Financial Distress and the Cross Section of Equity Returns*

Lorenzo Garlappi†
University of British Columbia

Hong Yan‡
University of South Carolina

First draft: November 2006
This draft: January 2010

*We are grateful to Aydoğan Alti, Kerry Back, Luca Benzoni, John Campbell, Murray Carlson, Allan Eberhart, Mike Gallmeyer, João Gomes (NBER discussant), Shingo Goto, Jennifer Huang, Timothy Johnson, Hong Liu (CICF discussant), Dmitry Livdan, Alexander Philipov (FEA discussant), Eric Powers, Jacob Sagi, Paul Tetlock, Sheridan Titman, Stathis Tompaidis, Sergey Tsyplakov, Raman Uppal, Lu Zhang (AFA discussant), and participants of the 2007 NBER Asset Pricing Program Meeting, the 2007 Financial Economics and Accounting Conference, the 2008 American Finance Association Annual Meeting, the 2008 China International Conference in Finance, the Fall 2009 JOIM Conference Series, and seminars at Brigham Young University, City University of Hong Kong, Hong Kong University, Hong Kong University of Science and Technology, McGill University, National University of Singapore, Singapore Management University, State Street Global Advisors, Temple University, Texas A&M University, Texas Tech University, the University of British Columbia, the University of Calgary, the University of Hong Kong, the University of Illinois at Urbana-Champaign, the University of Lausanne, the University of North Carolina at Charlotte, the University of South Carolina, the University of Texas at Austin, and the University of Toronto for helpful comments. We thank *Moody's KMV* for providing us with the data on *Expected Default Frequency*™ (EDF™). Tao Shu provided excellent research assistance. We are responsible for the remaining errors in the paper.

†Department of Finance, Sauder School of Business, University of British Columbia, Vancouver, BC V6T 1Z2, Canada. Email: lorenzo.garlappi@sauder.ubc.ca

‡Department of Finance, Moore School of Business, University of South Carolina, Columbia, SC 29208, USA. Email: yanh@moore.sc.edu.

Financial Distress and the Cross Section of Equity Returns

Abstract

In this paper, we provide a new perspective for understanding cross-sectional properties of equity returns. We explicitly introduce financial leverage in a simple equity valuation model and study the asset pricing implications of strategic default and potential shareholder recovery upon resolution of financial distress. Our model is capable of simultaneously explaining a number of empirical regularities regarding financial distress and the cross section of stock returns, such as lower returns for distressed stocks, the strengthening of the book-to-market effect for firms with high default likelihood, and the concentration of momentum profits among low credit quality firms. The model further predicts that: (i) the relationship between value premium and default probability is hump-shaped, and (ii) momentum profits are stronger among nearly distressed firms with significant prospects for shareholder recovery. Using data on *Expected Default Frequency (EDF)* from *Moody's KMV*, we find support for the novel predictions of our model, thus confirming the importance of shareholder recovery as a pervasive mechanism for a better understanding of cross-sectional stock return "anomalies."

JEL Classification Codes: G12, G14, G33

Keywords: Financial distress, shareholder recovery, value premium, momentum.

1 Introduction

Financial distress is frequently invoked to justify the existence of "anomalous" cross-sectional properties of equity returns such as the size effect and the value premium (*e.g.*, Fama and French (1996)). The existing empirical evidence, however, presents a complex picture that eludes a coherent and unifying explanation. On the one hand, Griffin and Lemmon (2002) and Vassalou and Xing (2004) show that the book-to-market and the size effects are concentrated in high default risk firms, thus lending credence to the conjecture that investors demand a premium for bearing distress risk. On the other hand, Dichev (1998) and Campbell, Hilscher, and Szilagyi (2008) document that high-default-risk firms tend to have *lower* future stock returns, hence casting doubt on the notion of a market compensation for distress risk. Furthermore, recent work by Avramov, Chordia, Jostova, and Philipov (2007) indicates that profits of momentum strategies that buy "winners" and sell "losers" are remarkably concentrated among a small subset of firms with low credit ratings, therefore adding a new dimension to the complex relationship between financial distress and cross-sectional properties of equity returns.

In this paper we show that these seemingly incongruent empirical patterns can be understood within an equity valuation model that explicitly accounts for financial leverage and recognizes that shareholders, by strategically defaulting on their debt, may recover part of the residual firm value upon the resolution of financial distress.[1] The resolution of financial distress includes debt restructuring and debt-equity exchange that do not necessarily lead to formal bankruptcy filings. It is therefore important to note that *shareholder recovery* is a broader concept than that of "violation of absolute priority" in bankruptcy proceedings.[2] In a recent study, Morellec, Nikolov, and Schürhoff (2008) estimate that, among U.S. firms over the period from 1992 to 2004, the average shareholder recovery is about 20% of the asset value at the time of financial distress.

[1]Financial distress includes instances of missed payments, modified terms and structure of debt in private workouts, and, ultimately, bankruptcy filings. In this paper, we use the terms "default" and "financial distress" interchangeably.

[2]In the case of bankruptcy filings, deviations from the absolute priority rule have been documented in the past by Franks and Torous (1989), Eberhart, Moore, and Roenfeldt (1990), Weiss (1991), and Betker (1995). Bharath, Panchapegesan, and Werner (2009) argue that deviations from the absolute priority rule in bankruptcy proceedings have become less prevalent in recent years. However, they also show that firms that do file for Chapter 11 protection are in a much worse financial condition than in earlier years. This can imply that other firms resolve their financial distress without going through the bankruptcy proceedings and that equity holders of firms in bankruptcy would have little bargaining power.

We first develop our main intuition in a simple model in which we take capital structure and investment decisions as given. We then verify the robustness of our intuition in a more general model in which firms endogenously make investment and financing decisions based on their existing capital and debt levels. As in Berk, Green, and Naik (1999) and several papers afterwards,[3] in our model equity beta is linked to firm characteristics such as the book-to-market ratio. The inclusion of financial leverage allows us to show how leverage amplifies the book-to-market effect, thus providing a rationale for the findings of Griffin and Lemmon (2002) and Vassalou and Xing (2004), who document a stronger book-to-market effect in highly levered stocks.

More important, we show that the likelihood of shareholder recovery from firms in financial distress, a feature largely ignored in the previous asset pricing literature,[4] can fundamentally alter the riskiness of equity as default probability rises. All else being equal, at low levels of default probability, higher leverage increases equity beta. At high levels of default probability, however, the possibility of debt renegotiation and subsequent asset redistribution upon financial distress actually de-levers the equity beta, and thus reduces the equity risk. As a consequence, in the presence of shareholder recovery, our model predicts that equity beta and expected return are hump-shaped in default probability.[5]

The hump-shaped relationship between expected return and default probability is capable of simultaneously explaining two known empirical regularities: the inverse relationship between expected return and default probability (Dichev (1998), Campbell, Hilscher, and Szilagyi (2008), Garlappi, Shu, and Yan (2008), and George and Hwang (2009))—the so-called "distress risk puzzle"—and the concentration of momentum profits in low-credit-quality stocks (Avramov, Chordia, Jostova, and Philipov (2007)). We further show that this hump-shaped relationship has novel predictions for the cross-sectional properties of both value premium and momentum profits. Specifically, (i) in the presence of shareholder recovery the value premium is hump-shaped in default probability; and (ii) among high default probability firms, momentum profits are larger for stocks with higher expected shareholder recovery.

[3]The related literature includes Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Cooper (2006), Gala (2006), Sagi and Seasholes (2007), and Novy-Marx (2008).

[4]Exceptions are Fan and Sundaresan (2000), who use this mechanism to study corporate bond spreads, and Garlappi, Shu, and Yan (2008), who adopt this feature to explain the negative relationship between default probability and stock returns.

[5]This pattern of betas is consistent with the observation in Campbell, Hilscher, and Szilagyi (2008), who illustrate that unconditional factor loadings on the excess market return and the value factor are hump-shaped (see their Figure 3).

The intuition for the pattern of value premium with respect to default probability is as follows. Consider two identical firms, A and B. Suppose firm A experiences a positive shock to its stock price and firm B a negative shock. As a consequence firm A will have a smaller book-to-market ratio and a smaller default probability than B. If we construct a portfolio that is long B and short A, then the expected return on this portfolio, $ER_B - ER_A$, will crucially depend on the relationship between expected return and default probability. If this relationship is monotonically increasing, as it happens when shareholders of both firms do not expect any recovery upon financial distress, then the spread $ER_B - ER_A$ is always positive. In contrast, if the relationship between expected return and default probability is hump-shaped, the sign of the spread will depend on where these two firms are located on the default probability spectrum. For low levels of default probability, expected returns are increasing in default probability and thus the spread $ER_B - ER_A$ is positive. However, for high levels of default probability, expected returns are decreasing in default probability and hence the spread is negative. This simple argument suggests a value spread that is hump-shaped in default probability in the presence of possible shareholder recovery upon financial distress.

A similar argument can be made to justify our empirical prediction regarding the relative strength of momentum profits. When the relationship between expected returns and default probability is hump-shaped, a shock to prices as reflected in the realized return can have different implications for expected future returns. When the probability of default is low, a negative shock to the stock price (low realized return) increases the default probability, and hence leads to a higher expected return. This results in a negative autocorrelation in returns. In contrast, when the probability of default is high, a negative shock to the stock price leads to a lower expected return and hence a positive autocorrelation in returns. Our model thus implies that return continuation should be more pronounced among firms with high default probability, a prediction that finds empirical support in Avramov, Chordia, Jostova, and Philipov (2007). Moreover, our theory suggests that it is not financial distress *per se* that causes momentum strategies to be more profitable. Rather, momentum profits increase with the prospect for shareholder recovery among nearly distressed firms.

To correctly interpret our analysis of momentum strategies, it is important to keep in mind that momentum is not equivalent to positive autocorrelation in returns, as pointed out by Lewellen (2002). The profitability of momentum strategies is mostly a cross-sectional phenomenon which may have a number of plausible explanations, including behavioral and liquidity-based ones.[6] Our emphasis is on "enhanced momentum strategies," *i.e.*, momentum strategies that focus on a set of stocks whose returns exhibit positive autocorrelations, similar to those studied by Sagi and Seasholes (2007). Unlike their mechanism that relies on growth options, our model shows that financial leverage and shareholder recovery upon financial distress are important to generate economically relevant enhanced momentum profits.

In addition to the supporting empirical evidence cited above, our theory is consistent with other recent empirical studies that attempt to directly model and measure individual firms' conditional betas. Exploiting the exogenous variation in creditor protection provisions across countries, Favara, Schroth, and Valta (2009) find that equity beta increases with the degree of credit protection and decreases with the likelihood of shareholders' recovery as measured by bargaining power and liquidation costs. Furthermore, O'Doherty (2009) argues that the effect of financial distress can be explained by conditional betas.

The novel contribution of our work is to recognize that the patterns of betas discussed above have implications for the variation of value premium and momentum profits across the spectrum of default probabilities. Our empirical analysis is thus focused on documenting this link and verifying the predictions of our theory on the cross-sectional properties of these two anomalies. For this purpose, we use as a measure of default probability the market-based *Expected Default Frequency (EDF)*, acquired directly from *Moody's KMV*. Our dataset, available at the monthly frequency, spans from 1969 to 2007.

To verify our prediction about the existence of a hump-shaped relationship between value premium and default probability, we form portfolios of stocks sorted on book-to-market ratios and default probability. Contrary to Griffin and Lemmon (2002) and Vassalou and Xing (2004), our results indicate that the value premium is hump-shaped instead of monotonically increasing in default probability: it increases when levels of EDF are low and declines sharply at very high levels of EDF. This hump-shaped pattern is robust to traditional risk adjustment procedures that account for market, size, book-to-market, momentum, and liquidity factors. We demonstrate that the discrepancy between our results and those previously documented in the literature stems from the procedure of sample selection and portfolio formation.

[6]See, *e.g.*, Daniel, Hirshleifer, and Subrahmanyam (1998) and Asness, Moskowitz, and Pedersen (2009).

To examine our conjecture about the relative strength of momentum profits across different levels of shareholder recovery for firms with high levels of default probability, we refine the Jegadeesh and Titman (1993) momentum profits by forming portfolios according to default probability and proxies for shareholder recovery, such as asset value, R&D expenditure, and degree of industry concentration. The results are strongly supportive of our theory. At high levels of default probability, momentum profits are considerably stronger when shareholder recovery is high. In contrast, shareholder recovery does not play any significant role at low levels of default probability. We further revisit the analysis of Avramov, Chordia, Jostova, and Philipov (2007) using our broader dataset and confirm that momentum profits are stronger in stocks with higher default probability. In particular, after adjusting for traditional risk factors, the enhanced momentum profits are significantly positive only among firms that rank in the top EDF quintiles. Finally, we find that momentum profits load positively on the size factor at low levels of default probability but negatively at high levels of default probability. This is indicative of the fact that at low leverage levels, the enhanced momentum profits are likely originating from small firms with growth opportunities, as suggested by Sagi and Seasholes (2007), while at high leverage levels, the enhanced momentum profits are most likely coming from potential shareholder recovery, a feature often associated with large firms.

The rest of the paper proceeds as follows. In Section 2 we present a simple valuation model for levered equity and develop the main intuition for the effect of shareholder recovery on equity beta, the value premium, and momentum profits. In Section 3 we extend the model to include optimal capital structure decisions and growth options and examine the robustness of the theoretical intuition in this more general framework. In Section 4 we use data on default probabilities from Moody's KMV to empirically confirm our predictions on the links between value premium, momentum profits and default probability. Section 5 concludes. All proofs are collected in Appendix A. Details of the general model of Section 3 and its numerical analysis are presented in Appendix B.

2 A simple model of levered equity returns

In this section we construct a stylized valuation model of levered equity in order to develop the main economic intuition underlying the connection between financial distress and cross-sectional properties of equity returns. To keep the analysis as simple as possible we take a firm's capital structure as given and ignore growth options and financing frictions. In Section 3 we generalize this setup by allowing for endogenous investment and financing decisions at the firm level over the business cycle.

2.1 The firm

We consider a representative firm producing one unit of output per period of time. The real price of output at time t is e^{p_t} where the log price p_t is assumed to follow a mean-reverting (Ornstein-Uhlenbeck) process

$$dp_t = (\mu^{\mathbb{P}} - \zeta p_t)dt + \sigma dW_t^{\mathbb{P}}, \tag{1}$$

with $\mu^{\mathbb{P}}$, ζ, and σ being firm-specific constants, and $dW_t^{\mathbb{P}}$ the increment of a standard Brownian motion under the physical measure $\mathbb{P}$. When the degree of mean reversion ζ goes to zero, process (1) collapses to a geometric Brownian motion. The mean reversion case is more realistic when dealing with competition in the product market and relates more closely to the general model that we discuss in Section 3. The geometric Brownian motion case, however, has the advantage of analytical tractability. As we will show later, the main intuition of the model is nevertheless unaffected by the nature of the process describing the output price.

The production of output requires an operating cost of c per unit of time. The firm finances its operations through a perpetual bond that pays a constant coupon of l per unit of time. The profit after interest service is thus $e^{p_t} - c - l$, which accrues to equity holders as long as the firm is operating.

When the firm is in financial distress, which we take to be the instance in which shareholders either enter into strategic renegotiation with debt holders or file for bankruptcy, we assume that equity holders can recover a fraction $\eta \in [0, 1]$ of the firm's residual value $R(p_t)$, a non-negative quantity that may depend on the underlying price p_t. This assumption is a reduced-form representation of asset redistribution as a consequence of strategic renegotiation between creditors and shareholders upon financial distress (*e.g.*, Fan and Sundaresan (2000)).[7] Many cases of financial distress are resolved through debt reorganization in private workouts, with only

[7] In the structural model of Fan and Sundaresan (2000), η is the product of shareholder bargaining power and liquidation costs, both taken to be deterministic quantities. While it is possible to consider the case of a stochastic η, adding this layer of complexity does not alter the basic intuition.

a fraction of them actually going through bankruptcy filings.[8] In their structural estimation of a dynamic capital structure model that incorporates such debt renegotiations, Morellec, Nikolov, and Schürhoff (2008) find that the parameter η has a wide cross-sectional variation among U.S. firms with a mean around 20% of firm value at the time of distress.

