

The Volatility Surface: Errata

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- **Page xxiii** “what it is that analysis of stock prices can tell **us** about how options ought to be priced”.
- **Page 7** In the first equation, the coefficient of

$$S \frac{\partial V}{\partial S}$$

should be $+r$ not $-r$. Moreover dZ_2 in the fourth line of the equation should have a prefactor η . Thus the equation at the top of page 7 should now read:

$$\begin{aligned} & d\Pi_1 - r \Pi_1 dt \\ = & \left\{ \frac{\partial V}{\partial t} + \frac{1}{2} v S^2 \frac{\partial^2 V}{\partial S^2} + \rho \eta v \beta S \frac{\partial^2 V}{\partial v \partial S} + \frac{1}{2} \eta^2 v \beta^2 \frac{\partial^2 V}{\partial v^2} + r S \frac{\partial V}{\partial S} - r V \right\} dt \\ & + \frac{\partial V}{\partial v} dv \\ = & \beta \sqrt{v} \frac{\partial V}{\partial v} \{ \phi(S, v, t) dt + \eta dZ_2 \} \end{aligned}$$

(Thanks to Jan Oblój of Imperial College)

- **Page 10** In the first equation, there should be no S in front of $\frac{\partial}{\partial S_T}$. The equation should read

$$\frac{1}{2} \frac{\partial^2}{\partial S_T^2} (\sigma^2 S_T^2 \varphi) - \frac{\partial}{\partial S_T} (\mu S_T \varphi) = \frac{\partial \varphi}{\partial T}$$

Also, the equation lower down on page 10:

$$\begin{aligned}\frac{\partial C}{\partial T} &= \frac{\sigma^2 K^2}{2} \varphi + \int_K^\infty dS_T \mu S_T \varphi \\ &= \frac{\sigma^2 K^2}{2} \frac{\partial^2 C}{\partial K^2} + \mu(T) \left(-K \frac{\partial C}{\partial K} \right)\end{aligned}$$

should be corrected to:

$$\begin{aligned}\frac{\partial C}{\partial T} &= \frac{\sigma^2 K^2}{2} \varphi + \int_K^\infty dS_T \mu S_T \varphi \\ &= \frac{\sigma^2 K^2}{2} \frac{\partial^2 C}{\partial K^2} + \mu(T) \left(C - K \frac{\partial C}{\partial K} \right)\end{aligned}$$

(Thanks to Jan Oblój of Imperial College)

- **Page 12** The last sentence should read “equation (1.8) becomes” rather than “equation (1.4) becomes”.
(Thanks to Dan Pirjol of Markit)
- **Page 33** In equation (3.14), $\sqrt{v_t}$ should be inside the expectation. (3.14) would then read:

$$du_t = -\lambda(u_t - \bar{v})dt + \frac{\rho\eta}{2}u_t dt + \rho\eta \frac{x_T}{\hat{w}_T} d\hat{w}_t + \sqrt{1 - \rho^2} \eta \mathbb{E}[\sqrt{v_t} dW_t | x_T]$$

Also, the line just after equation (3.15) should read: “with $\hat{v}'_s := (v_0 - \bar{v}') e^{-\lambda' s + \bar{v}'}$.” (an extra subscript 0 on the v).

On the last line of page 33, we should replace the words “Brownian Bridge” with the words “Brownian Bridge-like”.

(Thanks to Jan Oblój)

- **Page 35** Formula (3.19) should read:

$$\begin{aligned}\sigma_{BS}(K, T)^2 &\approx \frac{\hat{w}'_T}{T} + \rho\eta \frac{x_T}{T^2} \int_0^T dt \int_0^t e^{-\lambda'(t-s)} ds \\ &= \frac{\hat{w}'_T}{T} + \rho\eta \frac{x_T}{\lambda'T} \left\{ 1 - \frac{(1 - e^{-\lambda'T})}{\lambda'T} \right\}\end{aligned}$$

– the current version has an extra factor $\frac{1}{T}$ inside the first integral.

Also, further down on the same page there are three references to:

$$\frac{\partial}{\partial x_t} \sigma_{BS}(K, T)^2$$

These should of course read

$$\frac{\partial}{\partial x_T} \sigma_{BS}(K, T)^2$$

(Thanks to Paul Jones of Imperial College)

- **Page 44** Second paragraph, “from equation (3.11)” should read “from equation (3.15)”.
(Thanks to Jan Oblój)
- **Page 49** “We note too that the Heston-**Nandi** model and its local volatility equivalent are single-factor”.
- **Page 57** Line 5 should read “The characteristic function for **an** exponential”.
(Thanks to Jan Oblój)
- **Page 61** There is a subscript BS missing in equation (5.9) which should read:

$$\begin{aligned} C_J(S, K, \Delta T) &\approx (1 - \lambda \Delta T) C_{BS}(Se^{\mu_J \Delta T}, K, \Delta T) + \lambda \Delta T C_{BS}(JS, K, \Delta T) \\ &= C_{BS}(Se^{\mu_J \Delta T}, K, \Delta T) + O(\Delta T) \end{aligned}$$

(Thanks to Jan Oblój)

- **Page 68** Factor T missing in the denominator of the last equation on the page. It should read

$$\begin{aligned} I(u, T) &= \frac{1}{T} \int_0^T e^{\gamma_v D(u, t)} dt \\ &= -\frac{\mathbf{1}}{\mathbf{T}} \frac{2\gamma_v}{p_+ p_-} \int_0^{-\gamma_v D(u, T)} \frac{e^{-z} dz}{(1 + z/p_+)(1 + z/p_-)} \end{aligned}$$

(Thanks to Roger Lord)

- **Page 89** 6 lines from the bottom, “earlier in Chapter 7” should read “earlier in Chapter 3”.
(Thanks to Jan Oblój)

- **Page 102** The expression for v_{loc} should read:

$$v_{loc} = \frac{\frac{\partial w}{\partial T}}{1 - \frac{k}{w} \frac{\partial w}{\partial k} + \frac{1}{4} \left(-\frac{1}{4} - \frac{1}{w} + \frac{k^2}{w^2} \right) \left(\frac{\partial w}{\partial k} \right)^2 + \frac{1}{2} \frac{\partial^2 w}{\partial k^2}}$$

(Thanks to John-Joe Hosking of Imperial College)

- **Page 113** There should be an extra ΔK so that the formula reads

$$2 N (d_2)|_{S=K} \Delta K$$

(Thanks to Craig Nelson)

- **Page 124** should have $MinCoupon = -12\%$ not $MinCoupon = -1\%$.
(Thanks to Jining Han)

- **Page 139** The definition of d_2 should have a minus sign:

$$z(k) = d_2 = -\frac{k}{\sigma \sqrt{T}} - \frac{\sigma \sqrt{T}}{2}$$

(Thanks to James LaDue of Imagine Software)

- **Page 141** The upper limit of the y -integrations should be ∞ not T .
(Thanks to Fahmi Zaidi)

- **Page 160** The VXB formula should read

$$\mathbb{E}_{T_1}[\sqrt{\langle x \rangle_{T_1, T_2}}] =: Y$$

(Thanks to Jining Han)

- **Page 177** Add an entry: “Stochastic differential equation (SDE), 4,7,26,43,44,52,57,87,91.”