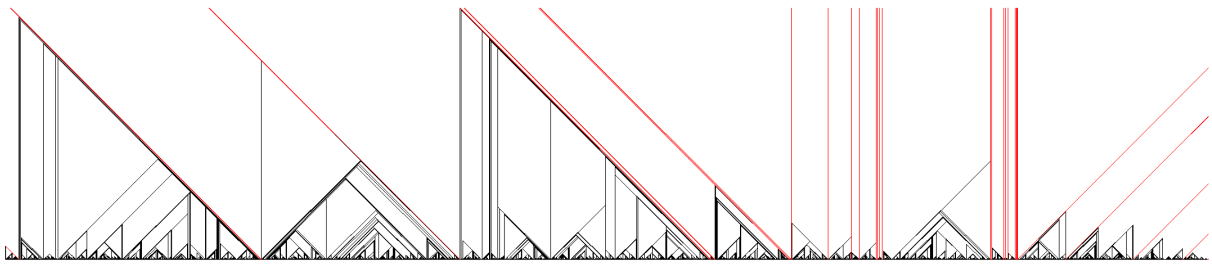

FIGURE 1. A graphical representation of symmetric three-velocity ballistic annihilation with $p = 1/4$ from [10]. Time is the y-axis and space is the x-axis.

Many intriguing features of BA were inferred by physicists in the 1990s [5, 8]. Perhaps the most fundamental quantity is the critical initial density of velocity-0 particles below which no particles persist for all time:

$$p_c = \sup\{p \colon \text{no particles survive}\}.$$

It was conjectured that $p_c = 1/4$ for BA, but at the time was never resolved in a rigorous probabilistic manner. A major difficulty is that the order in which collisions occur is sensitive to perturbations; changing the velocity of a single particle can have a cascading effect. This makes it difficult to compare processes with different parameters. In 2018 there was a breakthrough from Haslegrave, Sidoravicius, and Tournier that rigorously proved $p_c = 1/4$ [7]. It is important to better understand the reach, as well as limits, of this new approach. One way to do so is through the study of more general annihilating systems.

Inspired by earlier work from physicists [4, 11, 3], Benitez, Junge, Lyu, and Reeves (during the 2020 Baruch REU) extended BA dynamics to include collisions in which one of the particles involved sometimes survives the collision [2]. Fix parameters $0 \le a, b, x, y < 1$ with $2a + b \le 1$ and $x + y \le 1$. Denote left, right and stationary particles by $\overleftarrow{\bullet}$, $\vec{\bullet}$, and $\dot{\bullet}$, respectively. Using the notation $\bullet - \bullet \implies \Theta$ to denote a collision resulting in an outcome $\Theta \in \{\dot{\bullet}, \vec{\bullet}, \overleftarrow{\bullet}, \varnothing\}$, define *four parameter coalescing ballistic annihilation* (FCBA) to be systems with the following collision rules:

$$\vec{\bullet} - \overleftarrow{\bullet} \implies \begin{cases} \overleftarrow{\bullet}, & \text{with probability } a \\ \vec{\bullet}, & \text{with probability } a \\ \dot{\bullet}, & \text{with probability } b \\ \varnothing, & \text{else} \end{cases}, \tag{1}$$

$$\dot{\bullet} - \overleftarrow{\bullet} \implies \begin{cases} \overleftarrow{\bullet}, & \text{with probability } x \\ \dot{\bullet}, & \text{with probability } y \\ \varnothing, & \text{else} \end{cases}, \quad \text{and} \quad \vec{\bullet} - \dot{\bullet} \implies \begin{cases} \vec{\bullet}, & \text{with probability } x \\ \dot{\bullet}, & \text{with probability } y \\ \varnothing, & \text{else} \end{cases}.$$

See Figure 2 for an example. Note that BA is the special case $a = b = x = y = 0$.

The main result from [2] was a formula for p_c for a variety of parameter choices in FCBA.

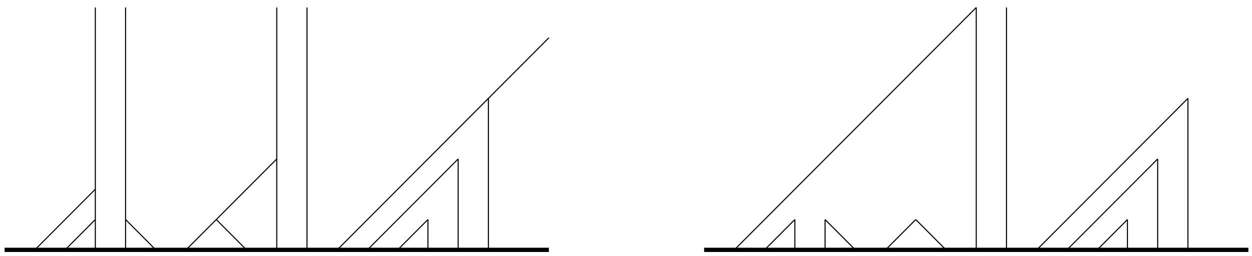

Figure 2. Graphical representations of coalescing ballistic annihilation (left) versus ballistic annihilation (right). Despite the same initial configuration, markedly different outcomes can occur.

Theorem. *Let* $p_c(a, b, x, y) := \sup\{p\colon$ *all particles perish in FCBA*$\}$. *Whenever* $y = 0$ *or* $a = b = x = 0$, *it holds that* $p_c = (\varphi/\psi)\mathbf{1}_{\varphi/\psi\in(0,1)}$ *with*

$$\begin{aligned}\varphi &= (1-y)^2(1-b(1-x)-y)\\ \psi &= 4-2a-2b-3x+2ax+3bx-bx^2-9y+5ay+2by\\ &\qquad +6xy-5axy-2bxy+8y^2-6ay^2-3xy^2+3axy^2-3y^3+3ay^3.\end{aligned}$$

The involved formula for p_c is not so enlightening on its own, but illustrates the complexity of these coalescing systems. A followup question is to demonstrate that there are phase transitions not just in the density of 0-velocity particles, but in the coalescence parameters.

Question. *Does FCBA undergo a phase transition when* p *is fixed, and the coalescence parameters from* (1) *are varied? Is there an exact formula for the transition location?*

For example, one would expect that once the probability y, that a stationary particle survives a collision with a moving particle, surpasses a critical threshold some stationary particles persist for all time. The hope is that the techniques from [2] can be extended. This would be interesting because it establishes a completely new regime in ballistic annihilation.

References

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