

# Volatility Pricing in the Stock and Treasury Markets

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## Abstract

Unexpected shifts in stock market volatility, often associated with financial crises, carry a significantly negative risk premium across the stock and Treasury markets, which suggests the existence of a unified pricing model for these markets. Results show that investors require a premium for holding the risky assets (stocks), which correlate negatively to volatility surprises, while they are willing to pay a premium for holding the safe assets (Treasury bonds), which correlate positively (the so-called "flight-to-safety"). Interestingly, when I break down stocks into large-cap and small-cap, I find that the former have a greater ability to weather volatility surprises, which partially accounts for their lower expected returns.

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Stock market volatility has been related for some time to business cycles and financial crises (Officer (1973), Schwert (1989a, 1989b, 1990), among others). This is because it tends to increase during turbulent times (see Figure 1). More importantly, recent work suggests that market volatility is a pervasive risk factor that explains the cross-section of expected stock returns (Ang, Hodrick, Xing, and Zhang (2006), and Adrian and Rosenberg (2008)). However, events associated with the recent financial crisis attest to the interdependence between financial markets, and to their joint sensitivity to extreme market volatility events.

Additionally, it has been documented that, while positive most of the time, the stock-Treasury bond return correlation becomes negative during recessions or financial crises (Connolly, Stivers, and Sun (2005), Campbell, Sunderam, and Viceira (2009), and Baele, Bekaert, and Inghelbrecht (2009)). The explanation lies in the fact that these assets have different risk-return profiles, that make them natural hedges for each other during times of heightened volatility (see Figure 2).

Furthermore, stocks and bonds should be priced using discounted future cash flows, discounted by the same stochastic discount factor. Accordingly, the literature has been moving towards developing a unified model for the stock and bond markets (Bekaert, Engstrom, and Grenadier (2005), Baker and Wurgler (2008), Bekaert, Engstrom, and Xing (2008), Campbell, Sunderam, and Viceira (2008), Lettau and Wachter (2009), Koijen, Lustig, and Van Nieuwerburgh (2010)).

Therefore, if stock market volatility is a state variable that captures dynamics in the investment opportunity set, then it should price assets across financial markets. Additionally, it should be related to the "flight-to-safety" phenomenon, which occurs in times of economic uncertainty, and induces investors to flee stocks and seek the safety of US Treasury securities, as we have experienced during the recent subprime mortgage meltdown.

I present new evidence consistent with the role played by volatility as a systematic source of

risk. Using data covering the period January 1952 to December 2008 for the 25 Fama-French size- and value-sorted portfolios and the 12 Fama Treasury portfolios, I document that volatility carries a significantly negative risk premium across the stock and Treasury markets. This suggests the existence of a unified pricing model for these markets. Further, I find that investors require a premium for holding the risky assets (stocks), which correlate negatively to volatility surprises, while they are willing to pay a premium for holding the safe assets (Treasury bonds), which correlate positively (the so-called "flight-to-safety").

From an investment perspective, it is important to understand the determinants of the required rates of return across markets. Extending this exercise by including the US Treasury market is even more important, since this market is very large and very important (it reached almost \$11 trillion at the end of my sample period). Furthermore, the role played by volatility across these markets is fundamental, since portfolio decisions are made by allocating funds between stocks and treasuries. To understand the negative volatility premium, it suffices to look at the most turbulent times, when market returns often are highly negative, while volatility rises in response to a series of volatility surprises. Assets with positive exposure to volatility surprises are safer during these times. To insure their portfolios against the double penalty of a declining stock market and a rise in volatility, investors are willing to pay a premium for holding them.

An in-depth analysis of the stock market shows another novel result; volatility explains to a large extent the size anomaly.<sup>1</sup> Specifically, while most of the stocks experience a drop in returns during times of heightened volatility, with small-cap stocks being the most affected, large-cap stocks seem to not be affected, or even to pay off in a couple of cases. The relatively greater ability of the

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<sup>1</sup>See for example Banz (1981), Reinganum (1981), and Fama and French (1992, 2008) for the size anomaly.

latter to weather volatility surprises, such as those often associated with financial crises, partially accounts for their lower expected returns. Across the book-to-market quintiles, I document between 3.20% and 4.32% lower premiums per annum for large-cap stocks versus small-cap stocks.

The Treasury market analysis shows that all of the volatility loadings are positive and statistically significant, and suggests incorporating stock market volatility into Treasury market models. Results imply that Treasuries pay off in times of heightened market volatility, which leads to higher demand and lower expected returns for these securities. For instance, a long-term Treasury bond has a 1.26% lower premium per annum because of its insurance against volatility shocks.

Interestingly, a 2-factor model with market return and volatility surprises has a similar explanatory power in the cross-section of stock returns when compared to a model with predictive variables: *aggregate dividend yield*, *term spread*, *default spread*, and *one-month Treasury-bill yield* (Petkova (2006)) ( $R_{adj}^2$  of 77.53% for the former, and 78.08% for the latter). This result reinforces the role of volatility surprises as an inter-temporal factor in Merton's (1973) Intertemporal Asset Pricing Model (*ICAPM*). But more importantly, the 2-factor model with market return and volatility surprises outperforms the Fama-French 3-factor model in explaining the cross-sectional variation in the size and value-sorted portfolios ( $R_{adj}^2$  of 77.53% versus 70.13%). This performance is impressive, since the test assets are sorted by exactly the characteristics used in building the risk factors in the Fama-French model.

The findings here are robust to the methodology used for capturing volatility surprises. The first approach consists of using the residual from an  $AR(1)$  model fitted to stock market volatility. The second approach uses the difference between predicted volatility (built using an *Asymmetric – GARCH* model) and realized volatility, and provides additional evidence that volatility surprises, however computed, are important sources of risk. The results are also robust when I control for

the *SMB*, *HML*, momentum and aggregate liquidity factors.

This paper, as far as I know, is among the first to look at the relation between equity market volatility and Treasury bond returns. The remainder of the paper is structured as follows. Section 1 presents the asset pricing model and builds unexpected volatility. Section 2 describes the test assets. Section 3 documents the empirical results. Section 4 links unexpected volatility to predictive variables, while Section 5 provides concluding comments. Hereafter, the term volatility refers to unexpected stock market volatility.

## 1 Methodology

The literature has been focusing for a while on stock market volatility as a systematic source of risk. That is because it has a strong countercyclical pattern; it peaks just before or during recessions, and it falls sharply late in recessions or early in recovery periods. Furthermore, stock market volatility has been related to the volatility of macro-economic fundamentals (Officer (1973), Schwert (1989a, 1989b), Hamilton and Lin (1996), Chen (2003), Vayanos (2004), Engle and Rangel (2005), and Engle, Ghysels, and Sohn (2006)), and has been shown to explain the cross-section of expected stock returns (Ang, Hodrick, Xing, and Zhang (2006), and Adrian and Rosenberg (2008)). Since the stock and Treasury markets are interconnected and jointly sensitive to market volatility risk, as evidenced by the most recent financial crisis, it is reasonable to investigate the volatility effect across these markets.

First, I compute stock market volatility,  $V_t$ , as in Anderson et al. (2003). I sum up squared daily market returns within a month, and then I take the square root of this quantity:

$$V_t = \sqrt{\sum_{i=0}^{\Delta} R_{m,t+i}^2}, \quad (1)$$

with  $\Delta$  representing the number of trading days within a month. Data cover the period January 1952 to December 2008. I plot  $V_t$  in Figure 1, while shading recessions and financial crises periods such as the recessions of 1954, 1958, 1961, 1981-1982, 1983, 1991, 2000-2001, the credit crunch of 1966, the Penn Central commercial paper debacle of May 1970, the oil crisis of November 1973, the stock market crash of October 1987, the Asian crisis from 1997, the Russian debt default from 1998, the burst of the hi-tech bubble in 2000, the World Trade attack of September 2001, the accounting scandals of 2002 (Enron, WorldCom), and the financial crisis of 2007-2008 (the bursting of the US housing bubble, accompanied by high default rates on subprime and other adjustable rate mortgages). As expected from a recession and financial crisis proxy, the graph documents that times of economic uncertainty tend to coincide with heightened volatility. Therefore, using such a long sample justifies volatility's role as a state variable proxying for bad time risk.