2.2 Equity valuation

Equity value is obtained in a risk-neutral valuation framework. Under the risk-neutral measure, $\mathbb{Q}$, the evolution of the log price, p_t, is

$$dp_t = (\mu^{\mathbb{Q}} - \zeta p_t)dt + \sigma dW_t^{\mathbb{Q}}, \tag{2}$$

where $\mu^{\mathbb{Q}}$ is the risk-adjusted drift, and

$$dW_t^{\mathbb{Q}} = \gamma dt + dW_t^{\mathbb{P}} \tag{3}$$

is a Brownian motion under the measure $\mathbb{Q}$ with γ being the market price of risk associated with the price process dp_t.[9] Denoting by $E^{\mathbb{Q}}$ the expectation under the risk-neutral measure $\mathbb{Q}$, the firm's equity value is given by:

$$V(p_t) = E^{\mathbb{Q}}\left[\int_0^{\tau_L} e^{-rs}(e^{p_{t+s}} - c - l)ds + \eta R(\underline{p})e^{-r\tau_L}\right], \tag{4}$$

where $\tau_L = \inf\left\{t : p_t = \underline{p}\right\}$ denotes the first time log price p_t hits the threshold $\underline{p}$, at which point the firm becomes distressed. For expositional convenience, we will use the terms "financial distress" and "default" interchangeably. The threshold $\underline{p}$ is chosen optimally by shareholders.[10] The integrand in equation (4) represents the stream of profits received by equity holders until default. The last term represents the present value of shareholder recovery upon default, which

[8] See, *e.g.*, Gilson, John, and Lang (1990) and Franks and Torous (1994). Hotchkiss, John, Mooradian, and Thorburn (2008) provide an excellent review of the recent literature.

[9] This is equivalent to assuming the existence of the following pricing kernel M_t:

$$\frac{dM_t}{M_t} = -rdt - \gamma dW_t^{\mathbb{P}},$$

where $\gamma = \frac{\mu^{\mathbb{P}} - \mu^{\mathbb{Q}}}{\sigma}$ and r is the instantaneous risk-free rate.

[10] The endogenous choice of default boundary by shareholders is a common feature in theoretical models (see, *e.g.*, Black and Cox (1976) and Leland (1994)). Empirically, Brown, Ciochetti, and Riddiough (2006) show that default decisions are endogenous responses to anticipated restructuring outcomes.

is a fraction η of the residual value $R(\underline{p})$. The following proposition characterizes the equity value and the endogenous default boundary.

Proposition 1 *Assume that the log price evolves according to the Ornstein-Uhlenbeck process (1). Then, the equity value (4) is given by*

$$V(p_t) = \begin{cases} V^U(p_t) - \frac{l+c}{r} + A \cdot H\left(-\frac{r}{\zeta}, -\frac{\mu^{\mathbb{Q}} - \zeta p_t}{\sigma\sqrt{\zeta}}\right), & \text{if } p_t > \underline{p} \\ \eta R(p_t), & \text{if } p_t \leq \underline{p} \end{cases}, \tag{5}$$

where

$$V^U(p_t) = \int_0^\infty e^{-r\tau}\left[\exp\left(p_t e^{-\zeta\tau} + \left(1 - e^{-\zeta\tau}\right)\frac{\mu^{\mathbb{Q}}}{\zeta} + \sigma^2\left(\frac{1 - e^{-2\zeta\tau}}{4\zeta}\right)\right)\right] d\tau, \tag{6}$$

$H(v, z)$ is the generalized Hermite function of order v given in (A5), and the quantities A and $\underline{p}$ are constants that are determined by the following value-matching and smooth-pasting conditions:

$$V(\underline{p}) = \eta R(\underline{p}) \tag{7}$$

$$V'(\underline{p}) = \eta R'(\underline{p}). \tag{8}$$

The equity value in Proposition 1 has an intuitive structure. Before default, $p_t \geq \underline{p}$, equity value is equal to the present value of the unlevered risky cash flow, $V^U(p_t)$, minus the capitalized value of the liabilities, $(c + l)/r$, plus the present value of A units of the limited liability option $H(-r/\zeta, -(\mu^{\mathbb{Q}} - \zeta p_t)/\sigma\sqrt{\zeta})$. The default trigger $\underline{p}$ is optimally chosen by shareholders who anticipate the potential recovery $\eta R(p_t)$ when p_t falls below $\underline{p}$.

It appears that, in the expression of equity value (5), financial leverage l does not have a substantially distinct role from operating leverage c. This observational equivalence between the two forms of leverage stems from the exogenous nature of both c and l in this simple model and will be resolved in the general model in Section 3. However, it is important to point out that, even with exogenous operating and financial leverages, financial leverage serves an entirely different *contractual* role than that of operating leverage. The contractual obligation of shareholders to bondholders is binding, and the outcome of the strategic interaction between them crucially determines the potential payoff to shareholders upon financial distress. In the absence of financial leverage, there is no renegotiation as equity holders have all claims.

The expression for the equity value in Proposition 1 simplifies considerably in the limit as the mean reversion parameter ζ vanishes, *i.e.*, the price p_t evolves according to a geometric Brownian motion. In order to obtain a fully closed form solution, in the next corollary we assume that the residual firm value $R(p_t)$ is affine in the product price e^{p_t}, *i.e.*, $R(p_t) = a + be^{p_t}$ $a, b > 0$. This choice includes situations in which, upon the resolution of financial distress, equity holders receive either a fixed payout ($b = 0$) or a stake in the unlevered firm ($a = 0$, $b = 1/\delta$, as in Fan and Sundaresan (2000)). The choice, however, does not affect the underlying intuition, as we will discuss later.

Corollary 1 *Suppose the mean reversion parameter* $\zeta \to 0$ *in (1). Let* $\delta \equiv r - \mu^{\mathbb{Q}} - \frac{\sigma^2}{2} > 0$, *and assume that the residual firm value upon default is* $R(p_t) = a + be^{p_t}$ *with* $a \geq 0$ *and* $0 \leq b \leq \frac{1}{\eta\delta}$. *Then the equity value (4) is given by*

$$V(p_t) = \begin{cases} \frac{e^{p_t}}{\delta} - \frac{c+l}{r} + Ae^{\phi\, p_t}, & \textit{if } p_t > \underline{p} \\ \eta(a + be^{p_t}), & \textit{if } p_t \leq \underline{p} \end{cases}, \tag{9}$$

where

$$\phi = \frac{1}{2} - \frac{2(r-\delta) + \sqrt{\left(\sigma^2 - 2(r-\delta)\right)^2 + 8\sigma^2 r}}{2\sigma^2} < 0 \tag{10}$$

and the constants $\underline{p}$ *and* A *are, respectively,*

$$\underline{p} = \log\left(\frac{\eta a + \frac{c+l}{r}}{\left(\frac{1}{\delta} - \eta b\right)\left(1 - \frac{1}{\phi}\right)}\right) > 0, \qquad A = \frac{1}{\phi}\left(\eta b - \frac{1}{\delta}\right) e^{\underline{p}(1-\phi)} > 0. \tag{11}$$

Note that in equation (9) the quantity e^{p_t}/δ is the limit of the value $V^U(p_t)$ in (6), and the value of the limited liability option, $e^{\phi p_t}$, is the limit of the generalized Hermite function $H(-r/\zeta, -(\mu^{\mathbb{Q}} - \zeta p_t)/\sigma\sqrt{\zeta})$ in Proposition 1, as $\zeta \to 0$. It is easily shown that the distress threshold, $\underline{p}$, is increasing in η. This is consistent with the findings in Bharath, Panchapegesan, and Werner (2009) that, in recent years, shareholder recovery in Chapter 11 proceedings is much lower and hence firms filing for bankruptcy tend to be in much worse financial conditions.

The condition $b \leq 1/(\eta\delta)$ in the above corollary ensures that the number of limited liability put options is non-negative ($A \geq 0$ in (11)) and that the default threshold $\underline{p}$ is well-defined.

Substituting the expression of A in (9) we obtain, for $p_t \geq \underline{p}$,

$$V(p_t) = \frac{e^{p_t}}{\delta} - \frac{c+l}{r} + \pi_t \left[\frac{e^{\underline{p}}}{\phi} \left(\eta b - \frac{1}{\delta} \right) \right] > 0, \tag{12}$$

where

$$\pi_t = e^{\phi(p_t - \underline{p})} \in [0, 1] \tag{13}$$

is the risk-neutral probability of default. The quantity in square brackets in equation (12) can be thought of as the payoff from the limited liability option when it expires in the money with probability π_t.

Due to the availability of a closed-form solution, the geometric Brownian motion case is useful for studying the main mechanism linking financial distress and properties of equity return. In what follows we will rely on the equity value in Corollary 1 to derive analytical relationships between default probability and equity return characteristics, including equity beta and return autocorrelation. We will then use Proposition 1 to verify numerically that such relationships are robust to different specifications of the stochastic process describing the evolution of product prices.

2.3 Equity beta, expected return, and default probability

Our main focus in this simple framework is to examine the effects of leverage and default probability on a firm's expected return. In the model, the product price p_t is the only state variable. Following a standard argument, we measure the risk β_t of equity with respect to p_t as

$$\beta_t = \frac{d \log V(p_t)}{dp_t}. \tag{14}$$

Hence, the (instantaneous) expected return on equity is given by

$$ER_t = r + \beta_t \lambda, \tag{15}$$

where $\lambda = \mu^{\mathbb{P}} - \mu^{\mathbb{Q}}$ denotes the risk premium associated with the price process p_t. Note that β_t in expression (15) is *not* the CAPM beta, and this stylized model is silent about the systematic risk structure of the product price process. For ease of exposition we will nevertheless refer to the quantity in (14) as the "equity beta" because, in our setting, this is the only determinant of equity risk.[11]

Using the expression for equity value derived in Proposition 1 we can compute equity beta from (14). Because its default threshold is not available in closed form, the mean-reversion case does not lend itself to further analytic characterization of equity beta. For the geometric Brownian motion case of Corollary 1, however, we can obtain a decomposition of equity beta that highlights the interaction between a firm's book-to-market ratio and its default probability, as described in the following corollary.

Corollary 2 *Assume that the price process p_t follows a geometric Brownian motion, and that the firm's residual value $R(p_t)$ upon default is as specified in Corollary 1. Then the levered equity beta can be expressed as*

$$\beta_t = 1 + \underbrace{\left(\frac{(c-l)/r}{V(p_t)}\right)}_{\text{BE/ME}} \underbrace{\left(\frac{c+l}{c-l}\right)\left(1 - \pi_t \frac{\eta a r + c + l}{c+l}\right)}_{\text{Distress}}. \tag{16}$$

The firm's revenue beta is normalized to 1. The term labeled "BE/ME" represents the equity book-to-market ratio. Because of the lack of an explicit account for capital in this simple model, we take the capitalized value of fixed cost, c/r, as a proxy for the book value of assets, following Carlson, Fisher, and Giammarino (2004). Similarly, we also use the capitalized value of coupons, l/r, as a proxy for the book value of debt. The quantity $(c-l)/rV(p_t)$ can hence be interpreted as a proxy for the equity book-to-market ratio. In the general model of Section 3, we explicitly account for installed capital and obtain a measure of the book-to-market ratio similar to that used in empirical work.

The part labeled "Distress" in (16) captures the impact of financial leverage and distress on equity beta. Financial leverage directly affects equity beta through the limited liability provision. This is reflected in the negative sign appearing in front of the default probability π_t in (16). This negative sign might suggest that equity risk is always declining with default probability. This

[11] Assuming a (conditional) CAPM risk structure, we can obtain the CAPM beta of the equity based on the covariance of the p_t process and the pricing kernel in the economy. Hence the expected return on equity may be further expressed as

$$ER_t = r + \beta_t \cdot SR \cdot \rho \cdot \sigma,$$

where SR is the maximal Sharpe ratio attainable in the economy, and $-\rho$ is the correlation of the price process p_t with the pricing kernel in the economy. This implies that the risk premium λ associated with the output price p_t is $\lambda = SR \cdot \rho \cdot \sigma$.

argument, however, is not accurate because it neglects the indirect effect of financial leverage on β_t through equity value and default probability.

More important, the effect of financial leverage is crucially dependent on the magnitude of shareholder recovery, as captured by the parameter η. In the absence of shareholder recovery, *i.e.*, when $\eta = 0$, it is possible to show that, as the firm approaches default, *i.e.*, $\pi_t \to 1$, the equity value $V(p_t)$ approaches zero at a faster rate than the quantity $1 - \pi_t$, causing β_t to increase with default probability and to explode to infinity as π_t tends to one. In contrast, in the presence of shareholder recovery, *i.e.*, when $\eta > 0$, the equity value $V(p_t)$ is bounded away from zero as $\pi_t \to 1$. If the beta of the residual firm value is finite, and equity holders receive a fraction of this residual value upon the resolution of distress, equity will progressively become less risky as the default boundary is approached. This implies that, for sufficiently high levels of default probability, the equity risk declines with π_t. The following corollary formally proves this intuition for the case in which the underlying price, p_t, follows a geometric Brownian motion.

Corollary 3 *Suppose the underlying price process follows a geometric Brownian motion and the residual firm value upon default is specified as in Corollary 1. Then,*

1. *If $\eta = 0$, equity beta and expected return are monotonically increasing in default probability π_t, with $\lim_{\pi_t \to 1} \beta_t = \infty$.*

2. *If $\eta > 0$, equity beta and expected return are increasing in default probability when π_t is small and decreasing in default probability when $\pi_t \to 1$.*

Corollary 3 provides a characterization of equity beta that is valid when the underlying price process follows a geometric Brownian motion. To assess the robustness of this result throughout the entire spectrum of default probabilities and for the case in which the underlying price process is mean reverting, we resort to a numerical analysis.

In Figure 1 we report the relationship between equity beta and default probability. Panel A presents the case in which there is no shareholder recovery upon financial distress ($\eta = 0$), while Panel B presents the case in which shareholders are capable of recovering 2% of the asset value upon distress. These graphs serve the purpose of demonstrating qualitative patterns which are robust to a wide range of parameter choices. In both panels, the left graph refers to the case in which the log price p_t follows a geometric Brownian motion (GBM) while the right graph refers to the case with a mean-reverting Ornstein-Uhlenbeck (OU) process. Each graph is obtained by choosing different price levels of $p_t \geq \underline{p}$ and recording the corresponding values for beta and 1-year ahead default probability.[12] For our purposes, varying p_t is qualitatively equivalent to considering a cross section of firms with different characteristics (leverage, operating cost, scale, etc.). The ultimate goal is to produce a cross section of "distances to default" to which we match the corresponding betas.

We choose the salvage value $R(p_t)$ to be the book value of asset c/r, *i.e.*, $a = c/r$ and $b = 0$ in the characterization of $R(p_t)$ in Corollary 1. This choice allows us to measure shareholder recovery as a fraction η of the book value of assets, as it is frequently reported in empirical studies (see, *e.g.*, Eberhart, Moore, and Roenfeldt (1990)). As we will argue later, imposing a constant salvage value does not affect the qualitative nature of our results.

Panel A of Figure 1 shows that when there is no possibility for shareholder recovery, *i.e.*, $\eta = 0$, the equity beta increases monotonically with default probability. As the default boundary is approached, the equity beta explodes and the equity value goes to zero, as stated in Corollary 3. In contrast, Panel B shows that when the expected shareholder recovery is set at a modest level of $\eta = 2\%$ of the book asset value, the equity beta (and hence the expected return) exhibits a hump shape with respect to default probability. This finding is consistent with the empirical evidence in Dichev (1998), who documents a distinct hump in the stock return relationship with accounting-based measures of distress (Altman's Z and Ohlson's O scores), and in Campbell, Hilscher, and Szilagyi (2008), who illustrate hump-shaped factor loadings with respect to default likelihood.[13]

The intuition for the hump shapes in the graphs is as follows. As financial leverage amplifies the risk of assets for equity, at relatively low levels of distress likelihood, equity beta increase with leverage, and hence with default probability. At high levels of distress likelihood, however, the prospect of recovering a fraction of assets, which have a lower beta than the levered equity, increasingly counterweighs the leverage effect in determining the risk of equity. As the firm inches

[12] While for our theoretical derivations we refer to π_t as the "probability of default," in our numerical analysis we adhere to the industry practice and adopt a definition derived under the real probability measure. For the geometric Brownian motion the default probability under the real measure $\mathbb{P}$ is available in closed form and is provided in Lemma 1 of Appendix A (equation (A7)). For the mean-reverting case, we discretize the Ornstein-Uhlenbeck process using Tauchen's quadrature (see Tauchen (1986)) with 100 grid points and numerically compute the T-period ahead default probability. Notice, however, that the use of the risk-neutral probability of default π_t does not alter any of the properties we derive in this section since the two quantities are monotonically related.

[13] The empirical evidence for the hump may have a different appearance than that of Figure 1 because, as most stocks cluster at low levels of default probability, portfolio sorting procedures usually tend to stretch out the hump shape to higher levels of default probability.

closer to the point of distress, the likelihood of recovery "mutates" the risk of levered equity into the risk of the underlying, safer, asset $R(p_t)$. When $R(p_t)$ is modeled as the (constant) book value of assets, the equity beta is in fact converging to zero at high levels of default probability, as shown in Panel B. Note, however, that, when $\eta > 0$, the relation between beta and default probability is bound to be hump-shaped, regardless of the form of the residual value $R(p_t)$, as long as this quantity is positive and has a finite beta.