Next, I assume an  $AR(1)$  model for the stock market volatility, with the residual being the surprise (or, the unexpected) component,  $UV_t$ :

$$V_t = \mu_V + \phi_1^V V_{t-1} + UV_t. \quad (2)$$

In this specification,  $UV_t$  represents the difference between the value of volatility at time  $t$  and its conditional expectation, which is the measure theoretically proposed by Ang. et al (2006).

For comparison reasons, I fit the same model to the updated implied volatility ( $VIX$ ) series downloaded for the period January 1990 to December 2008 from the Chicago Board of Options Exchange (CBOE). Table B1 (see Appendix B) reports the results. The  $AR(1)$  coefficient estimate for the  $VIX$  series is larger than the coefficient estimate for the realized volatility series (the former

is equal to 0.86, while the latter is equal to 0.68). This explains why taking simple innovations in VIX, as in Ang et al. (2006), suffices to capture unexpected moves in this index, but the same approach would not adequately capture the unexpected component of the stock market volatility<sup>2</sup>.

Next, based on Merton's (1973) model, I assume that the cross-section of asset returns has two sources of risk. The first source is the classic market beta, and the second one is the exposure to a state variable associated with unexpected volatility. Formally:

$$E[R_i^e] = \lambda_m \beta_i^m + \lambda_{UV} \beta_i^{UV}, \quad i = 1, 2, \dots, n, \quad (3)$$

where the regressors in equation (3) are the slope coefficients in the return generating process:

$$R_{i,t}^e = \alpha_i + \beta_i^m R_{m,t}^e + \beta_i^{UV} UV_t + \epsilon_{i,t}, \quad i = 1, 2, \dots, n, \quad (4)$$

with  $n$  being the number of test assets. The  $R_i^e$  and  $R_m^e$  represent excess returns on the asset  $i$  and the market portfolio, respectively. Following Ang et al. (2006), I assume time invariant betas in equation (4).

To check for robustness in volatility pricing effects, I build a second (unexpected) volatility factor, that is the difference between predicted and realized volatility. To predict volatility, I use an underlying model that captures the following stylized facts: volatility increases after a drop in stock prices (Black (1976), French, Schwert, and Stambaugh (1987), Nelson (1990, 1991)), and is persistent (Schwert (1989b), French, Schwert, and Stambaugh (1987)). In addition, financial data usually produce non-normal residuals, which is a consequence of the leptokurtosis in returns. Furthermore, Black (1976), among others, pointed out an asymmetry in the stock market data,

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<sup>2</sup> Actually, fitting the AR(1) model to VIX over the sample used in Ang et al. (2006) leads to an AR(1) coefficient estimate of 0.98.

with negative shocks to returns driving up volatility. Also, a positive autocorrelation of order one is usually present in an index' return series (Campbell, Lo and MacKinley (1997)). To address all these issues, I use a *GJR Asymmetric Student – GARCH* model with an *AR(1)* specification in the mean equation:<sup>3</sup>

$$\begin{aligned}
 R_{m,t} &= \mu + \rho R_{m,t-1} + \eta_t \\
 \eta_t &= \varepsilon_t \sqrt{h_t}, \\
 \varepsilon_t / I_{t-1} &\sim Student(v), \quad t = 1, \dots, T. \\
 h_t &= \alpha + \phi h_{t-1} + \theta^+ \eta_{t-1}^{2+} + \theta^- \eta_{t-1}^{2-}, \\
 \eta_t^{2+} &= \eta_t^2 1_{\{\eta_t > 0\}}, \quad \eta_t^{2-} = \eta_t^2 1_{\{\eta_t < 0\}}
 \end{aligned} \tag{5}$$

The model described by equation (5) accommodates the asymmetry in the news impact curve. The degrees of freedom variable for the *Student-t* distribution,  $v$ , is meant to capture the excess kurtosis present in the market return data. Following Bauwens and Lubrano (1998), I consider a *Half-Cauchy* prior for  $v$  and flat priors on finite intervals for all the other parameters (see Appendix A for details on the model).

To estimate equation (5), I employ the *Griddy – Gibbs sampler*, which is a very popular *Markov Chain Monte Carlo* method (see Appendix B for details on the estimation). Then, I use the posterior estimates from model (5) to predict stock market volatility (see the posterior estimates in Table B2 of Appendix B). I plot in Figure 1 this series next to the realized volatility series. The two volatility series track each other closely, result due to the *GARCH* predicting equation being a weighted average of past squared returns, with slowly declining weights.

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<sup>3</sup>Partial surveys of the big literature on volatility models can be found in Pagan and Schwert (1990), Bollerslev, Chou and Kroner (1992), Engle (1982), Hentschel (1995), Ghysels, Harvey and Renault (1996), Campbell, Lo and MacKinley (1997, Chap. 12) and Tsay (2001), among others.

## 2 The Test Assets

From an investment perspective, understanding the role played by volatility across markets is fundamental, since portfolio decisions are made by allocating funds between stocks and treasuries. These assets have different risk-return profiles that make them natural hedges for each other in bad economic times. It is useful thus to look at the realized correlation between the stock and Treasury markets, to have a better understanding of what happens during recessions and financial crises. Therefore, using 1-year rolling windows, I compute the correlation between the return on the stock market portfolio and the return on the 10-year Treasury bond, on a monthly basis, over the period January 1952 to December 2008. I plot the time series of the realized stock-bond return correlations in Figure 2. The graph shows that, while positive most of the time, the correlation becomes negative during recessions or financial crises. This result is in line with the results in Campbell, Sunderam, and Viceira (2009) (see Figure 4 in their paper), and with the results in Baele, Bekaert, and Inghelbrecht (2009) (see Figure 1 in their paper), and motivates me to formally investigate volatility pricing effects across these financial markets.

The Treasury market test assets are the 12 Fama Treasury bond portfolios with a maturity interval of six months. The returns are downloaded from CRSP. The first portfolio has return maturities from one to six months, while the last one has return maturities greater than one hundred and twenty months. Only non-callable, non-flower notes and bonds are included in these portfolios. Portfolio returns are computed as an equal-weighted average of the unadjusted holding period returns of individual bonds. The stock market test assets are the 25 size- and value-sorted portfolios of Fama and French (1992, 1993). Value-weighted returns are obtained from Kenneth French's Web site at Dartmouth.

If stock market volatility proxies for the common underlying sources of macroeconomic risk, then it should be priced across these financial markets. Further, if the test assets in one of the markets have a factor structure (as may be the case with the stock market portfolios), and if the variables tested have some correlation with these factors, then one obtains a good model fit (see Lewellen, Nagel and Shanken (2006)). To address these points, the pricing implications of the 2-factor model (3) are investigated across the two financial markets.

I report the average excess returns for the test assets in Table I. I present the Treasury market results in Panel A. According to the liquidity preference hypothesis, bond returns should increase with maturity, because long-term bonds are more sensitive to interest rate risk. This monotonic pattern can be observed in my sample, with average bond excess returns (in excess of the one-month T-bill rate) increasing with maturity from 4bp to 22bp per month, all statistically significant.

Panel B presents the stock market results. With the exception of two of the small-growth portfolios, the returns are statistically significant. Consistent with the previous literature, stocks sorted based on size and book-to-market over the period 1952-2000 display a sizeable dispersion in returns. I focus here on the size anomaly. Except for the growth stocks, there is a roughly monotonic pattern in portfolio returns in the size dimension, with the small-cap stocks earning historically larger average excess returns when compared to the large-cap stocks.