The hump shape in the relationship between levered equity beta and default probability, a consequence of shareholder renegotiation power in the event of financial distress, has interesting implications for the cross-sectional properties of equity returns. In the next two subsections we elaborate on how such a relationship affects two widely studied cross-sectional anomalies: the value premium and the momentum in stock returns.

2.4 Value premium and financial distress

The decomposition of equity beta in Corollary 2 illustrates that the cross-sectional variation in betas is attributable to the interaction between the "book-to-market" and "financial distress" effects. The way in which beta depends on default probability has implications for the relationship between default probability and the value premium, *i.e.*, the return spread between stocks with high vs. low book-to-market ratios. We claim that if beta is monotonically increasing and convex in default probability, as in Panel A of Figure 1, then the value premium is also positive and increasing. In contrast, if beta is hump-shaped in default probability, as in Panel B, then the value premium is positive for low levels of default probability and negative otherwise.

The intuition for the predicted patterns of the value premium at different levels of default probability is as follows: Suppose we have two firms with identical book values, default probabilities, and stock prices. One of them experiences a negative shock to its stock price, while the other a positive shock and hence a higher stock price. The first stock will then have a larger book-to-market ratio and a higher default probability than the second stock. If both stocks have $\eta = 0$, then from Panel A of Figure 1, the difference between their expected returns, *i.e.*, the value spread, should be positive and upward sloping with respect to default probability. In contrast, if both firms have $\eta > 0$, then Panel B of Figure 1 indicates that the return spread between high book-to-market stocks and low book-to-market stocks should be positive for low levels of default probability and negative for high levels.

Empirically, the value premium is computed by forming portfolios of stocks with different book-to-market ratios. Given the cross-sectional heterogeneity in the degree of shareholder recovery, η, the above argument allows us to conjecture, and numerically confirm, that value spreads should be humped with respect to default probability, *i.e.*, upward sloping at low levels of default probability and downward sloping at high levels. We verify that this intuition is robust in the general model of Section 3 and supported by the data in the empirical analysis of Section 4.

2.5 Momentum and financial distress

The relation between equity beta and default probability is also important for understanding return autocorrelation and the properties of momentum strategies.

Equity returns exhibit a positive return autocorrelation if the expected return increases with realized returns. Intuitively, return autocorrelation can be thought of as the slope obtained in regressing the instantaneous change in the expected return on the realized return. Assuming that the underlying state variable p_t follows the stochastic process in (1), using Itô's lemma and the definition of β_t given in (14), the covariance between changes in expected returns and realized returns is given by $\lambda\sigma d\beta_t/dp_t \times \sigma\beta_t$, and the variance of realized returns is $\sigma^2\beta_t^2$. Combining these two quantities, we obtain the following expression for autocorrelation in returns

$$AC(p_t) = \frac{\lambda}{\beta_t}\frac{d\beta_t}{dp_t} = \frac{\lambda}{\beta_t}\frac{d^2\log(V(p_t))}{dp_t^2}, \tag{17}$$

where the last equality follows from the definition of β_t. A positive return autocorrelation on the stock level is a sufficient, though not necessary, condition for the profitability of momentum strategies. Hence, strategies based on portfolios of stocks with positive autocorrelations should produce enhanced momentum profits.[14]

For the case in which the underlying price process follows a geometric Brownian motion, we can explicitly derive an expression for the autocorrelation coefficient and analyze its relationship with default probability, as stated in the following corollary.

[14] It is well known that momentum is not the same as positive return autocorrelation. In fact, Lewellen (2002) documents negative autocorrelations at the *portfolio* level, while our model focuses on autocorrelations of *individual* stock returns. As Lewellen explicitly recognizes, his finding "will have little to say directly about individual-stock momentum" (p. 542). Our theory simply helps to identify how autocorrelation changes in the cross section, depending on firm characteristics such as default probability, and implies that momentum profits should be stronger when stock-level autocorrelations are positive and large. These type of strategies are what Sagi and Seasholes (2007) call "enhanced momentum strategies."

Corollary 4 *Suppose the log price p_t follows a geometric Brownian motion and the residual firm value upon default is specified as in Corollary 1. Then the autocorrelation in equity return is given by*

$$AC(p_t) = \lambda \left[1 - \beta_t - \pi_t \frac{\phi(\eta ar + c + l)}{r\beta_t V(p_t)} \right]. \tag{18}$$

If $\eta = 0$, $AC(p_t) < 0$ always. If $\eta > 0$, there exists a $p^*(\eta)$ such that $AC(p^*(\eta)) = 0$. For all $\underline{p} \leq p_t < p^*(\eta)$, $AC(p_t) > 0$, and for all $p_t > p^*(\eta)$, $AC(p_t) < 0$. Furthermore, $p^*(\eta)$ is increasing in η.

This corollary highlights the crucial role of financial distress and the ensuing potential recovery for equity holders in the determination of stock return continuation. The corollary states that return autocorrelation is positive only in the presence of shareholder recovery $\eta > 0$ and for sufficiently high levels of default probability, *i.e.,* $p_t < p^*(\eta)$. The intuition behind this result stems from the humped relationship between expected return and default probability discussed above. Because beta is a hump-shaped function of default probability, as the firm edges toward default with a declining stock price, the ex-ante risk level of equity decreases. Similarly, as the firm moves away from the brink of bankruptcy, its stock price rises, as does its equity risk. Both scenarios depict a return pattern that exhibits a positive autocorrelation. Because this mechanism applies *only* to firms with high default probability and $\eta > 0$, the risk dynamic we highlight here is consistent with the recent empirical finding of Avramov, Chordia, Jostova, and Philipov (2007) that the momentum effect in stock returns is driven primarily by firms with low credit ratings.

There are three points worth noting. First, our model is capable of endogenously generating positive return autocorrelations among firms with low credit quality and high expected shareholder recovery upon default even when the underlying product price process is not predictable, as it happens when p_t follows a geometric Brownian motion. Our result stems uniquely from the existence of a hump-shaped relationship between beta and default probability, which is also present in the case of a mean-reverting price process, as shown in Panel B of Figure 1. Therefore the mechanism described in Corollary 4 extends naturally beyond the geometric Brownian motion case.

Second, as pointed out by Johnson (2002) and Sagi and Seasholes (2007), autocorrelation in equity returns is positive if the log equity value is convex in the log price p_t, *i.e.*,

$d^2 \log(V(p_t))/dp_t^2 > 0$, as long as the risk premium λ and β_t are positive. In our model, log convexity is obtained because of the presence of positive shareholder recovery when $\eta > 0$ and the output price $p_t < p^*(\eta)$, which implies that the default probability π_t is sufficiently high. Figure 2 illustrates this point by plotting the log equity value in the absence of shareholder recovery (Panel A) and with positive shareholder recovery (Panel B). As demonstrated in the figure, a positive value of η dramatically changes the curvature of the log equity value, compared to the case of no shareholder recovery. In Panel B, obtained for a value of $\eta = 2\%$, the log equity value is convex for low levels of the log price p_t and concave elsewhere. Low levels of log price are states with high default probabilities. Shareholder recovery introduces log convexity in equity value when default probability is high, hence generating positive autocorrelation. In contrast, in Sagi and Seasholes (2007) growth options and the absence of financial leverage are instrumental for inducing log convexity in equity values, and hence positive return autocorrelation. Therefore, the two mechanisms for generating enhanced momentum profits are complementary.

Finally, the fact that $p^*(\eta)$ is increasing in η implies that, when shareholder recovery is high, positive autocorrelation persists over a larger range of prices p_t. This property has implications for both the persistence and the strength of momentum profits and leads to a novel prediction on the cross-sectional variation of enhanced momentum strategies. Because for nearly distressed stocks the persistence of their positive return autocorrelation increases with the prospect of shareholder recovery upon distress, momentum profits can be enhanced by concentrating on stocks with stronger prospects of shareholder recovery. This prediction is shown to be robust in the general model of Section 3 and confirmed in the empirical analysis of Section 4.

2.6 Discussion

The simple model discussed in this section yields insights into several puzzling pieces of empirical evidence. First, because of the interaction between leverage and book-to-market in the determination of beta, for most firms—with the exception of low credit quality firms for which shareholders expect a non-zero recovery value in distress renegotiations—the risk of assets-in-place to equity holders is amplified by financial leverage, implying that the magnitude of the book-to-market effect is stronger for more heavily levered firms. This is consistent with the evidence that the value premium is most significant for firms with high default probability (see, *e.g.*, Griffin and Lemmon (2002), Vassalou and Xing (2004), and Chen (2009)).

Second, our model is useful for understanding the results of Hecht (2004) and Choi (2009) that firm-level asset returns do not exhibit strong cross-sectional patterns, such as the book-to-market and momentum effects. As our model shows, these patterns are generally enhanced by the presence of leverage, and their magnitude in asset returns may be too small to be statistically and economically significant. The impact of financial leverage on cross-sectional returns is also alluded to in Ferguson and Shockley (2003), who argue that the SMB and HML factors in the Fama-French three-factor model are instruments for measurement errors in equity beta because of the absence of debt in the proxy market portfolio. While compelling, this argument ignores the time-varying nature of beta as well as its dependence on firms' characteristics, as highlighted in our framework. Moreover, the nonlinearity in the equity payoff introduced by leverage and the limited liability option in our model helps provide a plausible justification for the relationship between conditional skewness and stock returns documented in Harvey and Siddique (2000).

More important, our simple model shows that accounting for potential shareholder recovery upon financial distress produces a rich set of implications for cross-sectional properties of stock returns. The resulting hump-shaped relationship between expected return and default probability leads to a testable new prediction of a humped value premium with respect to default probability. It also provides an explanation for the recently documented evidence on the concentration of momentum profits in low credit quality firms and further predicts that momentum profits will be stronger for nearly distressed firms with higher expectations of shareholder recovery.

3 A general model of levered equity returns

The simple model of the previous section provides the basic intuition for a number of cross-sectional patterns in stock returns. In this section we generalize the model to account for endogenous investment and financing decisions in order to verify the robustness of its predictions.

We adopt the neoclassical Lucas-Prescott framework and construct a stationary economic environment that can be used as a laboratory for analyzing the effect of shareholder recovery upon financial distress on the cross section of equity returns. In this environment, each firm is characterized by a production technology generating cash flows that are subject to both economy-wide and firm-specific shocks. The firm's manager maximizes equity value by optimally choosing (i) the level of capital investment, (ii) financing through a mix of debt and equity, and (iii) whether or not to default. The basic structure of the model is similar to that used in several recent papers and is described in detail in Appendix B.[15]

We solve the model numerically by using value function iteration after discretization of the state space. Details of the solution methodology are provided in Appendix B.2.1. The model contains a total of 19 parameters, summarized in Table 1. While it would be ideal to calibrate these parameters by matching relevant moments via, for example, a simulated method of moments (SMM) methodology, the large dimensionality of the state space makes this approach computationally infeasible.[16] To calibrate the model, we follow instead Livdan, Sapriza, and Zhang (2008) and Gomes and Schmid (2009), who base their parameter choice on the values used in the existing macro and finance literature (*e.g.*, Gomes (2001), Cooley and Quadrini (2001), Cooper and Ejarque (2003), Zhang (2005), and Hennessy and Whited (2005, 2007)). Our choice of parameter values is summarized in Table 1. The model is solved on a monthly basis.

For each firm, the solution of the model consists of the firm's equity value and the associated optimal investment, financing, and default policies over the state space. Knowledge of these quantities allows us to analyze the cross-sectional properties of equity returns and the effect of shareholder recovery η under a stationary distribution of the underlying state variables. We construct the cross section by bootstrapping expected returns from the stationary distribution and form portfolios according to the model-implied default probability (see details in Appendix B.2.2). According to the intuition developed in Section 2, the absence of shareholder recovery leads to expected returns that are increasing in default probability, while the presence of such recovery leads to expected returns that are hump-shaped in default probability.

Figure 3 confirms these findings in the general model of this section. Panel A presents the case of no shareholder recovery ($\eta = 0$), while Panel B reports the result for the case with expected shareholder recovery equal to 10% of the residual firm value defined in (B35). As it emerges clearly from the figure, the case of no recovery leads to a monotonically increasing relation between expected return and default probability that "explodes" in the highest decile when default becomes almost certain. On the other hand, in the presence of expected shareholder

[15]See, for example, Gomes and Schmid (2009), Li (2008), Livdan, Sapriza, and Zhang (2008), and Obreja (2006), although none of them has considered the feature of shareholder recovery.

[16]The problem is characterized by four state variables. After discretization, the state space contains a total of 6,630,000 grid points. See Appendix B.2.1 for details.

recovery upon distress, the relation between expected return and default probability is humped, increasing at low levels of default probability and decreasing at high levels of default probability. These patterns are consistent with the implications of the simple equity valuation model as well as with the empirical results presented in Garlappi, Shu, and Yan (2008).

The simple model of Section 2 predicts that the value premium should be increasing in default probability in the absence of shareholder recovery and humped in default probability in the presence of shareholder recovery. We construct the value premium in our stationary economy by following a similar bootstrapping methodology as the one used for expected returns (see Appendix B.2.3 for details). Figure 4 confirms that the prediction of the simple model of Section 2 is valid also for the general model of this section. Panel A reports the value premium on a monthly basis for the case of no shareholder recovery, while Panel B considers the case of expected shareholder recovery equal to 10% of the residual value defined in (B35). The presence of shareholder recovery substantially affects the pattern of the value premium conditional on default probability. The value premium is positive and increasing in the absence of shareholder recovery, while it is hump-shaped when shareholder recovery is present, turning negative in the highest default probability decile.

Finally, according to the simple model of Section 2, in the presence of possible shareholder recovery, the humped relationship between expected return and default probability implies the concentration of momentum profits in low credit-quality firms, and no momentum in the absence of shareholder recovery. To verify whether this conjecture is also true in the general model of this section, we generate momentum profits by computing the spread between the model-implied expected return of winner and losers, as described in Appendix B.2.4, and report the results in Figure 5. As before, Panel A considers the case of no shareholder recovery while Panel B refers to the case of shareholder recovery equal to 10% of the residual value defined in (B35). The figure illustrates that momentum profits are positive and significant only for firms with shareholder recovery and with high default probability. The range of momentum profits runs from 2% to 12% annually, depending on the level of default probability, which is comparable to empirical estimates. For firms without shareholder recovery, the momentum strategy does not work, as it would only generate losses, as indicated in Panel A.[17]

[17]The large magnitude of losses is attributable to the explosive nature of expected returns as default probability approaches 1.

In summary, the analysis in this section confirms that the intuition developed within the simple model of Section 2 is robust in a more general framework which allows firms to optimally choose their capital structure and investment levels. One caveat is that this analysis focuses on stratified samples along the dimension of shareholder recovery, η. In reality, stocks have a wide range of η, as estimated in Morellec, Nikolov, and Schürhoff (2008). While numerically simulating this cross section of stocks is infeasible at the present time, we verify our predictions directly with real data in the empirical analysis of the next section.

4 Empirical analysis

Recent empirical studies have provided evidence consistent with the existence of a hump-shaped relationship between equity betas and default probability as predicted by our model. Garlappi, Shu, and Yan (2008) show that, among U.S. firms, the relations between stock returns and default probability exhibit divergent patterns depending on their respective measures of shareholder recovery, similar to those discussed in our model. Moreover, Favara, Schroth, and Valta (2009), using international data to analyze the resolution of insolvency procedures across countries, document that equity beta increases with the degree of credit protection and decreases with shareholders' bargaining power and liquidation costs, both contributing to shareholder recovery.

Our contribution in this paper is to recognize that these patterns linking equity beta and default probability have important implications for understanding the profitability of value and momentum strategies. In this section, we empirically verify the novel predictions of our model regarding the cross-sectional properties of the value premium and momentum profits.

4.1 Data

To gauge a firm's default probability, we use a market-based measure—the *Expected Default Frequency* (EDF)—obtained directly from *Moody's KMV* (MKMV hereafter). The data is available at a monthly frequency. A firm's EDF measure represents an assessment of the likelihood of default for that firm within a year. This measure is constructed from market-traded stock prices based on the Vasicek-Kealhofer model (Kealhofer (2003a,b)) which adapts the Black and Scholes (1973) and Merton (1974) framework and is calibrated through a comprehensive database of historical default experiences.[18]

We match the EDF database with the CRSP and COMPUSTAT databases, *i.e.*, a stock needs to have data in all three databases to be included in our analysis. Specifically, for a given month, we require a firm to have an EDF measure and an implied asset value in the MKMV dataset; stock price, shares outstanding, and returns data from CRSP; and accounting numbers from COMPUSTAT for firm-level characteristics. We limit our sample to non-financial U.S. firms.[19] We also drop from our sample stocks with a negative book-to-market ratio. Our baseline sample contains 1,615,664 firm-month observations and spans from February 1969 to November 2007.[20]

Summary statistics for the EDF measure are reported in Table 2. The average EDF measure in our sample is 3.30% with a median of 1.07%. Panel A shows that 75% of firms have a default probability less than 3.5%. One should note that MKMV assigns an EDF score of 20% to all firms with an EDF measure larger than 20%. Around 5% of the firms are in this group at any given time.