### **3 Empirical Results**

The first step in the asset pricing exercise is to estimate the factor loadings in the first-pass regression (4). Table II Panel A) reports the *OLS* estimates with autocorrelation and heteroskedasticity robust standard errors for the Treasury market. I estimate positive and significant market betas in Panel

A across all bond portfolios. Since model (4) assumes time-invariable market betas, and since the stock-Treasury bond correlation is positive most of the time (see Figure 2), it is not surprising that the estimated market betas for Treasury securities are positive. Further, I find an increasing pattern in market betas across Treasury bonds, which is in line with the findings in Fama and French (1993). Specifically, long maturity bonds have a market beta of 0.11 ( $t\text{-stat} = 3.26$ ), while short maturity bonds have a market beta close to zero (expected, since the latter are considered to be riskless securities).

Interestingly, I find that all of the volatility loadings are positive and statistically significant, which translates into treasuries paying off in times of increased economic uncertainty, when volatility rises. Further, I find a strictly increasing pattern in volatility loadings across the different maturity bonds, with short maturity bonds having a volatility beta of 0.01, while long maturity bonds have a volatility beta of 0.17. As a side note, while the 2-factor model fits the long-end of the Treasury curve well, leading to an insignificant intercept in the time series, it does not do such a great job on the short-end.

The above results imply that long maturity bonds provide a hedge against volatility risk. Therefore, they should be in high demand, and should have lower expected returns when compared to short-maturity bonds. Why do we then observe higher rates of return required by investors on long maturity Treasuries? The answer is that long term bonds also have a higher market beta, which makes them relatively riskier when compared to short term bonds.

Table II Panel B) reports the estimates for the stock market. All of the market betas are positive and statistically significant, and controlling for value, small-cap stocks have larger market betas than large-cap stocks. Thus, small-cap stocks are more affected by downside risk. Meanwhile, the majority of the volatility loadings are negative and statistically significant. The latter result was

expected, since stocks experience low returns during turbulent times.<sup>4</sup>

Interestingly, I find that volatility loadings increase with size in a strictly monotonic fashion. Small-cap stocks have a significantly negative volatility exposure, while large-cap stocks are less affected by volatility surprises, holding the market return fixed. This result is in align with the findings in Coval and Shumway (2010). Further, there is a sizeable spread in volatility loadings between small-cap and large-cap stocks. For example, among the growth firms, the small-cap stocks have a volatility beta of -0.42 ( $t\text{-stat} = -2.71$ ), while the large-cap stocks have a beta of 0.16 ( $t\text{-stat} = 4.02$ ). Among the value firms, the small-cap stocks have a volatility beta of -0.50 ( $t\text{-stat} = -5.14$ ), while the large-cap stocks have a beta of 0.02 ( $t\text{-stat} = 0.21$ ). Therefore, small-cap stocks also have a larger exposure to volatility risk. Note that, while the model outlined in equation (4) explains between 59% and 89% of the time series variation in portfolio excess returns, there is no clear picture regarding volatility loadings in the value dimension.

Figure 3 plots the average portfolio excess returns in Panel A, and the volatility loadings in Panel B. The graphs show a nice alignment in the size dimension between portfolio returns and volatility loadings. The two panels visually capture the fact that small-cap stocks require a higher rate of return, while they have a larger exposure to volatility risk.

Note that, since both stocks and bonds have positive exposure to market risk, *CAPM* can not explain by itself the "flight-to-safety" phenomenon. Instead, it is the opposite exposure to volatility risk that partially accounts for this effect.

All these findings propel me to next formally investigate the volatility pricing effects across the stock and Treasury markets. To this end, I estimate factors' risk premiums using Fama-MacBeth (1973) regressions of excess returns on factor betas in the second-pass regression (3). I use the

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<sup>4</sup>This finding is also in Adrian and Rosenberg (2008), and Moise (2002).

statistical distribution of the estimated prices of risk to assess whether these factors are priced in the cross-section of stock returns. Although my results mitigate the "errors-in-variables in the regressors" problem by using returns from broad-based portfolios as units of observation, I also report *t-statistics* based on the Shanken (1992) correction, to address concerns arising from this issue.

If the factors form a basis for the space of returns, and if the factors are traded in the market place, then their risk prices should be close in value to their means (Cochrane (2001)).<sup>5</sup> It is reassuring to see that this is indeed the case here. Table III reports the risk premium estimates. The first row of this table together with Table IV Panel A highlight a novel result; the estimated market risk premium (equal to 49 bp per month,  $t\text{-stat} = 2.71$ ) is similar to the realized analog. This is a result that the literature has had a hard time obtaining. My finding may be due to the fact that the stock market is incomplete, and adding treasuries to it leads to a better performance of asset pricing models, which translates into an estimated market risk premium that makes economic sense. I document another interesting result in the first row: the *CAPM* explains a good part of the cross-sectional variation in returns in the joint markets ( $R_{adj}^2$  is equal to 55.45%).

The second row in Table IV supports the role of volatility as a risk factor priced across financial markets. Although there is on average a significant trade-off between market risk and return over the period January 1952 to December 2008 across these markets, a non-beta measure of risk like volatility plays an important and apparently systematic role, with a noticeable influence on assets' expected returns. Controlling for market risk, volatility has an estimated risk premium of -62bp per month ( $t\text{-stat} = -2.30$ ).

My results suggest that during bad times investors require a premium for holding the risky

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<sup>5</sup>I refer only to traded factors here, thus excluding the non-traded volatility factor.

assets (stocks), which correlate negatively with volatility, and they are willing to pay a premium for holding the safe assets (Treasury bonds), which correlate in a positive fashion (the so-called "flight-to-safety"). This implies that in the *ICAPM* world the market portfolio is not mean-variance efficient with respect to the universe of common stocks, and suggests adding a position that takes into account stock market volatility. Since investors are not fully insured against systematic volatility risk, the premium they pay for assets that covary positively with volatility reflects their attempts to reduce this risk exposure. For instance, Treasury bonds have lower premiums since they pay off in times of heightened volatility. As an example, a long-term Treasury bond has a 1.26% ( $=12*0.17*(-0.62)$ ) lower premium per annum because of its insurance against volatility shocks. In the stock market results suggest that investors demand large-cap stocks during turbulent times, since small-cap stocks expose them to larger downside and volatility risks. In accordance to rational pricing, this hedging demand drives up the price for such stocks, and leads to lower expected returns. Specifically, large-cap stocks have between 3.20% ( $=12*(-0.38-0.05)*(-0.62)$ ) and 4.32% ( $=12*(-0.42-0.16)*(-0.62)$ ) lower premium per annum when compared to small-cap stocks, across the book-to-market quintiles. Overall, my results are in line with the findings in Baker and Wurgler (2008), who show that government bonds comove most strongly with stocks of large, mature firms.

### **3.1 *Controlling for other Risk Factors***

Fama and French (1996) rely on their three-factor model with market return, *SMB* and *HML* for explaining stock return anomalies related to firm characteristics.<sup>6</sup> Other factors proposed as

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<sup>6</sup>HML and SMB are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). HML is the difference between high book-to-market-stocks portfolios and low book-to-market-stocks portfolios, with similar

candidates for state variables within the *ICAPM* framework are the momentum factor (*MOM*) of Jegadeesh and Titman (1993) and the liquidity factor of Pastor and Stambaugh (2003).<sup>7</sup> Innovations in the latter factor are denoted here by *IPS*. Since the data for *IPS* are available only starting with August 1962, tests involving this factor span a slightly shorter time period.

I document the average returns for the traded risk factors in Table IV Panel A, and I present the cross-correlation matrix for these factors and volatility in Panel B. There is a significantly negative correlation between volatility and market return (equal to  $-39\%$ ), which was expected since rises in volatility tend to correspond to downturns in the market. There is also a significant correlation between volatility and *SMB*, although not large in value (equal to  $-29\%$ ). This is in line with the results reported in the previous section, that show that returns of small stocks are more (negatively) affected by volatility risk. There is also a significant correlation between volatility and aggregate liquidity (equal to  $-43\%$ ), which suggests that times of heightened market volatility tend to coincide with times of low aggregate liquidity, as we have recently experienced.