Because many empirical studies exclude stocks with a per-share price lower than $5 out of concern for liquidity and market microstructure issues, we examine separately distributions of the EDF measure in the subsample with a minimum per-share price of $5 and in the subsample containing only stocks with a per-share price lower than $5. Panel A of Table 2 shows that low-priced stocks tend to have much higher default probabilities, with a mean EDF of 6.89%, and more than 50% of them have a higher than 4% chance to default in a year. For high-priced stocks, default probabilities are generally low with a mean EDF of 1.15%, and 90% of them have a less than 2.84% probability to default within a year. This implies that low-priced stocks on average have a greater risk of financial distress.

In our empirical examination below, we group stocks evenly into ten deciles according to their EDF value in each month. To better understand the property of each group, Panel B of

[18]See Crosbie and Bohn (2003) for details on how MKMV implements the Vasicek-Kealhofer model to construct the EDF measure.

[19]Financial firms are identified as firms whose industrial code (SIC) are between 6000 and 6999.

[20]We follow Shumway (1997) and Shumway and Warther (1999) to deal with the problem of delisted firms. Specifically, whenever available, we use the delisting return reported in the CRSP datafile for stocks that are delisted in a particular month. If the delisting return is missing but the CRSP datafile reports a performance-related delisting code, then we impute a delisted return of either −30% (NYSE and AMEX stocks) or −55% (NASDAQ stocks) in the delisting month.

Table 2 presents the time series average of the mean and maximum EDF measures in each EDF decile for both the full sample and the subsample with a minimum per-share price of \$5. In addition, we also report the Amihud (2002) measure of illiquidity for each decile portfolio to indicate the liquidity difference across decile portfolios. From the values reported in Panel B, it is evident that the biggest difference between the full sample and the subsample of stocks with per-share price of \$5 or higher occurs in the portfolios with high levels of default probability. For example, the average EDF in the top EDF decile is 14.79% for the full sample and only 5.21% for the subsample of stocks with high per-share prices. Similarly, the Amihud illiquidity measure is 11.47 in the tenth decile for the full sample and only 0.58 in the tenth decile of the subsample. Finally, the mean values of the maximum EDF of decile portfolios indicate that the cutoff EDF values for deciles are not evenly spaced, especially for stocks with higher per-share prices.

4.2 Value premium and default probability

We first examine how value premium changes with default probability. In each month we sort all stocks in our full sample into ten deciles according to their EDF scores and, independently, into three terciles according to their book-to-market ratios. We then compute both value-weighted and equal-weighted returns of each portfolio in the month after portfolio formation and record the time-series average of the value premium, *i.e.*, the return spread between the top book-to-market portfolio and the bottom book-to-market portfolio. The results are reported in Table 3.

For the full sample, the value premium initially rises with default probability and then starts to decline at high levels of default probability. For value-weighted returns, the value premium rises from 0.05% per month in the first EDF decile to 1.60% in the eighth decile and then drops to 1.07% in the last decile. This hump-shaped pattern is more pronounced with equally-weighted returns, where the value premium rises from 0.11% per month in the first EDF decile to 1.37% in the eighth decile and then drops to 0.44% in the last decile. This hump shape in the relationship between value premium and default probability is consistent with the prediction of our theoretical model.

Table 3 further shows that the pattern in raw returns persists in risk-adjusted returns obtained from the CAPM ("CAPM α"), the Fama and French (1993) three-factor model ("FF α"), and the momentum-augmented four-factor model of Carhart (1997) ("4-Factor α"), respectively.

This pattern is present even after accounting for the Pastor and Stambaugh (2003) liquidity factor ("5-Factor α"). Moreover, in order to mitigate the liquidity effect and market microstructure issues concerning stocks with high default probabilities and/or low prices per-share, we record and analyze the return patterns in the second month after portfolio formation as suggested in Da and Gao (2008). The results are very similar to those reported in Table 3 and thus omitted here to save space.

The above results appear to contradict the existing evidence in the literature that documents a larger value spread among firms with higher default probability (*e.g.*, Griffin and Lemmon (2002)). This discrepancy is, however, illusory and has much to do with the procedure of sample selection and/or with the coarseness of the sorts used to classify stocks into portfolios. A frequently used sample filtering rule is to exclude stocks with per-share price less than $5 to avoid potential market microstructure issues. As illustrated in Table 2, this filtering rule exactly excludes many stocks with high levels of default probability. Therefore, it is likely that the extant empirical evidence reflects the variation of the value premium over a limited range of default likelihood where the value premium increases with default probability, as indicated by our theory. Alternatively, some studies, such as Griffin and Lemmon (2002) and Vassalou and Xing (2004), sort stocks into quintile portfolios according to their default likelihood, a coarse classification that may elude finer features in the relationship between default probability and value spread.

To verify that data selection and classification can affect the results examined here, we first sort all stocks, including the low-priced ones, into quintiles, instead of deciles, according to their EDF scores and repeat the procedure for Table 3. In Panel A of Figure 6, we present the value spread based on value-weighted returns of book-to-market sorted portfolios. The panel shows that with a coarse rank of default probability quintiles, we only observe a generally positive association between default likelihood and value spread, as reported in the previous studies. We then restrict our sample of stocks to those with per-share price of $5 or higher and repeat the same procedure of portfolio formation and return recording used in Table 3. The results, presented in Panel B of Figure 6, confirm that for this subset of stocks the value premium is indeed increasing in EDF scores.

Since Table 2 shows a close link between high default probabilities and low stock prices, the results illustrated in Panel B of Figure 6 could be the consequence of still low levels of default probability in the high EDF deciles among high-priced stocks. Indeed, Table 2 shows that the 75 percentile EDF score for high-priced stocks is 1.36%, compared to 10.68% for low-priced stocks and 3.47% for the full sample, respectively. If our prediction is robust, then it should also apply to high-priced stocks with high levels of default probability that are comparable to those of low-priced stocks. To examine this conjecture, we use the breakpoints for the ten EDF portfolios based on the full sample, including low-priced stocks, and sort only stocks with per-share price of $5 or higher into their respective portfolios.[21] The value spread presented in Panel C of Figure 6 does exhibit a striking hump shape that is absent in Panel B. This finding demonstrates the validity of our prediction and at the same time mitigates the concern for the illiquidity problem typically associated with low-priced stocks.

4.3 Momentum profits, shareholder recovery, and default probability

Our model predicts that momentum profits in stock returns are likely to be more pronounced for firms with high levels of default probability. This is consistent with the evidence in Avramov, Chordia, Jostova, and Philipov (2007), who document that among stocks with *S&P* firm-level credit ratings, those with poor credit ratings are most important in generating momentum profits. Furthermore, our model yields a unique prediction regarding how expected shareholder recovery can affect the cross-sectional pattern of momentum.

In order to test the prediction of our model, we first need proxies for the shareholder recovery parameter η. One component of shareholder recovery is a measure of shareholders' "bargaining power": shareholder recovery is larger when shareholders' bargaining power is greater. To capture this dimension, we rely on two proxies used in the literature to measure shareholder bargaining power: (i) asset size and (ii) R&D expenditures. This choice is supported by earlier studies of Franks and Torous (1994) and Betker (1995) who document that deviation from the absolute priority rule is positively related to firm size, and of Opler and Titman (1994) who show that firms with high costs of R&D suffer the most in financial distress and may be subject to liquidity shortage that diminishes the bargaining power of shareholders in renegotiation. In our empirical analysis, we take firms with larger asset bases or lower R&D expenditures as more likely to have a larger η.

[21] We thank an anonymous referee for this suggestion.

A second component of shareholder recovery is the "liquidation cost" incurred if financial distress leads to liquidation of a firm's assets. All else equal, shareholder recovery is high when liquidation costs are high because high liquidation costs provide incentives for debt holders to participate in debt renegotiations in order to avoid the deadweight costs, and hence increase the likelihood and the amount of shareholder recovery. To capture this aspect of the renegotiation process, we rely on the concept of asset specificity. As Shleifer and Vishny (1992) argue, when a firm's assets are specific or unique to a particular industry, they are likely to be subject to substantial fire-sale discounts in liquidation auctions. Therefore, all else being equal, liquidation costs increase when a firm's assets are more specific. We gauge a firm's asset specificity using a measure of industry concentration: the Herfindahl index of sales in an industry.[22] Firms in a more concentrated industry are likely to have more specific assets, and hence larger liquidation costs, *i.e.*, a larger η.

In summary, our empirical analysis on momentum strategies is based on three proxies of shareholder recovery: asset size, R&D expenditure, and industry concentration.[23] Based on these proxies, our theory then predicts that, among firms with high levels of default probability, momentum profits should be stronger when (i) asset size is large, or (ii) R&D expenditure is small, or (iii) industry sales concentration is high.

The methodology we follow to construct momentum profits is adapted from the "6-1-6" strategy in Jegadeesh and Titman (1993). At the beginning of each month t, we sort stocks independently into: (i) five quintiles based on their returns over the formation period $t-7$ to $t-2$, (ii) three terciles based on their EDF measures at time $t-2$, and (iii) three terciles based on a proxy for η at time $t-2$. Following the sort, the "winner" portfolio makes a fixed \$1 value-weighted investment in the top quintile stocks and sells stocks that were similarly added to the portfolio at the beginning of month $t-6$. The "loser" portfolio is defined similarly in the

[22]The Herfindahl index on sales in industry j at time t is defined as

$$SalesHfdl_j(t) = \sum_{i=1}^{N_j(t)} s_{ij}^2(t),$$

where $s_{ij}(t)$ denotes the sales of firm i at time t as a fraction of total sales in industry j, and $N_j(t)$ denotes the number of firms in industry j at time t.

[23]Garlappi, Shu, and Yan (2008) also use the book-to-market ratio as a proxy for liquidation costs, as a low book-to-market ratio may imply that the firm is worth more as a going concern than the book value of its existing assets. Even though the results using this proxy are consistent with the model prediction, we refrain from using it here because of the multiple roles the book-to-market ratio plays that may confound its interpretation in this context.

bottom quintile stocks. Momentum profits are defined as the difference between the returns of the winner and loser portfolios over the six-month period from $t+2$ to $t+7$, after skipping a month post portfolio formation. We report monthly average returns in Table 4.

Panel A of Table 4 shows that for firms with high EDF scores (subpanel labeled "High EDF") and large asset bases, winners outperform losers by 1.9% per month over the next six-month period. Among those high EDF firms with small asset bases, past winners outperform past losers by 0.79% per month. This difference of 1.1% per month is significant at the 5% confidence level. This pattern persists when we examine the Fama-French three-factor adjusted alpha for the momentum profits which yields a difference of 1.2% per month between large firms and small firms. When we add the momentum factor to the Fama-French adjustment, the alpha for firms with small asset values becomes zero, indicating that there is no enhanced momentum profit for these firms. For large firms, however, there is still a sizeable enhanced momentum profit of 0.79% per month. These results are consistent with our prediction regarding the role of shareholder recovery in producing enhanced momentum profits among stocks with high default probabilities. Notice that for firms with low EDF scores none of these patterns are present, as shown in the left-hand side of Panel A. In fact, asset size has the opposite effect on both raw and risk-adjusted momentum profits, *i.e.*, enhanced momentum profits are weaker for large firms. This finding is consistent with the argument of Sagi and Seasholes (2007) who attribute momentum to growth options in absence of financial leverage, usually associated with small firms.

Panel B of Table 4 demonstrates that firms with high EDF scores and low R&D expenditures experience strong momentum in stock returns, but firms with high R&D expenditures and similar credit profiles do not. Again, this pattern is robust to the risk adjustment according to both the Fama-French three-factor model and the momentum-augmented four-factor model. Furthermore, none of these patterns are present among firms with low EDF scores, as the left-hand side of Panel B illustrates. In Panel C, we test whether the liquidation costs aspect of shareholder recovery has an impact on momentum profits. As the panel shows, high EDF firms in a more concentrated industry, *i.e.*, high *SalesHfdl* portfolios, have stronger momentum in stock returns than similar firms in a more competitive industry, and the statistical significance remains strong despite the coarse nature of the Herfindahl measure. This pattern is even stronger after risk-adjustment, but it disappears when we restrict our analysis to low EDF firms. In summary, the evidence presented in Table 4 strongly supports the prediction of our model regarding the importance of shareholder recovery for financially distressed firms in enhancing the profitability of momentum strategies.

Our large database of expected default frequencies also allows us to provide a comprehensive picture of the relationship between momentum profits and default probabilities. In their study, Avramov, Chordia, Jostova, and Philipov (2007) rely on a sample of stocks for which a firm-level *S&P* credit rating is available. This sample represents a small subset of the entire cross section of stocks. In contrast, our sample of stocks with EDF measure covers virtually all publicly traded stocks in the CRSP database. To better understand the relationship between momentum profits and default probability, in Table 5 we report both the unconditional momentum profits (column labeled "Uncond.") and the momentum profits conditional on the level of EDF. The unconditional momentum profit is 0.8% per month, statistically significant at the 1% level and comparable in magnitude to the documented evidence in the literature. Interestingly, adjusting for risk within a four-factor model which includes the momentum factor (UMD) produces a negative alpha of −0.23%, statistically significant at the 5% level. The factor loadings, indicate that a large portion of the momentum profits is accounted for by the momentum factor.

The remaining columns of Table 5 show that the conditional momentum profits range from 0.7% a month in the low-EDF quintile to 1.54% a month in the high-EDF quintile. The difference of 0.84% per month is both economically and statistically significant. When we adjust momentum profits with the four-factor model, the pattern remarkably persists. In fact, for low EDF quintiles, the alpha turns negative, but it increases to 0.52% per month for the highest EDF quintile, strongly indicating that increasing distress risk contributes to enhanced momentum profits that are not captured by the momentum factor. This is indeed consistent with our prediction. Moreover, the table also shows that while momentum profits load strongly on the momentum factor with a factor loading close to 1, they are almost orthogonal to the market factor. Their loadings on the HML factor are quite uniform across EDF quintiles.

One striking observation, however, is that for low EDF quintiles, momentum profits load positively on the SMB factor, while for the highest EDF quintile, the loading becomes negative, albeit statistically insignificant.[24] Moreover, the difference between SMB loadings in the top and

[24]In unreported results based on equal-weighted returns, the overall patterns are similar to the value-weighted results of Table 5, but the SMB loading is negative and statistically significant for the highest EDF quintile, similar in magnitude to that of unconditional momentum profits in the table.

bottom quintiles is statistically significant at the 5% level. The implication of this finding is that for lightly levered firms, small stocks drive the enhanced momentum strategy, while for heavily levered firms, large stocks contribute more to the enhanced momentum profits. Therefore, for low EDF firms, growth options matter as in Sagi and Seasholes (2007), while for high EDF firms, potential shareholder recovery, which is more likely for larger firms, plays an indispensable role in producing enhanced momentum profits, as predicted by our model.

5 Conclusion

Recent empirical evidence strongly suggests that financial distress is instrumental in understanding cross-sectional properties of stock returns. While this seems to confirm the conjecture of Fama and French (1992) that the book-to-market effect and other cross-sectional "anomalies" are related to the risk of financial distress, efforts toward finding a distress risk factor have unveiled puzzling empirical patterns.

In this paper, we propose a new perspective for understanding the empirical regularities in the cross section of equity returns. We explicitly introduce financial leverage in a simple equity valuation model and investigate how the possibility of shareholder recovery upon financial distress affects the relationship between a firm's expected return and its likelihood of encountering such distress. Within this simple framework, we derive three important insights.

First, the presence of potential shareholder recovery upon financial distress alters the risk structure of equity and causes the expected return to be *hump-shaped* in default probability. Second, this non-monotonic relationship between risk and default probability leads to hump-shaped value spreads with respect to default probability. Third, the same hump-shaped relationship between expected return and default probability predicts that momentum profits should be enhanced among firms with both high default likelihood and strong prospects for shareholder recovery upon financial distress. These predictions are robust in a general model with endogenous investment and financing choices.

Using the *Expected Default Frequencies (EDF)* from Moody's KMV as a market-based measure of default probability, we find empirical support for the novel predictions of our theory in the data. Specifically, the value premium is hump-shaped in default probability and momentum profits are stronger for stocks with high likelihood of default and larger prospects for shareholder recovery. These results complement and corroborate recent empirical evidence that documents patterns of conditional betas consistent with the predictions of our model.

At a more general level, the perspective we offer underscores the importance of financial leverage and the resolution of financial distress in asset pricing models for levered equity. Our model highlights the role of shareholder recovery upon financial distress as a pervasive mechanism for understanding the variation of both value premium and momentum profits in the cross section. While in this paper we focus our attention on these two major cross-sectional regularities in equity returns, our framework promises to be a useful platform for understanding a broader set of cross-sectional properties of both stock and bond returns.