Next, I extend model (4) to:

$$R_{i,t}^e = \alpha_i + \beta_i^m R_{m,t}^e + \sum_s \beta_i^s F_{s,t} + \epsilon_{i,t}, \quad i = 1, 2, \dots, n, \quad (6)$$

where  $\beta_i^s$  represents the loading associated with the generic factor  $s$ .

As evidenced in Figure 1, financial crises are times of heightened volatility. But another important aspect of financial crises, especially present in the most recent one, is a reduction in market liquidity, as there is usually an increased risk aversion among market makers. Therefore, a detailed

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weighted-average size, while *SMB* is the difference between the returns on small-stocks portfolios and those of big-stocks portfolios, with similar weighted-average book-to-market equity.

<sup>7</sup>MOM is built as the average return on the two high prior (months 2-12) return portfolios minus the average return on the two low prior return portfolios.

look at a time series model that also controls for liquidity deserves further attention.

In the Treasury market, the question is whether stock market liquidity has an effect in the time series of bond returns in the presence of volatility risk, especially in light of recent literature that documents that a stock market based liquidity measure like *IPS* is priced in the cross-section of Treasury bond returns (Li, Wang, Wu, and He (2009)). Analysis shows opposite signs for the loadings of volatility and liquidity on bond returns, result due to the fact that heightened volatility states of the world tend to coincide with lower liquidity ones. More importantly, volatility continues to have a significant effect in the time series of Treasury bond returns, while liquidity seems to lose its explanatory power in the presence of volatility risk. The only Treasury bonds where liquidity still has significant loadings are the short maturity ones (up to 12 months).<sup>8</sup>

In the stock market, I find that, except for the quintile of large-cap stocks, value stocks have larger liquidity loadings, which suggests that the value anomaly is related to some extent to liquidity risk.<sup>9</sup><sup>10</sup> However, with one exception, these loadings are statistically insignificant. Interestingly, the strictly increasing pattern in volatility loadings in the size dimension continues to be present when liquidity is included in the model. Additionally, the statistical significance of the volatility loadings is unchanged. Interestingly, although not strictly monotonic, I also find a slightly increasing pattern in volatility loadings in the value dimension, with value stocks being more affected by volatility risk, when liquidity is included in the model.

In the next step, I perform cross-sectional tests, where I control for the effects of the risk factors

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<sup>8</sup>Results are not reported here in the interest of brevity.

<sup>9</sup>Results are not reported here in the interest of brevity.

<sup>10</sup>Seminal references include Graham and Dodd (1934), Basu (1977, 1983), Ball (1978), and Rosenberg, Reid, and Lanstein (1985) for the value anomaly.

mentioned above. To this end, I extend the asset pricing model in equation (3) to:

$$E[R_i^e] = \lambda_m \beta_i^m + \sum_s \lambda_s \beta_i^s, \quad (7)$$

where  $\lambda_s$  and  $\beta_i^s$  represent the risk premium and respectively, the loading associated with the generic factor  $s$  in the corresponding time series model (6). I present the estimates of the parameters in equation (7), for various values of  $s$ , in Table III, rows three through six. While the estimated market risk premium continues to be positive and statistically significant, the volatility pricing effect is also robust across the different model specifications. Its premium ranges from -62bp to -125bp per month.<sup>11</sup> However, a stringent test consists of assessing the empirical performance of volatility in the presence of the Fama-French factors. I find that volatility continues to have a significant risk premium of -0.87bp per month ( $t$ -stat = -2.62) when the *SMB* and *HML* factors are included in the model. Across all model specifications, I find that *SMB* has an insignificant risk premium, while *HML*, *MOM* and *IPS* have significant risk premiums. The estimated risk premiums for the traded factors are close to the realized analogs (the exception being the *MOM* factor, result probably due to the factors not being completely orthogonal to each other). Further, the volatility effect seems to be orthogonal to the effects of the *SMB* and *HML* factors, suggesting that volatility is a novel source of risk.

The  $R_{adj}^2$  statistic from the cross-sectional regression of average excess returns on the risk factors' loadings is used for comparing the relative performance of the different asset pricing models (Jagannathan and Wang (1996)). Results show some increase in the model's explanatory power when *CAPM* is augmented with volatility, from 55.45% to 60.68%. Volatility also slightly enhances the explanatory power of the Fama-French (1992, 1993) model, from 86.95% to 88%.

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<sup>11</sup>Market volatility is not a traded factor. Nonetheless, one can project volatility on the space of returns to obtain a traded factor, as in Moise (2002), or as in Ang et al. (2006).

### 3.2 Robustness Checks

This section ensures that results are robust with respect to the volatility measure used. To this end, I build a second (unexpected) volatility factor, that is the difference between predicted and realized market volatility, as described in Section 1. I call it  $PRED\_UV$ . I present the correlation of this new (unexpected) volatility factor with the other risk factors in Table IV. Similar to the first volatility measure, it has a negative and significant correlation with the  $SMB$  and  $IPS$  factors (-27% and -41%, respectively).

I plot the loadings of this new volatility factor on the 25 size- and value-sorted portfolios in Figure 3, Panel C. A visual inspection of this graph shows an increasing, strictly monotonic pattern in the volatility loadings on stock returns, when going from small-cap to large-cap firms. Again, small firms have a larger exposure to volatility risk.

Next, I report the results for the asset pricing tests performed using the new volatility measure in the last two rows of Table III. I estimate a significant risk premium for the second volatility factor, equal to -84bp per month ( $t\text{-stat} = -2.81$ ) in a 2-factor model specification that controls for the market beta, and -91bp per month ( $t\text{-stat} = -2.21$ ) in a 5-factor model specification that also controls for the  $SMB$ ,  $HML$  and momentum effects. As in Section 3.2, I find that  $SMB$  has an insignificant risk premium, while the rest of the factors have significant risk premiums. I conclude that my previous findings regarding the volatility pricing effect across financial markets, as well as its relation to the size anomaly, are robust with respect to the type of volatility measure used.

## 4 Horse-Race Tests in the Stock Market

In Merton's (1973) *ICAPM* the risk premiums are associated with the conditional covariances between asset returns and innovations in state variables that describe the time-variation in the investment opportunity set. Using *the aggregate dividend yield, the term spread, the default spread,* and *the one-month Treasury-bill yield* as predictors of stock returns, the literature has shown that the Fama-French factors lose their explanatory power for the cross-section of stock returns in the presence of these state variables (Hahn and Lee (2006), and Petkova (2006)). Therefore, the next test consists of comparing the explanatory power of volatility to that of a model with these predictive variables. To this end I construct the following: *the default spread, DEF* (the difference between the yields of a long-term corporate Baa bond and a long-term corporate Aaa bond); *the term spread, TERM* (the difference between the yields of a thirty-year and a one-year government bond); *the dividend yield* on the S&P500 value-weighted portfolio, *DY* (the sum of dividends over the last 12 months, divided by the level of the index); and *the one-month T-bill yield, R<sub>f</sub>*. The data series are downloaded from CRSP for the period January 1952 to December 2008. Similar to the previous literature, I build the shocks to the predictive variables as residuals from a first-order *VAR* system.

In the first step, I analyze the joint distribution of shocks to *DEF, TERM, DY, R<sub>f</sub>* and volatility. Table V Panel A presents the summary statistics for the actual predictive variables, while Panel B documents estimates from a regression of volatility on the contemporaneous shocks to the predictive variables. I find that *DEF, R<sub>f</sub>* and *DY* are significantly correlated with volatility (the first two variables are positively correlated, and the third one is negatively correlated). This result implies that volatility spans part of the set of the forward-looking state variables, and justifies its role as

an inter-temporal hedging factor in the *ICAPM*.

In the second step, I compare the performance of a two-factor model, consisting of market return and volatility, to the performance of a model with predictive variables. I report the results in Table V Panel C. Although most of the predictors are insignificant in this sample, a model with predictive variables explains a large portion of the cross-sectional variation in stock returns ( $R_{adj}^2$  of 78.08%).<sup>12</sup> Interestingly, the two factor model compares favorably to this model, having just a slightly lower  $R_{adj}^2$  (77.53% versus 78.08%).

I also find that, controlling for market risk, volatility has a significant risk premium of -80bp per month ( $t\text{-stat} = -2.59$ ) in the cross-section of stock returns. But more importantly, a 2-factor model with market return and volatility outperforms the Fama-French 3-factor model in explaining the cross-sectional variation in the size and value-sorted portfolios ( $R_{adj}^2$  of 77.53% versus 70.13%). This performance is impressive, given that the test assets are sorted by exactly the characteristics used in building the risk factors in the Fama-French model. Also, the volatility effect is robust with respect to the *SMB* and *HML* factors. Its pricing effect is orthogonal to those of these factors, and by augmenting the Fama-French 3-factor model with volatility, a model with higher explanatory power is achieved ( $R_{adj}^2$  increases from 70.13% to 77.21%).

## 5 Conclusions

The stock and Treasury markets are interconnected, and they are jointly sensitive to unexpected shifts in stock market volatility. For instance, the recent financial market turmoil, which has

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<sup>12</sup>Multicollinearity may be the explanation behind this result. However, since the point of this exercise is to compare the 2-factor model with the model with predictive variables used in the existing literature, I keep all the predictors in the model.

translated into heightened volatility, has spurred investors to flee stocks and seek the safety of US Treasury securities, which in turn led to one of the worst periods for holding stocks to coincide with one of the best periods to hold Treasuries. These observations motivated me to formally investigate volatility pricing effects across these financial markets.

I find that volatility carries a significantly negative risk premium across the stock and Treasury markets. This implies that, when faced with a downturn in the economy that results in tighter credit markets, investors are willing to forgo expected returns to get downside protection. Further, I find that investors are willing to pay a premium for holding the safe assets (Treasury bonds), which correlate positively to volatility surprises and thus, provide insurance against volatility shocks, while they require a premium for holding the risky assets (stocks), which correlate negatively (the so-called "flight-to-safety").

Interestingly, asset pricing tests performed in the joint markets lead to an economically sound estimate for the market risk premium (which is positive, while the literature usually finds it to be negative), close in value to the realized analog. They also show that the cross-markets volatility effect is robust with respect to other risk factors and to the type of volatility measure used.

A detailed analysis of the stock market shows that large-cap stocks seem to be less affected by volatility risk. Their relatively greater ability to weather volatility surprises, such as those often associated with financial crises, accounts for their lower expected returns. More importantly, I find that a 2-factor model with market return and volatility surprises outperforms the Fama-French 3-factor model in explaining the cross-sectional variation in the 25 size- and value-sorted portfolios.

In sum, my findings suggest that the size premium has been tied to economic fundamentals, and that the same premium is found in the term premium of the Treasury curve for the more than 50 years over which I have data. Merton's (1973) model suggests using factors that capture

unanticipated changes in the opportunity set as hedging instruments. Volatility is clearly related to such changes. By capturing the key role played by volatility in asset pricing as a hedging factor, and by extending its pricing implications outside the stock market, I have taken one step forward in showing that a unified asset pricing model may be build for the stock and Treasury markets.

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## Appendix A: The Volatility Model

The model used in this study is the *GJR model* of Glosten, Jagannathan and Runkle (1993), which is designed to introduce asymmetry into the model:

$$\begin{aligned}
 R_{m,t} &= \mu + \rho R_{m,t-1} + \eta_t \\
 \eta_t &= \varepsilon_t \sqrt{h_t}, \\
 \varepsilon_t / I_{t-1} &\sim \text{Student}(v), \quad t = 1, \dots, T. \\
 h_t &= \alpha + \phi h_{t-1} + \theta^+ \eta_{t-1}^{2+} + \theta^- \eta_{t-1}^{2-}, \\
 \eta_t^{2+} &= \eta_t^2 1_{\{\eta_t > 0\}}, \quad \eta_t^{2-} = \eta_t^2 1_{\{\eta_t < 0\}}
 \end{aligned} \tag{A1}$$

The distribution of  $R_{m,t}$  is *Student* with mean zero and variance  $h_t^{1/2} v / (v - 2)$  given past information  $I_{t-1}$  and assuming  $v > 2$ . The  $\varepsilon_t$  sequence is independent and the initial variance is a known constant. Let  $\gamma$  denote the parameter vector in this model, with  $\gamma = (\mu, \rho, \alpha, \theta^+, \theta^-, \phi, v)$ . The posterior density for a sample of  $T$  observations is given by

$$\varphi(\gamma/R) \propto \varphi(\gamma) l(\gamma/R), \tag{A2}$$

with the likelihood function given by

$$l(\gamma/R) \propto \prod_{t=1}^T \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})} (v h_t^{1/2})^{-1/2} \left[ 1 + \frac{R_t^2}{v h_t^{1/2}} \right]^{-\frac{v+1}{v}}, \tag{A3}$$

where the prior density,  $\varphi(\gamma)$ , needs to respect the positivity restrictions on the parameters and the condition  $\phi < 1$ . Integrability of the posterior density depends in part on the integrability of the prior density. Given an integrable (or proper) prior and a non-pathological likelihood, the posterior will also be integrable. Examining the likelihood function (A3) it can be seen that, if  $h_t^{1/2}$

is strictly positive, since the *Student* density is finite and positive, no pathology appears. However, the posterior density of  $v$  is not integrable if one were to use a flat prior (see Bauwens and Lubrano (1998)). For the posterior density of  $v$  to be integrable, the prior information must be such that the posterior is forced to go to zero quickly enough in the tail. The prior at the right tail should be at least  $O(v^{1+d})$ , with  $d$  being small and positive, e.g.  $1/v^2$  (improper prior obtained by being flat on  $1/v$ ). This prior must be truncated to the interval  $(m, \infty)$ , with  $m$  being small and positive, to avoid causing problems at the left tail. This approach avoids the problem of  $l(\gamma/R)/v^2$  approaching infinity as  $v$  approaches zero. Therefore, for a proper prior for  $v$ , I use a *half – right Cauchy* centered at 0:<sup>13</sup>

$$\varphi(v) \propto (1 + v^2)^{-1} \quad (v > 0). \tag{A4}$$

For the rest of the parameters in the model I use a uniform prior.

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<sup>13</sup>Some of the other possibilities for the prior on  $v$  include a flat prior on  $v$  over a finite range  $(0, M)$ , and also an exponential density (Geweke (1993)), which uses a subjective parameter chosen to fix the prior mean and variance of  $v$  (see Bauwens and Lubrano (1998) for a detailed discussion).

## Appendix B: The *Griddy-Gibbs Sampler*

The *Gibbs sampler* of Geman and Geman (1984) and Gelfand and Smith (1990) is a very popular *MCMC* method. Let  $\theta_1, \theta_2, \dots, \theta_n$  be a set of parameters that need to be estimated,  $X$  the available data, and  $M$  the model entertained. Suppose that the conditional distributions of each parameter given the others,  $f_i(\theta_i/\theta_{j \neq i}, X, M)$  are known, but the likelihood function of the model is hard to obtain. What I do is to draw a random number from each of these conditional distributions. For instance, if  $n = 3$ , let's consider  $\theta_{2,0}$  and  $\theta_{3,0}$  two arbitrary starting values of  $\theta_2$  and  $\theta_3$ . Then

1. I draw a random sample  $f_1(\theta_1/\theta_{2,0}, \theta_{3,0}, X, M)$ , call it  $\theta_{1,1}$ ;
2. I draw a random sample  $f_2(\theta_2/\theta_{3,0}, \theta_{1,1}, X, M)$ , call it  $\theta_{2,1}$ ;
3. I draw a random sample  $f_3(\theta_3/\theta_{2,1}, \theta_{1,1}, X, M)$ , call it  $\theta_{3,1}$ .