A Appendix: Proofs

Proof of Proposition 1

Consider first the value of an unlevered firm $V^U(p_t)$ that has no debt and operating costs and receives a continuous stream of cash flow, e^{p_t}. From (2), the value of this firm is

$$V^U(p_t) = \int_0^\infty E^{\mathbb{Q}}\left[e^{p_{t+\tau}}\right] d\tau = \int_0^\infty e^{-r\tau}\left[\exp\left(p_t e^{-\zeta\tau} + \left(1 - e^{-\zeta\tau}\right)\frac{\mu^{\mathbb{Q}}}{\zeta} + \sigma^2 \frac{1 - e^{-2\zeta\tau}}{4\zeta}\right)\right] d\tau. \tag{A1}$$

Now consider the equity value of a firm that issues debt with perpetual coupon l and faces operating costs of c per unit of time. Ignoring the limited liability option, the equity value $V^L(p_t)$ of this firm is

$$V^L(p_t) = V^U(p_t) - \frac{l+c}{r}. \tag{A2}$$

The value $V(p_t)$ of equity with limited liability is given by $V(p_t) = V^L(p_t) + U(p_t)$ where $U(p_t)$ represents the value of the limited liability option. The value of $U(p_t)$ satisfies the following ordinary differential equation (ODE):

$$\frac{1}{2}\sigma^2 U''(p) + (\mu^{\mathbb{Q}} - \zeta p)U'(p) - rU(p) = 0, \tag{A3}$$

whose solution is given by

$$U(p_t) = A \cdot H\left(-\frac{r}{\zeta}, -\frac{\mu^{\mathbb{Q}} - \zeta p_t}{\sigma\sqrt{\zeta}}\right) + B \cdot H\left(-\frac{r}{\zeta}, \frac{\mu^{\mathbb{Q}} - \zeta p_t}{\sigma\sqrt{\zeta}}\right), \tag{A4}$$

where $H(v, z)$ is the generalized Hermite function of order v (see *e.g.*, Abramowitz and Stegun (1972)):

$$H(v, z) = \frac{2^{v+1}}{\sqrt{\pi}} e^{z^2} \int_0^\infty e^{-t^2} t^v \cos\left(2zt - \frac{\pi v}{2}\right) dt. \tag{A5}$$

Imposing the boundary condition $\lim_{p_t \to \infty} U(p_t) = 0$ allows us to exclude the second Hermite function in (A4). Upon default, $p_t \leq \underline{p}(\eta)$, the equity holder recovers a fraction η of the residual value $R(p_t)$. Therefore, the value of the firm equity can be written as in (5), where the constants A and $\underline{p}(\eta)$ are obtained from the value-matching and smooth-pasting conditions (7)-(8). ∎

Proof of Corollary 1

Taking limit $\zeta \to 0$ in (1) we obtain the Geometric Brownian Motion case. The ODE describing the limited liability option is

$$\frac{1}{2}\sigma^2 U''(p) + \mu^{\mathbb{Q}} U'(p) - rU(p) = 0. \tag{A6}$$

The general solution to (A6) is of the form $U(p_t) = Ae^{\phi p_t}$. After imposing the boundary condition $\lim_{p_t \to \infty} U(p_t) = 0$ and setting $\mu^{\mathbb{Q}} = r - \delta - \frac{1}{2}\sigma^2$, the equity value $V(p_t)$ is given by equation (9) with ϕ given in (10). Solving the value-matching and smooth pasting conditions (7)-(8) for $\zeta \to 0$ yields the expressions of the constant A and the default threshold $\underline{p}$ in (11).

∎

Lemma 1 (Default probability under the physical measure) *Let p_0 be the current value of the product log price, evolving according to the process described in (1) with $\zeta \to 0$, and $\underline{p}$ the endogenously determined default trigger. The time 0 cumulative real default probability $\Pr_{(0,T]}$ over the time period $(0,T]$ is given by*

$$\Pr_{(0,T]}(p_0) = \mathcal{N}\left(h(T)\right) + e^{-\frac{2\omega}{\sigma^2}(p_0 - \underline{p})} \mathcal{N}\left(h(T) + \frac{2\omega T}{\sigma\sqrt{T}}\right), \tag{A7}$$

with $\omega = \mu - \frac{1}{2}\sigma^2 > 0$, $h(T) = \frac{\underline{p} - p_0 - \omega T}{\sigma\sqrt{T}}$, and $\mathcal{N}(\cdot)$ the cumulative standard normal function.

Proof: Direct application of the property of the hitting time distribution of a Geometric Brownian Motion, *e.g.*, Harrison (1985), equation (11), p. 14. ∎

Proof of Corollary 2

The equity β_t of a firm that is still operating at time t, *i.e.*, for $p_t > \underline{p}$, is

$$\beta_t = \frac{d\log V(p_t)}{dp_t} = \frac{1}{V(p_t)}\left(\frac{e^{p_t}}{\delta} + \pi_t\left(\eta b - \frac{1}{\delta}\right)e^{\underline{p}}\right) \tag{A8}$$

$$= 1 + \frac{1}{V(p_t)}\left[\frac{c+l}{r} + \pi_t\left(1 - \frac{1}{\phi}\right)\left(\eta b - \frac{1}{\delta}\right)e^{\underline{p}}\right], \tag{A9}$$

where (A8) follows by using the definition of risk-neutral probability of default (13) and (A9) obtains by isolating the expression of $V(p_t)$ in (12) for $p_t \geq \underline{p}$. The corollary follows after substituting the expression of $\underline{p}$ in (11) and rearranging terms. ∎

Proof of Corollary 3

Let us consider the case of $\eta = 0$ first. Because, from (13), π_t is inversely related to p_t, to show that $d\beta_t/d\pi_t > 0$ it is sufficient to show that $d\beta_t/dp_t < 0$. Using the expressions in equations (12) and (13) with $\eta = 0$, we obtain

$$\beta_t = \frac{d \log V(p_t)}{dp_t} = \frac{e^{p_t} - \pi_t e^{\underline{p}}}{\delta V(p_t)}. \tag{A10}$$

Hence,

$$\begin{aligned} \frac{d\beta_t}{dp_t} = \frac{d^2 \log V(p_t)}{dp_t^2} &= \frac{\delta V(p_t)(e^{p_t} - \phi\pi_t e^{\underline{p}}) - (e^{p_t} - \pi_t e^{\underline{p}})^2}{\delta^2 V^2(p_t)} \\ &= -e^{p_t + \underline{p}} \left(\frac{(\pi_t + \phi - 1) + \phi(\phi - 2)\pi_t - \phi(\phi-1)\pi_t e^{\underline{p} - p_t}}{\phi \delta^2 V^2(p_t)} \right) \\ &= -e^{p_t + \underline{p}} \left(\frac{\phi - 1}{\phi} \right) \left(\frac{1 + (\phi - 1)\pi_t - \phi \pi_t e^{\underline{p} - p_t}}{\delta^2 V^2(p_t)} \right), \end{aligned}$$

where to arrive at the second equality, we have used the expression $\frac{c+l}{r} = \frac{\phi-1}{\phi}\frac{e^{\underline{p}}}{\delta}$ derived from equation (11) with η set to 0. Therefore, to show that $d^2 \log(V(p_t))/dp_t^2 < 0$, we only need to show that

$$1 + (\phi - 1)\pi_t - \phi \pi_t e^{\underline{p} - p_t} > 0 \tag{A11}$$

for all p_t. Because p_t is bounded from below by $\underline{p}$, we first check if the equality (A11) holds when $p_t \to \underline{p}$. Let $\epsilon = p_t - \underline{p} > 0$. Using a second order Taylor expansion of $e^{-\epsilon}$ around ϵ, we obtain

$$e^{\underline{p} - p_t} = e^{-\epsilon} \approx 1 - \epsilon + \frac{1}{2}\epsilon^2 + o(\epsilon^2), \tag{A12}$$

$$\pi_t = e^{-\phi\epsilon} \approx 1 + \phi\epsilon + \frac{1}{2}\phi^2\epsilon^2 + o(\epsilon^2). \tag{A13}$$

Substituting the above expressions in (A11) and simplifying we have that the leading-order term is $\frac{1}{2}\phi(\phi - 1)\epsilon^2 > 0$. We then note that the derivative of the left-hand side of (A11) with respect to p_t is $\phi(\phi-1)(1-e^{\underline{p}-p_t}) > 0$. This proves that $d\beta/dp_t < 0$ and hence $d\beta/d\pi_t > 0$ always when $\eta = 0$. In particular, when $\pi_t \to 1$, using a second order Taylor expansion of $V(p_t)$

$$V(p_t) \approx -\frac{1}{2}\phi\,\epsilon^2\,\frac{c+l}{r} + o(\epsilon^2), \tag{A14}$$

and substituting (A13) in (A10), we have $\beta_t \approx \frac{2}{\epsilon} + 1 + \phi \to +\infty$ as $\pi_t \to 1$.

To analyze the case of $\eta > 0$, let us rewrite the beta expression in Corollary 2 as

$$\beta_t = 1 + \frac{c + l - \pi_t(\eta ar + c + l)}{rV(p_t)}. \tag{A15}$$

Then,

$$\frac{d\beta_t}{d\pi_t} = \frac{-(\eta ar + c + l)rV(p_t) - (c + l - \pi_t(\eta ar + c + l))r\frac{dV(p_t)}{d\pi_t}}{(rV(p_t))^2}. \tag{A16}$$

Applying the chain rule

$$\frac{dV(p_t)}{d\pi_t} = \frac{dV(p_t)}{dp_t}\left(\frac{d\pi_t}{dp_t}\right)^{-1} = \frac{dV(p_t)}{dp_t}\frac{1}{\phi\pi_t}, \tag{A17}$$

and the definition of $\beta_t = \frac{1}{V(p_t)}\frac{dV(p_t)}{dp_t}$ in (A16), we obtain

$$\frac{d\beta_t}{d\pi_t} = -\frac{1}{V(p_t)}\frac{(\eta ar + c + l)}{r} - \frac{(\beta_t - 1)\beta_t}{\phi\pi_t}. \tag{A18}$$

When $\pi_t \to 0$, *i.e.*, $p_t \gg \underline{p}$, the first term in (A18) goes to zero as $V(p_t) \to \infty$. The second term will be positive as $\beta_t > 1$ and $\phi < 0$. Therefore, β_t increases in π_t for small levels of π_t. When $\pi_t \to 1$, the first term in (A18) is negative because $V(p_t) > 0$. Furthermore, from (A15), $\beta_t - 1 \to -\frac{\eta a}{V(p_t)} < 0$. Because $\phi < 0$, the second term in (A18) is negative as well. Therefore, $\frac{d\beta_t}{d\pi_t} < 0$, *i.e.*, β_t decreases in π_t when $\pi \to 1$. ∎

Proof of Corollary 4

Because $\lambda > 0$, from (17), $AC(p_t) > 0$ if and only if $\theta_t \equiv \frac{1}{\beta_t}\frac{d\beta_t}{dp_t} > 0$. From (A18), we have

$$\begin{aligned}\theta_t &= \frac{1}{\beta_t}\frac{d\beta_t}{d\pi_t}\frac{d\pi_t}{dp_t} \\ &= 1 - \beta_t - \frac{1}{\beta_t V(p_t)}\frac{\phi(\eta ar + c + l)}{r}\pi_t.\end{aligned} \tag{A19}$$

We now show that when $\eta = 0$, $AC_t < 0$ always. If $\lambda > 0$ and $\beta_t > 0$, then it suffices to show that $d^2 \log(V(p_t))/dp_t^2 < 0$. This is true from the proof of Corollary 3.

For $\eta > 0$, when $\pi_t \to 0$, $\theta_t < 0$ because $\beta_t > 1$. When $\pi_t \to 1$, as shown in the proof of Corollary 3, $1 - \beta_t > 0$. Because $\phi < 0$, the second term in (A19) is also positive. Therefore, when $\eta > 0$ and $\pi_t \to 1$, $\theta_t > 0$.

We now show that the boundary for log-convexity of the equity value, $p^*(\eta)$, is increasing in η. From the expression of equity value (9) with $p_t > \underline{p}$, we have

$$\frac{d^2 \log V(p_t)}{dp_t^2} = \frac{\frac{(\phi-1)^2 A e^{(\phi+1)p_t}}{\delta} - \frac{c+l}{r}\left(\frac{e^{p_t}}{\delta} + \phi^2 A e^{\phi p_t}\right)}{V^2(p_t)}. \tag{A20}$$

Because $\frac{d^2 \log V(p^*(\eta))}{dp_t^2} = 0$, $p^*(\eta)$ satisfies

$$\frac{(\phi-1)^2 A e^{(\phi+1)p^*(\eta)}}{\delta} = \frac{c+l}{r}\left(\frac{e^{p^*(\eta)}}{\delta} + \phi^2 A e^{\phi p^*(\eta)}\right). \tag{A21}$$

Taking derivatives with respect to η on both sides of (A21), and denoting $X = e^{p^*(\eta)}$, we obtain

$$\begin{aligned}\frac{\partial X}{\partial \eta} &= \frac{\frac{\partial A}{\partial \eta}\left(\frac{(\phi-1)^2 X^{\phi}}{\delta} - \left(\frac{c+l}{r}\right)\phi^2 X^{\phi-1}\right)}{A\left((\phi-1)\phi^2\left(\frac{c+l}{r}\right)X^{\phi-2} - \frac{\phi(\phi-1)^2}{\delta}X^{\phi-1}\right)} \\ &= \frac{\frac{\partial A}{\partial \eta}X}{(1-\phi)A}\left(\frac{\frac{(\phi-1)^2}{\delta}X - \frac{c+l}{r}\phi^2}{\frac{\phi(\phi-1)}{\delta}X - \frac{c+l}{r}\phi^2}\right).\end{aligned} \tag{A22}$$

From the expressions in (11), $\frac{\partial A}{\partial \eta} > 0$. Moreover, because $\phi < 0$, $1-\phi > 1$, and $\frac{(\phi-1)^2}{\delta}X - \frac{c+l}{r}\phi^2 > \frac{\phi(\phi-1)}{\delta}X - \frac{c+l}{r}\phi^2$. Finally, because $p^* > \underline{p}$, it can be shown that $\frac{\phi(\phi-1)}{\delta}X - \frac{c+l}{r}\phi^2 > 0$. Hence

$$\frac{\partial X}{\partial \eta} > \frac{\frac{\partial A}{\partial \eta}X}{(1-\phi)A} > 0, \tag{A23}$$

implying that $\partial p^*(\eta)/\partial \eta > 0$. ∎

B Appendix: The general model of levered equity return

The structure of the model closely follows that in Gomes and Schmid (2009), with the following exceptions: (i) we explicitly allow for shareholder recovery in the event of default; (ii) we do not impose that investment is irreversible; and (iii) we explicitly model capital adjustment costs.

B.1 The firm's problem

Production technology

The production output $Y_{i,t}$ of firm i at time t is

$$Y_{i,t} = K_{i,t}^{\alpha} e^{X_t + Z_{i,t}}, \quad \alpha \in (0,1), \tag{B24}$$

where $K_{i,t}$ is the firm's capital level at the beginning of period t, α is the capital share in total output, chosen to be between 0 and 1 in order to obtain decreasing return to scale. The variables X_t and $Z_{i,t}$ in (B24) represent, respectively, the aggregate and firm-specific shocks to the output. These shocks are modeled as stationary Markov processes evolving according to the following auto-regressive processes

$$X_{t+1} = X_t + (1-\rho_x)(\overline{X} - X_t) + \sigma_x \varepsilon_{t+1}^{x}, \tag{B25}$$

$$Z_{i,t+1} = Z_{i,t} + (1-\rho_z)(\overline{Z} - Z_{i,t}) + \sigma_z \varepsilon_{i,t+1}^{z}, \tag{B26}$$

where $\overline{X}$ and $\overline{Z}$ are the long-run averages, ρ_x and ρ_z the autocorrelation coefficient and σ_x and σ_z the volatility coefficients. The innovations ε_{t+1}^{x} and $\varepsilon_{i,t+1}^{z}$ are normally distributed with mean zero and unit variance, $E[\varepsilon_{i,t}^{z}\varepsilon_{t}^{x}] = 0$ for all i, and $E[\varepsilon_{i,t}^{z}\varepsilon_{j,t}^{z}] = 0$ for all $i \neq j$. In period t, the firm's after-tax profit is

$$\Pi_{i,t} = (1-\tau)(Y_{i,t} - fK_{i,t} - F), \tag{B27}$$

where τ is the corporate tax rate, and f and F are the proportional and fixed costs, respectively.

Investment

Each period, the firm makes an investment decision that affects its capital stock in the next period according to the capital accumulation equation

$$K_{i,t+1} = I_{i,t} + (1 - \kappa)K_{i,t}, \tag{B28}$$

where $I_{i,t}$ is the amount of new investment at time t, and κ is the depreciation rate of the installed capital. Following Lucas (1967), we assume a quadratic adjustment cost for new investment, *i.e.*,

$$h(I_{i,t}) = \frac{\theta}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 K_{i,t}, \tag{B29}$$

where $\theta > 0$ is the adjustment cost coefficient.