This is a *Gibbs iteration*. The iteration can be repeated for  $n$  times, with  $n$  sufficiently large such that  $m < n$  initial random draws can be discarded. I get the *Gibbs sample* this way,  $(\theta_{1,m+1}, \theta_{2,m+1}, \theta_{3,m+1}), \dots, (\theta_{1,n}, \theta_{2,n}, \theta_{3,n})$ , which can be used to obtain the point estimates and the variances of the three parameters.

In the case when the conditional posterior distributions of the parameters don't have closed-form expressions, the *Gibbs sampler* implementation can become complicated. But Ritter and Tanner (1992) have a method to obtain draws in this case. It is called the *Griddy-Gibbs sampler*:

1. I choose a grid of points from a properly selected interval of  $\theta_i$ , say  $\theta_{i1} \leq \theta_{i2} \leq \dots \leq \theta_{im}$ . I evaluate the conditional posterior density function to obtain  $w_j = f(\theta_{ij}/\theta_{lk \neq ij}, X, M)$  for  $j = 1, \dots, m$ ;

2. I use  $w_1, \dots, w_m$  to obtain an approximation to the inverse cumulative distribution function of  $f(\theta_{ij}/\theta_{lk \neq ij}, X, M)$ ;

3. I draw a *Uniform*(0, 1) random variate and I transform the observation via the approximate inverse *CDF* to obtain a random draw for  $\theta_i$ .

The usual *Gibbs sampler* cannot be applied to the *GARCH* model even if the error term is (conditionally) normal. It requires analytical knowledge of the full conditional posterior densities. Regression models with *GARCH* errors do not contain this knowledge. To handle this, I apply a unidimensional deterministic integration rule to each coordinate of the posterior density in combination with the *Gibbs sampler*, as described by Bauwens and Lubrano (1998). The random draws of the joint posterior are then obtained by evaluating and inverting the full conditional densities.

I keep 5000 draws in the *Griddy – Gibbs sampler* and I consider the 1000 initial draws as the burn-in sample. The grid I do the search over is similar to the one used by Bauwens and Lubrano (1998):  $\mu \times \rho \times \alpha \times \phi \times \theta^+ \times \theta^- \times v \in (-0.60, 0.94) \times (0.00, 0.40) \times (0.01, 0.90) \times (0.35, 0.95) \times (0.00001, 0.50) \times (0.01, 0.70) \times (0.01, 30)$ . I report the posterior estimates in Table B1. The extent to which a volatility shock today feeds through into the next period's volatility is equal to 0.75. The leverage hypothesis of Black (1976) is also supported by the results, with only the coefficient for the negative shocks to returns being precisely estimated.

**Table B1**  
**Market Volatility**

This table presents the estimates of an AR(1) model applied to market volatility:  $V_t = \mu_V + \phi_1^V V_{t-1} + \varepsilon_t$ . I use the updated implied volatility series, VIX, available for the period January 1990 to December 2008, obtained from CBOE, and the realized market volatility series for the period January 1952 to December 2008. The latter measure is built using daily market return data from NYSE, AMEX, and NASDAQ maintained by CRSP. It is given by  $V_t = \sqrt{\sum_{i=0}^{\Delta} R_{m,t+i}^2}$ , where  $R_{m,t}$  denotes daily return on the stock market portfolio and  $\Delta$  represents the number of trading days in a given month. The estimation is done using the maximum likelihood method. The *t-statistics* are reported in parentheses.

	<i>Implied Volatility (VIX)</i>	<i>Realized Market Volatility</i>
$\hat{\mu}_V$	20.28 (11.16)	3.55 (18.99)
$\hat{\phi}_1^V$	0.86 (24.19)	0.68 (23.55)

**Table B2**  
**Posterior Estimates for the Asymmetric Student-GARCH(1,1) Model**

The model used is:

$$\left\{ \begin{array}{l} R_{m,t} = \mu + \rho R_{m,t-1} + \eta_t \\ \eta_t = \varepsilon_t \sqrt{h_t} \\ \varepsilon_t / I_{t-1} \sim \text{Student}(0,1,\nu) \quad , t=1, \dots, T. \\ h_t = \alpha + \phi h_{t-1} + \theta^+ \eta_{t-1}^{2+} + \theta^- \eta_{t-1}^{2-} \\ \eta_t^{2+} = \eta_t^2 1_{\{\eta_t > 0\}}, \eta_t^{2-} = \eta_t^2 1_{\{\eta_t < 0\}} \end{array} \right.$$

The  $R_t$  represents the monthly time series of returns on the stock market portfolio for the period January 1952 to December 2008. The results are computed using a *Griddy-Gibbs* sampling algorithm in which 5000 draws are kept and 1000 initial draws are the burn-in sample. A *flat prior* on finite intervals is used on all parameters except for the *prior* on  $\nu$ , which is *half-Cauchy*. *Standard errors* are reported in the parentheses.

	<i>Estimate</i>	<i>Std Err</i>
$\mu$	0.20	(0.03)
$\rho$	0.05	(0.03)
$\alpha$	0.06	(0.03)
$\theta^+$	0.05	(0.04)
$\phi$	0.75	(0.07)
$\nu$	8.61	(2.51)
$\theta$	0.20	(0.06)

**Table I**  
**Average Excess Returns for the Test Assets:**  
**The 25 Fama-French Portfolios and the 10 Fama Treasury Bond Portfolios**

Panel A reports the monthly average excess returns for the 10 Fama maturity-sorted Treasury bond portfolios while Panel B reports the monthly value-weighted average excess returns for the 25 size- and value-sorted portfolios of Fama and French (1992), in percentages. Data cover the period January 1952 to December 2008. The *t*-statistics are reported in parentheses.

*Panel A) Average Excess Returns for the Treasury Market Portfolios*

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<i>Maturity</i>											
<i>1-6 mo</i>	<i>7-12 mo</i>	<i>13-18 mo</i>	<i>19-24 mo</i>	<i>25-30 mo</i>	<i>31-36 mo</i>	<i>37-42 mo</i>	<i>43-48 mo</i>	<i>49-54 mo</i>	<i>55-60 mo</i>	<i>61-120 mo</i>	<i>&gt;120 mo</i>
0.04	0.06	0.09	0.09	0.11	0.12	0.13	0.13	0.14	0.11	0.14	0.22
(7.33)	(4.74)	(4.13)	(3.44)	(3.18)	(3.06)	(2.93)	(2.77)	(2.77)	(1.94)	(2.24)	(2.06)

*Panel B) Average Excess Returns for the Stock Market Portfolios*

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<i>Book-to-Market Equity (BE/ME) Quintiles</i>											
<i>Size Quintiles</i>	<i>Low</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>High</i>	<i>Low</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>High</i>	
<i>Average Excess Returns</i>						<i>t-stats</i>					
<i>Small</i>	0.19	0.73	0.76	0.98	1.06	(0.63)	(2.89)	(3.52)	(4.81)	(4.89)	
<i>2</i>	0.36	0.66	0.86	0.91	1.01	(1.37)	(3.05)	(4.45)	(4.78)	(4.76)	
<i>3</i>	0.47	0.71	0.75	0.84	0.96	(1.92)	(3.61)	(4.15)	(4.70)	(4.87)	
<i>4</i>	0.52	0.55	0.73	0.78	0.80	(2.41)	(2.93)	(4.02)	(4.39)	(3.99)	
<i>Big</i>	0.45	0.52	0.55	0.55	0.63	(2.51)	(3.12)	(3.43)	(3.38)	(3.44)	

**Table II**  
**Estimates of an Unconditional ICAPM**

The test assets are the 12 Fama maturity-sorted Treasury portfolios and the 25 Fama-French (1992) portfolios sorted on size and book-to-market equity. The time series of portfolios' excess returns ( $R_{i,t}^e$ ) are regressed on the excess market return ( $R_{m,t}^e$ ) and unexpected volatility,  $UV_t$ :

$$R_{i,t}^e = \alpha_i + \beta_i^m R_{m,t}^e + \beta_i^{UV} UV_t + \varepsilon_{i,t}, \quad i = 1, \dots, 37.$$

$UV_t$  is the residual from an AR(1) model applied to the time series of realized stock market volatility. Data cover the period January 1952 to December 2008. The estimation is done using *OLS*. The *t*-statistics are based on autocorrelation and heteroskedasticity corrected standard errors, and are reported in parentheses.

<i>Panel A) Factor Loadings in the Treasury Market</i>											
<i>Bond Maturity</i>											
<i>1- 6 mo</i>	<i>7- 12 mo</i>	<i>13- 18 mo</i>	<i>19- 24 mo</i>	<i>25- 30 mo</i>	<i>31- 36 mo</i>	<i>37- 42 mo</i>	<i>43- 48 mo</i>	<i>49- 54 mo</i>	<i>55- 60 mo</i>	<i>61- 120 mo</i>	<i>&gt;120 mo</i>
$\hat{\alpha}_i$											
0.04 (6.64)	0.05 (3.86)	0.07 (3.31)	0.08 (2.67)	0.09 (2.47)	0.10 (2.39)	0.10 (2.23)	0.11 (2.10)	0.12 (2.13)	0.08 (1.31)	0.11 (1.58)	0.16 (1.41)
$\hat{\beta}_i^m$											
0.00 (2.64)	0.02 (3.80)	0.02 (3.88)	0.03 (3.81)	0.04 (3.56)	0.04 (3.45)	0.04 (3.34)	0.04 (3.08)	0.05 (2.94)	0.06 (3.26)	0.07 (3.74)	0.11 (3.26)
$\hat{\beta}_i^{UV}$											
0.01 (2.49)	0.03 (3.85)	0.05 (4.37)	0.07 (4.54)	0.08 (4.34)	0.11 (4.78)	0.12 (4.64)	0.12 (4.34)	0.12 (4.25)	0.14 (4.54)	0.14 (3.62)	0.17 (2.14)
$R_{adj}^2$											
1.69	3.43	3.02	3.30	3.02	3.19	3.17	2.70	2.34	2.80	3.00	2.47

**Table II – Continued**

*Panel B) Factor Loadings in the Stock Market*

*Book-to-Market Equity (BE/ME) Quintiles*

<i>Size</i>	<i>Quintiles</i>					<i>Quintiles</i>				
<i>Quintiles</i>	<i>Low</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>High</i>	<i>Low</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>High</i>
	$\hat{\alpha}_i$					<i>t-stats for <math>\hat{\alpha}_i</math></i>				
<i>Small</i>	-0.47	0.17	0.27	0.52	0.59	(-2.35)	(1.04)	(1.97)	(3.90)	(3.97)
<i>2</i>	-0.30	0.13	0.39	0.45	0.52	(-2.06)	(1.09)	(3.74)	(4.23)	(4.00)
<i>3</i>	-0.16	0.19	0.29	0.39	0.49	(-1.46)	(2.18)	(3.10)	(4.11)	(3.82)
<i>4</i>	-0.07	0.04	0.25	0.32	0.31	(-0.84)	(0.46)	(2.88)	(3.47)	(2.58)
<i>Big</i>	-0.07	0.04	0.12	0.13	0.20	(-1.07)	(0.73)	(1.52)	(1.40)	(1.69)
	$\hat{\beta}_i^m$					<i>t-stats for <math>\hat{\beta}_i^m</math></i>				
<i>Small</i>	1.34	1.15	0.99	0.93	0.96	(25.45)	(22.91)	(21.58)	(20.71)	(19.29)
<i>2</i>	1.34	1.09	0.97	0.94	1.01	(33.33)	(29.57)	(24.47)	(24.67)	(20.70)
<i>3</i>	1.28	1.05	0.94	0.91	0.96	(38.28)	(36.02)	(27.29)	(25.38)	(21.71)
<i>4</i>	1.21	1.04	0.97	0.93	1.00	(49.65)	(38.51)	(31.77)	(27.73)	(22.44)
<i>Big</i>	1.04	0.95	0.87	0.86	0.89	(50.49)	(48.19)	(34.82)	(26.35)	(20.72)
	$\hat{\beta}_i^{UV}$					<i>t-stats for <math>\hat{\beta}_i^{UV}</math></i>				
<i>Small</i>	-0.42	-0.43	-0.38	-0.35	-0.50	(-2.71)	(-2.93)	(-3.58)	(-3.24)	(-4.81)
<i>2</i>	-0.26	-0.36	-0.34	-0.35	-0.40	(-2.11)	(-4.07)	(-4.03)	(-4.61)	(-3.75)
<i>3</i>	-0.25	-0.22	-0.24	-0.21	-0.22	(-2.50)	(-3.28)	(-3.24)	(-3.26)	(-2.56)
<i>4</i>	-0.10	-0.14	-0.19	-0.12	-0.13	(-1.52)	(-2.49)	(-2.32)	(-1.50)	(-1.66)
<i>Big</i>	0.16	0.11	0.05	0.16	0.02	(4.02)	(2.07)	(0.99)	(1.94)	(0.21)

*Book-to-Market Equity (BE/ME) Quintiles*

<i>Size</i>	<i>Quintiles</i>				
<i>Quintiles</i>	<i>Low</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>High</i>
	$R_{adj}^2$				
<i>Small</i>	59.45	62.26	64.12	63.78	62.14
<i>2</i>	73.22	75.58	74.74	73.17	68.41
<i>3</i>	78.71	82.24	79.16	75.09	68.17
<i>4</i>	85.29	86.77	82.15	76.96	69.74
<i>Big</i>	88.73	86.91	78.52	70.90	61.94

**Table III**  
**Risk Premiums**

Factors' risk premiums are estimated using the Fama-MacBeth (1973) procedure for the following asset pricing model:

$$E(R_{i,t+1}^e) = \lambda_0 + \lambda_m \beta_i^m + \sum_{s=1}^p \lambda_s \beta_i^s,$$

where  $\lambda_m$  is the market price of risk and  $\lambda_s$  is the price of risk associated with the generic factor  $s$ . The left-hand side variable is a vector of monthly excess returns for the 12 maturity-sorted Treasury portfolios and the 25 size- and value-sorted portfolios, and the betas represent factors' loadings estimated in the corresponding time series models.  $UV$  represents unexpected volatility, and is the residual from an  $AR(1)$  model applied to the time series of realized stock market volatility.  $PRED\_UV$  represents unexpected volatility, and is constructed as the difference between predicted and realized volatility (see equations (5) and (1)).  $MOM$  is the momentum factor of Jegadeesh and Titman (1993).  $HML$  and  $SMB$  are mimicking portfolios for book-to-market equity and size (zero-investment portfolios).  $IPS$  represents the innovations in the liquidity factor of Pastor and Stambaugh (2003). Results are reported in percentages, on a monthly basis. Data cover the period January 1952 to December 2008, except for the liquidity factor, which is available only starting with August 1962. The Fama-MacBeth  $t$ -statistics are reported in parentheses, while the Shanken (1992) corrected  $t$ -statistics are reported in brackets.