Financing

In order to finance new investment and distribution to shareholders, the firm also makes a choice of issuing either new equity (*i.e.*, negative dividends) or new debt, or a combination of both. As in Li (2008) and Gomes and Schmid (2009), we assume that the only debt instrument available to the firm is a one-period bond. At each date t the firm decides to issue a bond with promised principal $B_{i,t+1}$ and coupon $b_{i,t+1}$, to be repaid at time $t+1$. The debt is assumed to be issued at par, so at time t firm i is raising an amount equal to $B_{i,t+1}$. The firm is implicitly allowed to refinance all its liability in each period. Accounting for tax deductibility of the coupon payment, we define firm i's *total debt commitment* at time t as

$$D_{i,t} = B_{i,t} + (1 - \tau)b_{i,t}. \tag{B30}$$

This quantity represents the amount needed to service the debt issued by the firm in the *previous* period and coming due at time t.

If we assume that the net cash flow to the equity is paid out as dividend to equity-holders, the dividend at time t is then

$$c_{i,t} = \Pi_{i,t} + \tau\kappa K_{i,t} - I_{i,t} - h(I_{i,t}) - D_{i,t} + B_{i,t+1}. \tag{B31}$$

If $c_{i,t} < 0$, the firm can raise external financing through a seasoned equity offering. Following Gomes (2001) and Hennessy and Whited (2007), we assume that it is costless to increase debt, but costly to raise new equity. The cost of raising new financing through seasoned equity offering is assumed to be

$$\Lambda(c_{i,t}) = (\lambda_0 + \lambda_1(-c_{i,t}))\mathbf{1}_{c_{i,t}<0}, \tag{B32}$$

where λ_0 is the fixed cost and λ_1 represents the proportional cost. Therefore, the net dividend is

$$d_{i,t} = c_{i,t} - \Lambda(c_{i,t}). \tag{B33}$$

Equity valuation

A firm's equity value is the maximal present value of the discounted stream of dividends that the firm can achieve by altering its investment and financing policy. To evaluate cash flow, we assume a process for the pricing kernel $\mathbb{M}_{t,t+1}$ similar to that in Berk, Green, and Naik (1999) and Zhang (2005),

$$\mathbb{M}_{t,t+1} = \beta \exp\left\{\Gamma_t(X_t - X_{t+1})\right\}, \quad \Gamma_t = \gamma_0 + \gamma_1(X_t - \overline{X}), \tag{B34}$$

where $0 < \beta < 0$ is the time discount factor, and γ_0 and γ_1 constants.[25]

At any point in time a firm is entirely described by four state variables: the aggregate shock X_t, the firm-specific shock $Z_{i,t}$, the capital level K_t, and the total debt commitment $D_{i,t}$ defined in (B30). We denote by $S_{i,t} = \{X_t, Z_{i,t}, K_{i,t}, D_{i,t}\}$ the vector of state variables. Equity value, $V(S_t)$, is the solution of a dynamic programming problem with optimal investment financing and default choices. Unless the company optimally defaults at time t, these choices will result in a new level of capital $K_{i,t+1}$ and a new level of total debt commitment $D_{i,t+1}$. Hence the future levels of capital $K_{i,t+1}$ and total debt commitment $D_{i,t+1}$ are *control* variables at time t, and become *state* variables at time $t + 1$.

In the absence of shareholder recovery upon financial distress, the firm is financially viable as long as equity has a positive value, *i.e.*, $V(S_{i,t}) > 0$ and default occurs when $V(S_{i,t}) = 0$. In the presence of shareholder recovery, equity can extract a fraction η of the residual asset value upon financial distress, $R(S_t)$. Following Hennessy and Whited (2007), we model the residual

[25] The market price of risk is equal to $\lambda_{m,t} = Var_t[\mathbb{M}_{t,t+1}]/E_t[\mathbb{M}_{t,t+1}]$. Given the assumption of normality in the innovations of X_t, $\lambda_{m,t} = \beta(e^{\sigma_m^2} - 1)$, where $\sigma_m = \sigma_x[\gamma_0 + \gamma_1(X_t - \overline{X})]$.

asset value as

$$R(S_{i,t}) = \max\left\{\Pi_{i,t} + \tau\kappa K_{i,t} + \xi_1(1-\kappa)K_{i,t} - \xi_0, 0\right\}, \tag{B35}$$

where $\Pi_{i,t}$ is the after-tax profit defined in (B27), and $1-\xi_1$ and ξ_0 are the proportional and fixed distress costs, respectively. Firm i's equity value is therefore determined by the solution of the following Bellman equation:

$$V(S_{i,t}) = \max\left\{\eta R(S_{i,t}), \max_{\{K_{i,t+1}, D_{i,t+1}\}} \left\{d(S_{i,t}) + \mathbb{E}_t\left[\mathbb{M}_{t,t+1} V(S_{i,t+1})\right]\right\}\right\}. \tag{B36}$$

One potential difficulty in computing the net cash flow—$c_{i,t}$ in (B31), and, in turn, $d_{i,t}$ in (B33)—needed for the solution of the dynamic programming problem (B36) resides in the determination of the face value $B_{i,t+1}$ (and the coupon $b_{i,t+1}$) of the newly issued debt. As argued in Li (2008), the use of the total debt commitment $D_{i,t}$ as a state variable simplifies the problem considerably because we can avoid keeping track of the coupon $b_{i,t+1}$. Denoting by

$$\chi_{i,t+1} = \mathbf{1}_{\{V(S_{i,t+1}) > \eta R(S_{i,t+1})\}} \tag{B37}$$

the indicator function for the firm's solvency, we can in fact evaluate the market value of the bond as follows,

$$\begin{aligned} B_{i,t+1} &= \mathbb{E}_t\left[\mathbb{M}_{t,t+1}\left(\chi_{i,t+1}(b_{i,t+1} + B_{i,t+1}) + (1-\chi_{i,t+1})(1-\eta)R(S_{i,t+1})\right)\right] \\ &= \frac{\mathbb{E}_t\left[\mathbb{M}_{t,t+1}\left\{\chi_{i,t+1}\frac{D_{i,t+1}}{1-\tau} + (1-\chi_{i,t+1})(1-\eta)R(S_{i,t+1})\right\}\right]}{1 + \frac{\tau}{1-\tau}\mathbb{E}_t\left[\mathbb{M}_{t,t+1}\chi_{i,t+1}\right]}, \end{aligned} \tag{B38}$$

where the first equality considers the debt value in cases of solvency and default, respectively, and the last equation uses the definition of total debt commitment (B30) to express the coupon $b_{i,t+1}$ as a function of $D_{i,t+1}$ and $B_{i,t+1}$. The bond pricing equation (B38) involves only knowledge of the evolution of the state variables $S_{i,t}$ and will be used to determine the cash flows net of investment and financing defined in (B31).

B.2 Numerical analysis

B.2.1 Solution

We solve for the fixed point in the Bellman equation (B36) by using a standard value function iteration algorithm on a discretized grid space (see, *e.g.*, Judd (1998)). To this purpose we discretize the four-dimensional state space by choosing (i) 100 equally spaced grid points between 0 and 20 for both the capital level $K_{i,t}$ and the debt commitment level $D_{i,t}$, (ii) 17 grid points for the systematic shock $X_{i,t}$, and (iii) 39 grid points for the idiosyncratic shock $Z_{i,t}$. We use Tauchen (1986)'s quadrature method to choose the grid points for the systematic and idiosyncratic shocks. As a result, the state space is discretized into $17 \times 39 \times 100 \times 100 = 6,630,000$ grid points.

B.2.2 Equity expected return

To simplify notation, in the following, we will drop the subscript i from firm-specific quantities. The equity expected return is defined as

$$E_t[R_{t+1}] = \frac{\mathbb{E}_t[V(S_{t+1})]}{V(S_t) - d(S_t)}, \tag{B39}$$

where the expectation $\mathbb{E}_t$ is taken with respect to the probability measure induced by the Markov processes (B25) and (B26) for the systematic and idiosyncratic shocks and by the optimal investment and financing policies. We subtract the dividend $d(S_t)$ from the equity value in the denominator of (B39) because equity value in (B36) is *cum dividend*. From the stationary solution we compute recursively the τ-month ahead *probability of default* as follows:

$$p_\tau(S_t) = (1 - \chi(S_t)) + \chi(S_t) \cdot \mathbb{E}_t[p_{\tau-1}(S_{t+1})], \quad p_0(S_{t+1}) = 1 - \chi(S_{t+1}), \tag{B40}$$

where χ is the stationary default boundary (B37).

Figure 3 in the text is obtained by bootstrapping expected returns from the stationary distribution. Specifically, according to the processes (B25), the unconditional distribution for X_t is normal with mean $\overline{X}$ and variance $\sigma_x^2/(1-\rho_x^2)$. Similarly, the unconditional distribution for Z_t is normal with mean $\overline{Z}$ and variance $\sigma_z^2/(1-\rho_z^2)$. A firm is characterized by a point $S_t = \{X_t, Z_t, K_t, Z_t\}$ in the state space. We construct three representative panels of 10,000 firms each. Each panel corresponds to a different realization of the systematic state variable

X_t. We select these three points to be $\{\overline{X} - s_X, \overline{X}, \overline{X} + s_X\}$ where $\overline{X}$ is the long run mean and $s_X = \sigma_x/(1-\rho_x^2)^{1/2}$, the long run volatility of X_t. These values are chosen to represent three phases of the business cycle. For each realization of X_t we randomly select 100 points from the stationary distribution of Z_t and 10 points each for K_t and D_t, chosen uniformly from their respective support. After forming each panel, we sort firms into ten portfolios according to their default probability (B40) and compute the equally-weighted *expected* returns of the portfolios thus obtained. We repeat this procedure 500 times for each panel and compute the average expected return conditional on the realization of X_t. Figure 3 reports the unconditional expected return, obtained by weighting each conditional expected return by the long run probability of the chosen realization of X_t. Panel A presents the case of no shareholder recovery ($\eta = 0$), while Panel B reports the result for the case with expected shareholder recovery equal to 10% of the residual value $R(S_t)$ defined in (B35).

B.2.3 Value premium

From the stationary solution of the general model, we can construct the book-to-market ratio $BM(S_t)$ at each point S_t of the state space as

$$BM(S_t) = \frac{K_t - D_t}{V(S_t)}. \tag{B41}$$

To study the structure of the value premium in the cross section, we follow the bootstrap methodology in the previous subsection. For each realization of X_t we draw a panel of 10,000 firms by randomly selecting 100 points from the stationary distribution of Z_t and 10 points each for K_t and D_t chosen uniformly from their support. We then sort firms into ten portfolios based on their default probability (B40). We finally sort each of these portfolios into five sub-portfolios according to the firms' book-to-market ratio. Within each default probability decile, we compute the value premium as the spread between the expected returns of the highest and lowest book-to-market quintile. We repeat this procedure 500 times for each panel and aggregate the results from each panel by weighting them according to the long run probability of the chosen realization of X_t. Figure 4 in the text reports the average value premium in each default probability decile across the 500 repetitions.

B.2.4 Momentum profits

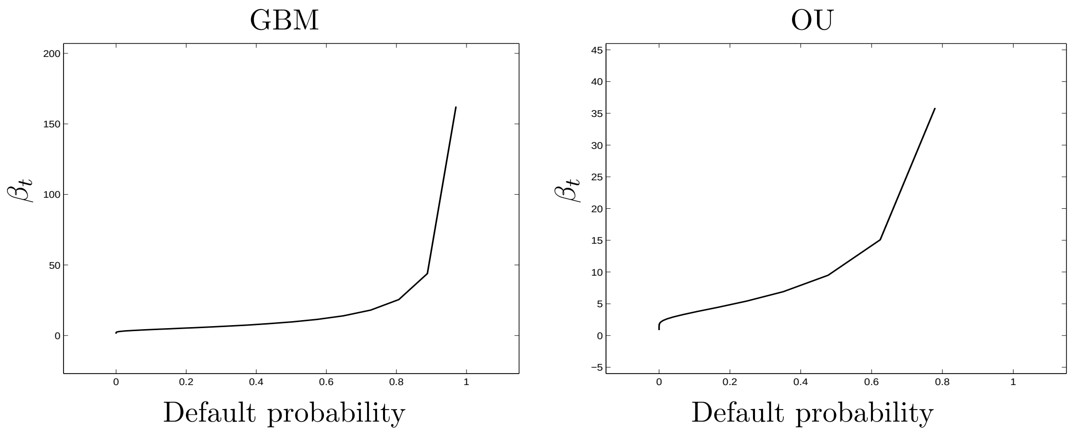
To construct momentum portfolios we generate a time series of realized returns that will determine winners and losers in each period. For this purpose, we follow the bootstrapping procedure of the previous two subsections. For each realization of X_t we draw a panel of 10,000 firms by randomly selecting 100 points from the stationary distribution of Z_t and 10 points each for K_t and D_t, chosen uniformly from their support. Based on the dynamics of the state variables X_t and Z_t, and the optimal investment and financing strategies derived from the solution of the model, each state S_t will evolve to a *future* state S_{t+1}. The realized return is hence $\frac{V(S_{t+1})}{V(S_t)-d(S_t)}$, which will be used to separate winners from losers. The expected return in the state S_{t+1} can be subsequently deduced directly from the stationary solution as discussed in Appendix B.2.2.

We construct momentum profits by sorting the panel of firms in state S_{t+1} into ten portfolios based on their default probability and, independently, into five portfolios based on the realized return from state S_t to state S_{t+1}. The bottom quintile represents the portfolio of *losers* and the top quintile is the portfolio of *winners*. The expected momentum profits are calculated as the difference in the equally-weighted expected returns of winners and losers in each default probability decile. We repeat this procedure 500 times for each panel and aggregate the results from each panel by weighting them according to the long run probability of the chosen realization of X_t. Figure 5 in the text reports the average monthly momentum profits in each default probability decile across the 500 repetitions.

Figure 1: Equity beta and default probability

The figure reports the equity β as a function of 1-year ahead default probability. The probability is computed according to equation (A7) in Lemma 1 of Appendix A, for the Geometric Brownian Motion (GBM), and obtained numerically for the Ornstein-Uhlenbeck (OU) case using Tauchen (1986) quadrature to discretize the process for p_t with 100 grid points. The parameter used for the graphs are: $\sigma = 0.3$, $\mu^{\mathbb{P}} = -\sigma^2/4$ (to normalize gross revenues from a unit of production to 1), $r = 0.02$, $\zeta = 0.1$ (OU), $\zeta = 0$ (GBM); $\lambda = 0.05$, $c = 1$. Panel A refers to the case of no expected shareholder recovery upon financial distress, $\eta = 0$, while Panel B refers to the case in which $\eta = 2\%$.

Panel A: No shareholder recovery ($\eta = 0$)



Panel B: Shareholder recovery ($\eta = 2\%$)

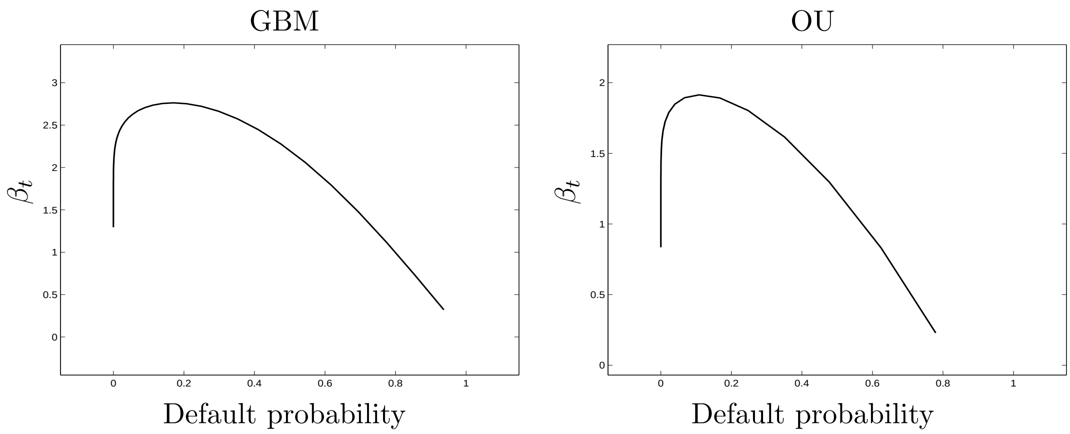
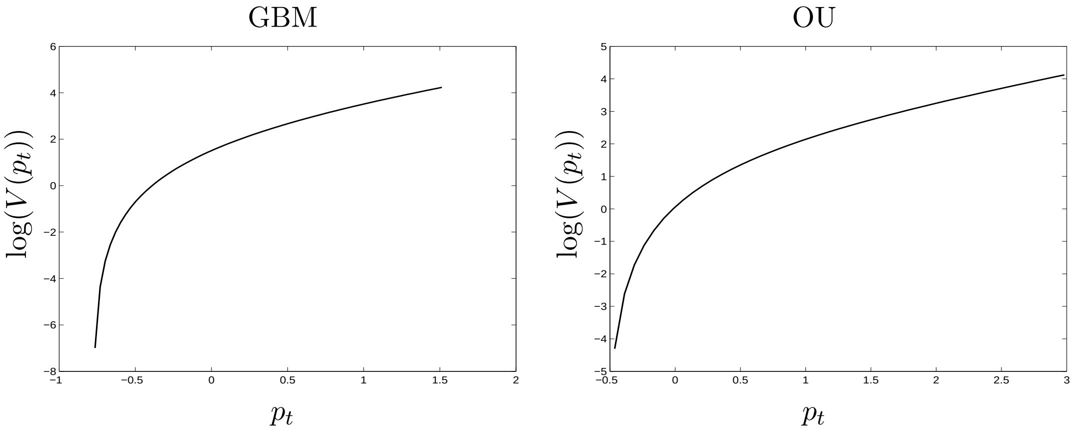


Figure 2: Shareholder recovery and log-convexity

The figure reports the logarithm of equity value $\log(V(p_t))$ for the Geometric Brownian Motion (GBM) and Ornstein-Uhlenbeck (OU) cases of Proposition 1 as a function of the underlying price p_t. Parameter values are the same as in Figure 1, and shareholder recovery value is set to be a fraction η of the book value of assets c/r.

Panel A: No shareholder recovery ($\eta = 0$)



Panel B: Shareholder recovery ($\eta = 2\%$)

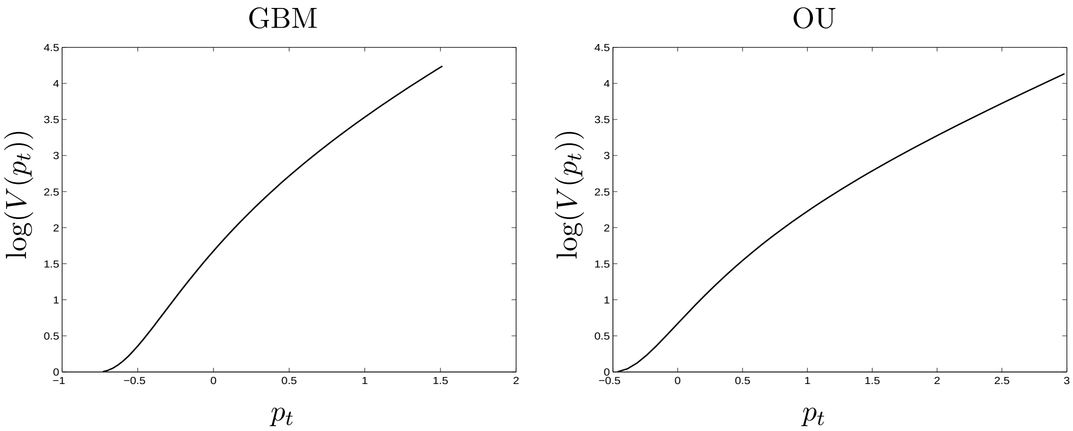


Figure 3: Expected return and default probability

The figure reports the monthly expected return as a function of default probability obtained from the general model of Section 3. Panel A refers to the case of no shareholder recovery, $\eta = 0$, while Panel B refers to the case in which shareholder recovery is set to $\eta = 10\%$ of the recovery value in (B35).

Panel A: No shareholder recovery ($\eta = 0$) Panel B: Shareholder recovery ($\eta = 10\%$)

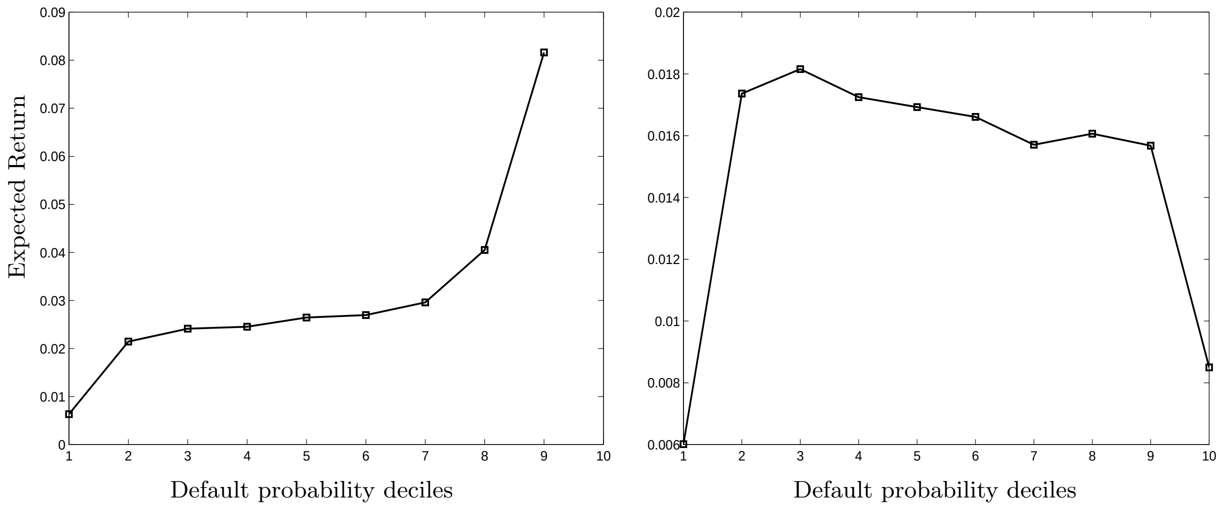


Figure 4: Value premium and default probability

The figure reports the monthly spread between the average expected returns of high book-to-market firms and that of low book-to-market firms within each decile of default probability in the cross section of firms generated from the stationary solution of the general model of Section 3. Panel A corresponds to the case with $\eta = 0$, while Panel B refers to the case with $\eta = 10\%$.

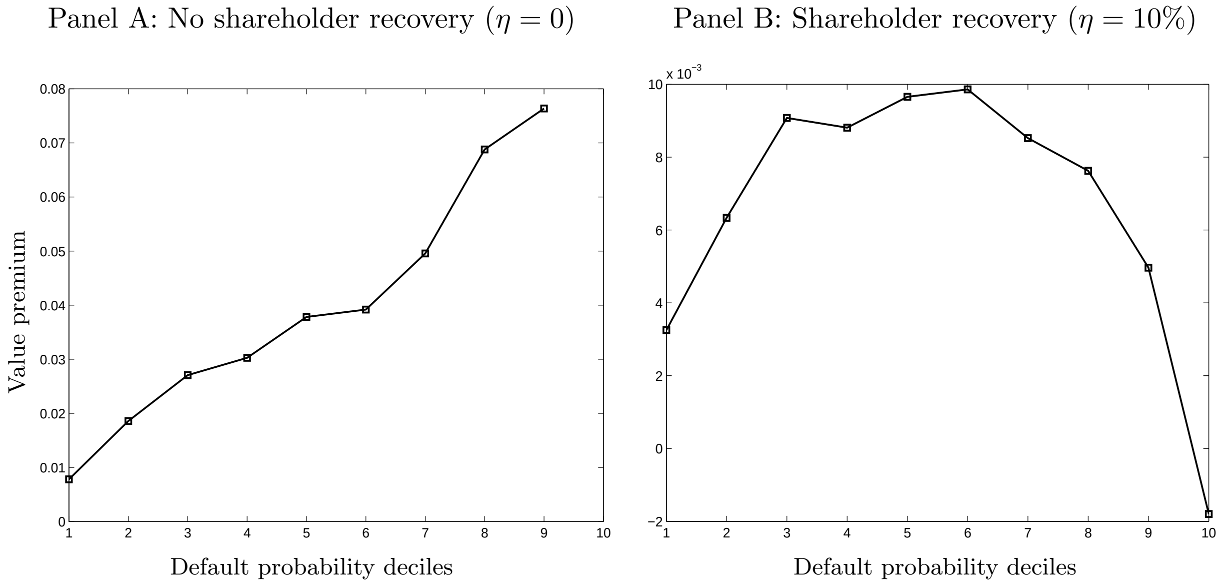


Figure 5: Momentum profits and default probability

The figure reports the monthly momentum profits as a function of default probability generated from the general model of Section 3. Panel A refers to the case of no shareholder recovery, $\eta = 0$, while Panel B refers to the case in which shareholder recovery is set to $\eta = 10\%$ of the recovery value in (B35).

Panel A: No shareholder recovery ($\eta = 0$) Panel B: Shareholder recovery ($\eta = 10\%$)

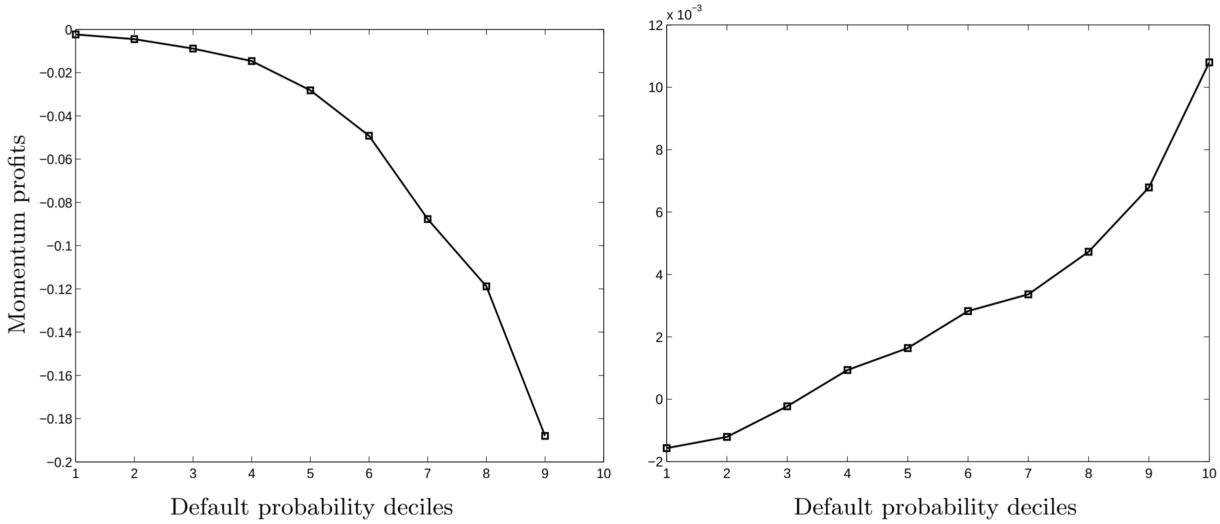
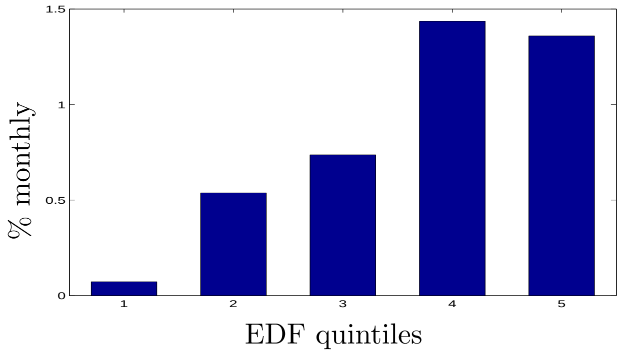


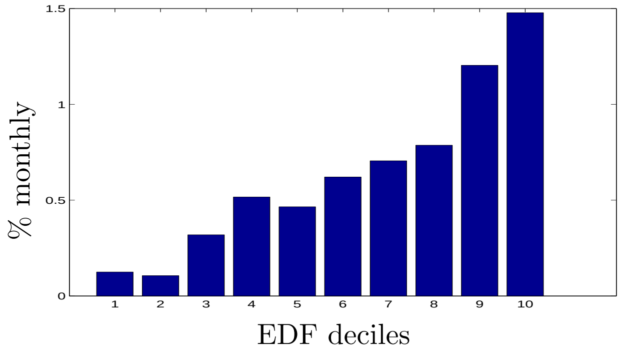
Figure 6: Value premium and default probability

The figure reports the value spread based on value-weighted returns of book-to-market sorted portfolios. Panel A considers the full sample of stocks from 1969 to 2007 and uses EDF quintiles. Panel B considers stocks with per-share prices of $5 or higher. Panel C also considers stocks with per-share prices of no less than $5 but computes the cutoffs for EDF deciles from the full sample of stocks.

Panel A: Full sample



Panel B: Price $\geq$ $5 subsample



Panel C: Price $\geq$ $5, EDF cutoffs from full sample

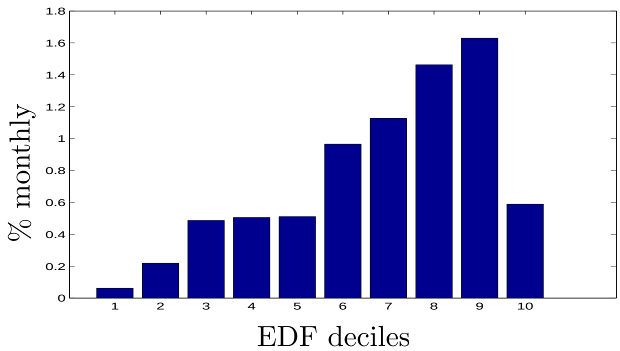


Table 1: Parameters for the general model

The table reports the parameters used in solving the model described in Section 3. Parameter values are consistent with those in Gomes and Schmid (2009), Li (2008), and Zhang (2005).

Parameter	Description	Value
$\overline{X}$	Long-run average of aggregate productivity	-3.100
σ_x	Conditional volatility of aggregate productivity	0.002
ρ_x	Persistence of aggregate productivity	0.983
$\overline{Z}$	Long-run average of firm-specific productivity	0.000
σ_z	Conditional volatility of firm-specific productivity	0.100
ρ_z	Persistence of firm-specific productivity	0.900
α	Capital share	0.650
δ	Capital depreciation	0.010
f	Variable cost of production	0.000
F	Fixed cost of production	0.034
θ	Adjustment cost	15.000
γ_0	Constant price of risk	50
γ_1	Time-varying price of risk	-1000
β	Time-preference coefficient	0.995
τ	Tax rate	0.350
ξ_0	Fixed bankruptcy cost	0.120
ξ_1	Liquidation value per unit of capital	0.900
λ_0	Fixed equity issuance cost	0.080
λ_1	Variable equity issuance cost	0.025

Table 2: Summary statistics of the EDF measure

Panel A reports the summary statistics for the entire sample and for the subsamples of stocks with a per-share price no less than \$5 and those with a per-share price smaller than \$5. Panel B reports summary statistics of the EDF measure and of the Amihud (2002) illiquidity measure across EDF-sorted portfolios. EDF quantities are expressed in percentage. "Average EDF" refers to the time series mean of average EDF measures within each EDF decile and "Max EDF" is the time series mean of maximum EDF measures within each EDF decile.

Panel A: EDF statistics

	Mean	Std	Min	p5	p10	p25	Median	p75	p90	Max
Full sample	3.30	5.19	0.02	0.05	0.09	0.30	1.07	3.47	10.44	20.00
Stocks with $p \geq \$5$	1.15	1.86	0.02	0.03	0.06	0.17	0.52	1.36	2.84	20.00
Stocks with $p < \$5$	6.89	6.75	0.02	0.26	0.54	1.52	4.10	10.68	20.00	20.00

Panel B: Summary statistics of EDF-sorted decile portfolios

	Low EDF	2	3	4	5	6	7	8	9	High EDF
Full Sample										
Average EDF	0.07	0.20	0.38	0.61	0.95	1.46	2.26	3.75	6.98	14.79
Max EDF	0.13	0.28	0.48	0.76	1.17	1.79	2.84	4.93	9.93	19.82
Average Amihud Measure	0.01	0.03	0.05	0.10	0.18	0.33	0.64	1.33	3.36	11.47
Stocks with price $\geq \$5$										
Average EDF	0.06	0.14	0.23	0.36	0.52	0.73	1.03	1.47	2.24	5.21
Max EDF	0.10	0.18	0.29	0.43	0.61	0.86	1.21	1.77	2.86	19.2
Average Amihud Measure	0.01	0.02	0.03	0.05	0.07	0.11	0.15	0.23	0.34	0.58

Table 3: Value premium and default probability

Each month, stocks are sorted independently into terciles of book-to-market ratios (BM) and deciles of MKMV's EDF scores (EDF). The table reports the time series average of value-weighted (VW) and equal-weighted (EW) returns of each portfolio obtained in the month after portfolio formation. Portfolio returns are expressed in percentage per month. CAPM-alpha, FF-alpha, 4-Factor alpha, and 5-Factor alpha refer to the value premium after controlling for risk according to, respectively, the CAPM market factor, the Fama-French three-factor model, the Carhart 4-factor model, and a 5-factor model that also includes the liquidity factor of Pastor and Stambaugh (2003).

	Low EDF 1	2	3	4	5	6	7	8	9	High EDF 10
					Panel A: Value-weighted					
B/M Ratio										
Low	0.94	0.96	0.71	0.93	0.74	0.40	0.06	-0.25	-0.49	-0.19
Medium	0.99	1.02	1.21	1.27	1.01	1.20	0.99	0.68	0.46	0.32
High	0.99	1.11	1.23	1.47	1.27	1.42	1.35	1.34	0.92	0.88
Value Premium										
Raw	0.05	0.15	0.52	0.54	0.53	1.03	1.29	1.60	1.41	1.07
t-stat	0.275	0.745	2.334	2.246	2.199	4.111	4.747	5.437	4.468	2.989
CAPM alpha	0.15	0.34	0.74	0.78	0.74	1.24	1.46	1.75	1.60	1.17
t-stat	0.789	1.841	3.700	3.622	3.352	5.349	5.566	6.141	5.263	3.273
FF alpha	-0.44	-0.22	0.14	0.17	0.20	0.71	0.96	1.16	1.17	0.67
t-stat	-3.103	-1.673	0.983	1.068	1.081	3.531	3.981	4.509	3.945	1.933
4-Factor alpha	-0.53	-0.35	0.01	0.05	-0.02	0.58	0.74	0.84	0.73	0.33
t-stat	-3.653	-2.596	0.038	0.304	-0.134	2.805	3.038	3.261	2.507	0.935
5-Factor alpha	-0.54	-0.37	0.02	0.12	-0.08	0.53	0.68	0.90	0.72	0.28
t-stat	-3.638	-2.704	0.150	0.726	-0.428	2.538	2.757	3.486	2.445	0.785
					Panel B: Equally-weighted					
B/M Ratio										
Low	1.03	0.96	0.78	0.78	0.58	0.51	0.39	0.16	0.28	1.71
Medium	1.03	1.14	1.31	1.33	1.28	1.41	1.15	1.08	0.82	1.91
High	1.13	1.19	1.43	1.50	1.48	1.55	1.62	1.53	1.48	2.14
Value Premium										
Raw	0.11	0.23	0.65	0.72	0.90	1.05	1.24	1.37	1.19	0.44
t-stat	0.651	1.206	3.469	3.517	4.605	5.012	6.110	6.271	4.703	1.498
CAPM alpha	0.28	0.47	0.88	0.96	1.12	1.28	1.45	1.56	1.38	0.60
t-stat	1.937	3.058	5.778	5.705	6.766	7.155	8.141	7.796	5.786	2.118
FF alpha	-0.19	-0.02	0.41	0.47	0.67	0.82	1.06	1.17	1.08	0.31
t-stat	-1.832	-0.248	4.138	4.172	5.452	5.873	6.883	6.546	4.676	1.101
4-Factor alpha	-0.22	-0.09	0.28	0.34	0.48	0.64	0.78	0.83	0.58	-0.07
t-stat	-2.148	-0.888	2.798	2.974	3.989	4.597	5.287	4.862	2.695	-0.244
5-Factor alpha	-0.21	-0.09	0.25	0.33	0.46	0.62	0.74	0.82	0.55	-0.10
t-stat	-1.967	-0.911	2.508	2.871	3.788	4.455	4.953	4.722	2.536	-0.339

Table 4: Momentum profits, default probability, and shareholder recovery

Each month, all stocks are sorted independently into terciles of EDF scores, terciles of a proxy for expected shareholder recovery, and quintiles of winners/losers according to past six-month returns. We skip a month after portfolio formation and follow the methodology of Jegadeesh and Titman (1993) to report momentum profits in the next six-month holding period. "Low EDF" refers to the bottom EDF quintile and "High EDF" to the top quintile. *AVL* is asset size, *R&D* is the ratio of R&D expenditure over total book asset, and *SalesHfdl* is the Herfindale index for sales within an industry. Portfolio returns are expressed in percentage per month. $W-L$ are the raw momentum profits, FF-α and 4-Factor α refer to momentum profits after controlling for risk according to the Fama-French 3-factor model and the Carhart 4-factor model, respectively.

Panel A: Momentum profits across AVL groups

	Low EDF						High EDF					
	W−L	t-stat	FF-α	t-stat	4-Factor α	t-stat	W−L	t-stat	FF-α	t-stat	4-Factor α	t-stat
AVL												
Low	1.39	3.599	1.49	3.780	0.93	2.372	0.79	2.784	0.84	2.882	0.00	-0.001
Med	1.22	5.130	1.39	5.795	0.57	3.065	1.64	4.687	1.76	4.893	0.67	2.227
High	0.73	2.814	0.84	3.117	-0.25	-1.466	1.90	3.767	2.03	3.940	0.79	1.685
High−Low	-0.66	-1.605	-0.66	-1.605	-1.18	-2.799	1.10	2.330	1.20	2.446	0.79	1.597

Panel B: Momentum profits across R&D groups

	Low EDF						High EDF					
	W−L	t-stat	FF-α	t-stat	4-Factor α	t-stat	W−L	t-stat	FF-α	t-stat	4-Factor α	t-stat
R&D												
Low	0.81	2.304	0.92	2.560	-0.17	-0.557	2.00	4.570	2.19	4.877	1.05	2.608
Med	1.10	3.486	1.16	3.532	0.16	0.570	0.47	1.099	0.51	1.170	-0.55	-1.384
High	0.65	1.857	0.86	2.381	-0.05	-0.141	0.26	0.455	0.61	1.051	-0.55	-0.991
High−Low	-0.16	-0.369	-0.07	-0.150	0.12	0.267	-1.75	-2.951	-1.58	-2.601	-1.60	-2.553

Panel C: Momentum profits across SalesHfdl groups

	Low EDF						High EDF					
	W−L	t-stat	FF-α	t-stat	4-Factor α	t-stat	W−L	t-stat	FF-α	t-stat	4-Factor α	t-stat
SalesHfdl												
Low	0.85	2.801	0.87	2.772	-0.16	-0.630	1.20	3.089	1.23	3.082	0.15	0.434
Med	0.72	2.554	0.77	2.667	-0.15	-0.624	1.01	2.656	1.17	2.992	0.20	0.572
High	0.75	2.441	0.96	3.053	-0.14	-0.608	2.14	5.302	2.27	5.497	1.21	3.296
High−Low	-0.10	-0.308	0.10	0.299	0.01	0.035	0.94	2.234	1.04	2.393	1.06	2.379

Table 5: Momentum profits and default probability

The column labeled "Uncond." reports momentum profit computed according to the "6-1-6" procedure in Jegadeesh and Titman (1993). The remaining columns report momentum profits similarly computed within EDF quintiles. To obtain these values, each month, all stocks are sorted independently into quintiles of EDF scores and quintiles of winners/losers according to past six-month returns. We skip a month after portfolio formation. The returns of each portfolio for the subsequent six-month period are recorded and averaged through time. Portfolio returns are expressed in percentage per month. Momentum alphas are obtained after controlling for risk according to the Carhart 4-factor model.

	Uncond.			EDF			Diff
		1	2	3	4	5	
Raw profits	0.80	0.70	0.70	0.97	1.06	1.54	0.84
t-stat	3.145	2.772	2.830	4.301	4.309	4.780	2.715
4-Factors alphas	-0.23	-0.28	-0.32	0.09	0.17	0.52	0.80
t-statistic	-2.022	-1.675	-2.109	0.588	0.956	2.042	2.463
Factor loadings							
UMD	1.186	1.049	1.050	0.905	0.934	1.094	0.045
t-stat	45.313	27.466	29.422	25.314	22.670	18.378	0.596
MKT	-0.045	0.013	0.058	0.042	-0.021	-0.013	-0.027
t-stat	-1.664	0.337	1.578	1.128	-0.486	-0.218	-0.343
HML	0.185	0.206	0.223	0.209	0.241	0.295	0.089
t-stat	4.594	3.502	4.061	3.792	3.802	3.213	0.763
SMB	-0.092	0.102	0.227	0.145	0.146	-0.090	-0.192
t-stat	-2.654	2.021	4.785	3.060	2.675	-1.138	-1.924

References

Abramowitz, Milton, and Irene A. Stegun, 1972, Handbook of mathematical functions, *National Bureau of Standards: Applied Mathematics Series*, U.S. Government Printing Office.

Amihud, Yakov, 2002, Illiquidity and stock returns: Cross-section and time-series effects, *Journal of Financial Markets* 5, 31–56.

Asness, Clifford S., Tobias J. Moskowitz, and Lasse H. Pedersen, 2009, Value and momentum everywhere, Working Paper, University of Chicago and New York University.

Avramov, Doron, Tarun Chordia, Gergana Jostova, and Alexander Philipov, 2007, Momentum and credit rating, *Journal of Finance* 62, 2503–2520.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.

Betker, Brian, 1995, Management's incentive, equity bargaining power, and deviation from absolute priority, *Journal of Business* 68, 161–184.

Bharath, Sreedhar, Venky Panchapegesan, and Ingrid Werner, 2009, The changing nature of Chapter 11, Working Paper, University of Michigan, GSAM and Ohio State University.

Black, Fischer, and John C. Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351–367.

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–659.

Brown, David, Brian Ciochetti, and Timothy J. Riddiough, 2006, Theory and evidence on the resolution of fiancial distress, *Review of Financial Studies* 19, 1357–1397.

Campbell, John Y., Jens Hilscher, and Jan Szilagyi, 2008, In search of distress risk, *Journal of Finance* 63, 2899–2939.

Carhart, Mark, 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577–2603.

Chen, Jason, 2009, Firm life expectancy and the heterogeneity of the book-to-market effect, *Journal of Financial Economics,* forthcoming.

Choi, Jaewon, 2009, Rethinking the conditional CAPM: the impact of financial leverage, Working Paper, Pennsylvania State University.

Cooley, Thomas F., and Vincenzo Quadrini, 2001, Financial markets and firm dynamics, *American Economic Review* 91, 1286–1310.

Cooper, Ilan, 2006, Asset pricing implications of nonconvex adjustment costs and irreversibility of investment, *Journal of Finance* 61, 139–170.

Cooper, Russel W., and João Ejarque, 2003, Financial frictions and investment: A requiem in Q, *Review of Economic Dynamics* 6, 710–728.

Crosbie, Peter J., and Jeffrey R. Bohn, 2003, Modeling default risk, *Moody's KMV* Available at http://www.defaultrisk.com/pp_model_35.htm.

Da, Zhi, and Pengjie Gao, 2008, Clientele change, liquidity shock, and the return on financially distressed stocks, *Journal of Financial and Quantitative Analysis,* forthcoming.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and over-reaction, *Journal of Finance* 53, 1839–1886.

Dichev, Ilia D., 1998, Is the risk of bankruptcy a systematic risk?, *Journal of Finance* 53, 1131–1147.

Eberhart, Allan C., William T. Moore, and Rodney L. Roenfeldt, 1990, Security pricing and deviation from absolute priority rule in bankruptcy proceedings, *Journal of Finance* 45, 1457–1469.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

——— , 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

——— , 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55–84.

Fan, Hua, and Suresh M. Sundaresan, 2000, Debt valuation, renegotiation, and optimal dividend policy, *Review of Financial Studies* 13, 1057–1099.

Favara, Giovanni, Enrique Schroth, and Philip Valta, 2009, Is shareholders' strategic default behavior priced? Evidence from the international cross-section of stocks, Working paper, University of Lausanne.

Ferguson, Michael F., and Richard L. Shockley, 2003, Equilibrium 'anomalies', *Journal of Finance* 58, 2549–2580.

Franks, Julian R., and Walter N. Torous, 1989, An empirical investigation of U.S. firms in renegotiation, *Journal of Finance* 44, 747–779.

——— , 1994, A comparison of financial restructuring in distressed exchanges and chapter 11 reorganizations, *Journal of Financial Economics* 35, 349–370.

Gala, Vito D., 2006, Investment and returns, Working Paper, London Business School.

Garlappi, Lorenzo, Tao Shu, and Hong Yan, 2008, Default risk, shareholder advantage, and stock returns, *Review of Financial Studies* 21, 2743–2778.

George, Thomas J., and Chuan-Yang Hwang, 2009, A resolution of the distress risk and leverage puzzles in the cross section of stock returns, *Journal of Financial Economics,* forthcoming.

Gilson, Stuart C., Kose John, and Larry H. Lang, 1990, Troubled debt restructuring: An empirical study of private reorganization of firms in default, *Journal of Financial Economics* 27, 315–353.

Gomes, João F., 2001, Financing investment, *American Economic Review* 90, 1263–1285.

——— , Leonid Kogan, and Lu Zhang, 2003, Equilibrium cross-section of returns, *Journal of Political Economy* 111, 693–732.

Gomes, João F., and Lukas Schmid, 2009, Levered returns, *Journal of Finance* forthcoming.

Griffin, John M., and Michael L. Lemmon, 2002, Book-to-market equity, distress risk and stock returns, *Journal of Finance* 57, 2317–2336.

Harrison, Michael J., 1985, *Brownian Motion and Stochastic Flow Systems* (John Wiley and Sons: New York, N.Y.).

Harvey, Campbell R., and Akhtar Siddique, 2000, Conditional skewness in asset pricing tests, *Journal of Finance* 55, 1263–1295.

Hecht, Peter, 2004, The cross section of expected firm (not equity) returns, Working Paper, Harvard Business School.

Hennessy, Christopher, and Toni Whited, 2005, Debt dynamics, *Journal of Finance* 60, 1129–1165.

———, 2007, How costly is external financing? Evidence from a structural estimation, *Journal of Finance* 62, 1705–1745.

Hotchkiss, Edith S., Kose John, Robert M. Mooradian, and Karin S. Thorburn, 2008, Bankruptcy and the resolution of finanacial distress, *Handbook of Empirical Corporate Finance* 2, Edited by B. Espen Eckbo (Elsevier B.V.).

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Johnson, Timothy C., 2002, Rational momentum effects, *Journal of Finance* 57, 585–608.

Judd, Kenneth, 1998, *Numerical Methods in Economics* (MIT Press).

Kealhofer, Stephen, 2003a, Quantifying credit risk I: Default prediction, *Financial Analysts Journal* January/February, 30–44.

———, 2003b, Quantifying credit risk II: Debt valuation, *Financial Analysts Journal* May/June, 78–92.

Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

Lewellen, Jonathan, 2002, Momentum and autocorrelation in stock returns, *Review of Financial Studies* 15, 533–563.

Li, Erica X., 2008, Corporate governance, the cross section of returns, and financing choices, Working Paper, University of Michigan.

Livdan, Dimitry, Horacio Sapriza, and Lu Zhang, 2008, Financially constrained stock returns, *Journal of Finance* forthcoming.

Lucas, Robert E. Jr., 1967, Adjustment costs and the theory of supply, *Journal of Political Economy* 75, 321–334.

Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.

Morellec, Erwan, Brosi Nikolov, and Norman Schürhoff, 2008, Dynamic capital structure under managerial entrenchment: Evidence from a structural estimation, Working Paper, University of Lausanne.

Novy-Marx, Robert, 2008, Competition, productivity and the cross section of expected returns, Working Paper, University of Chicago.

Obreja, Iulian, 2006, Financial leverage and the cross-section of stock returns, Working Paper, Carnegie Mellon.

O'Doherty, Michael S., 2009, Does the conditional CAPM explain the financial distress anomaly?, Working Paper, University of Iowa.

Opler, Tim, and Sheridan Titman, 1994, Financial distress and corporate performance, *Journal of Finance* 49, 1015–1040.

Pastor, Lubos, and Robert Stambaugh, 2003, Liquidity risk and expected returns, *Journal of Political Economy* 111, 642–685.

Sagi, Jacob S., and Mark S. Seasholes, 2007, Firm-specific attributes and the cross-section of momentum, *Journal of Financial Economics* 84, 389–434.

Shleifer, Andrei, and Robert W. Vishny, 1992, Liquidation values and debt capacity: A market equilibrium approach, *Journal of Finance* 47, 1343–1366.

Shumway, Tyler, 1997, The delisting bias in CRSP data, *Journal of Finance* 52, 327–340.

——— , and Vincent A. Warther, 1999, The delisting bias in CRSP's Nasdaq data and its implications for the size effect, *Journal of Finance* 54, 2361–2379.

Tauchen, George, 1986, Finite state Markov-chain approximations to univariate and vector autoregressions, *Economics Letters* 20, 177–181.

Vassalou, Maria, and Yuhang Xing, 2004, Default risk in equity returns, *Journal of Finance* 59, 831–868.

Weiss, Lawrence A., 1991, The bankruptcy code and violations of absolute priority, *Journal of Applied Corporate Finance* 4, 71–78.

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.