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{PRED\_UV}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	$\hat{\lambda}_{IPS}$	$R^2_{adj}$
0.15 (3.49) [3.47]	0.49 (2.71) [2.71]							55.45
0.21 (5.03) [4.68]	0.32 (1.87) [1.86]	-0.62 (-2.46) [-2.30]						60.68
0.29 (5.48) [3.64]	0.12 (0.62) [0.58]	-0.77 (-2.81) [-2.34]					0.06 (3.93) [2.63]	61.87
0.10 (2.65) [2.58]	0.37 (2.15) [2.15]			0.15 (1.27) [1.27]	0.44 (4.18) [4.18]			86.95
0.17 (4.48) [3.86]	0.31 (1.86) [1.85]	-0.87 (-3.02) [-2.62]		0.18 (1.53) [1.51]	0.43 (4.09) [4.06]			88.00
0.11 (3.33) [1.96]	0.48 (2.87) [2.77]	-1.25 (-4.14) [-2.46]		0.19 (1.62) [1.52]	0.47 (4.41) [4.17]	4.14 (6.28) [3.75]		93.98
0.23 (5.44) [4.89]	0.29 (1.70) [1.69]		-0.84 (-3.11) [-2.81]					63.43
0.08 (2.87) [1.78]	0.51 (3.05) [2.97]		-0.91 (-3.45) [-2.21]	0.16 (1.40) [1.37]	0.48 (4.51) [4.31]	4.05 (6.05) [3.85]		92.57

**Table IV**  
**Average Returns and Cross-Correlations for Risk Factors**

The  $R_m^e$  is the excess market return. The *MOM* is the momentum factor of Jegadeesh and Titman (1993). The *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). *UV* represents unexpected volatility, and is the residual from an *AR(1)* model applied to the time series of realized stock market volatility. *PRED\_UV* is the second unexpected volatility measure, built as the difference between realized and predicted volatility. *IPS* represents the innovations in liquidity factor of Pastor and Stambaugh (2003). Data cover the period January 1952 to December 2008 (the exception is the *IPS* factor, which is available only starting with August 1962). Results are reported in percentages, on a monthly basis. \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% levels, respectively. The *t-statistics* are reported in parentheses.

*Panel A) Average Returns for the Traded Factors*

	$R_m^e$	<i>SMB</i>	<i>HML</i>	<i>MOM</i>
Average	0.49	0.18	0.39	0.87
<i>t-stat</i>	(3.01)	(1.57)	(3.83)	(6.05)

*Panel B) Cross-Correlation Matrix for all Factors*

	$R_m^e$	<i>UV</i>	<i>PRED_UV</i>	<i>SMB</i>	<i>HML</i>	<i>MOM</i>	<i>IPS</i>
$R_m^e$	1.00						
<i>UV</i>	-0.39***	1.00					
<i>PRED_UV</i>	-0.42***	0.84***	1.00				
<i>SMB</i>	0.26***	-0.29***	-0.27***	1.00			
<i>HML</i>	-0.33***	0.09**	0.06	-0.24***	1.00		
<i>MOM</i>	-0.08*	-0.06	-0.05	-0.02	-0.15***	1.00	
<i>IPS</i>	0.34***	-0.43***	-0.41***	0.17***	-0.12**	-0.03	1.00

**Table V**  
**Predictive Variables: Descriptive Statistics, Contemporaneous Relations with Unexpected Volatility, and Horse-Race Tests in the Stock Market**

Panel A presents the descriptive statistics for the actual predictors. Panel B documents the contemporaneous time series relation between shocks to predictors and volatility. The shocks are computed as the residuals from a first-order VAR system: *DEF* represents shocks to *the default spread* (which is computed as the difference between the yields of a long-term corporate Baa bond and a long-term corporate bond Aaa), *TERM* represents shocks to *the term spread* (which is computed as the difference between the yields of a thirty-year and a one-year government bond), *DY* represents shocks to *the dividend yield* on the S&P500 value-weighted portfolio (which is computed as the sum of dividends over the last 12 months, divided by the level of the index), and  $R_f$  represents shocks to *the one-month T-bill yield*. Panel C estimates factors risk premiums, using the Fama-MacBeth (1973)

procedure, in the following asset pricing model:  $E(R_{i,t+1}^e) = \lambda_0 + \lambda_m \beta_i^m + \sum_{s=1}^p \lambda_s \beta_i^s$ , where  $\lambda_m$  is the market risk premium

and  $\lambda_s$  is the risk premium for the generic factor  $s$ . The left-hand side variable is a vector of excess returns for the 25 size- and value-sorted portfolios, and the betas represent factors loadings from the corresponding time series models. *UV* represents unexpected volatility. Data cover the period January 1952 to December 2008. The *F*-test with its corresponding *p*-value for the joint significance of the factor estimates is reported in Panel B. The Fama-MacBeth *t*-statistics are reported in parentheses, while the Shanken (1992) corrected *t*-statistics are reported in brackets, in Panel C.

*Panel A) Descriptive Statistics for Predictors*

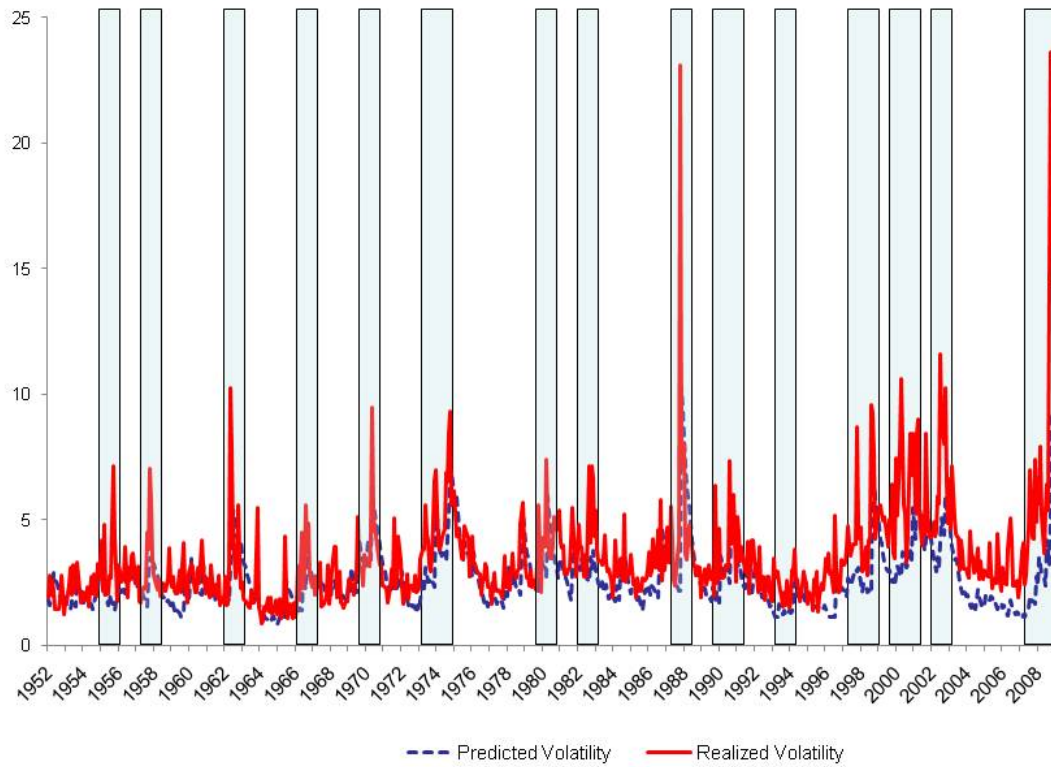
	<i>DEF</i>	<i>TERM</i>	<i>DY</i>	$R_f$
<i>Avg</i>	0.95	0.08	3.34	0.40
<i>Std Dev</i>	0.43	2.65	1.21	0.23
<i>Skewness</i>	1.68	0.66	0.17	1.09
<i>Kurtosis</i>	3.86	3.91	-0.55	1.72

*Panel B) Contemporaneous Regression of Unexpected Volatility on Shocks to Predictors*

$\hat{\beta}_0$	$\hat{\beta}_m$	$\hat{\beta}_{DEF}$	$\hat{\beta}_{TERM}$	$\hat{\beta}_{DY}$	$\hat{\beta}_{Rf}$	$R^2_{adj}$	<i>F</i>
0.06	-0.24	1.83	0.03	-3.03	2.65	19.86	34.86
(1.00)	-(9.62)	(3.38)	(1.28)	-(4.60)	(3.02)		(<0.0001)

*Panel C) Horse-Race Tests in the Stock Market*

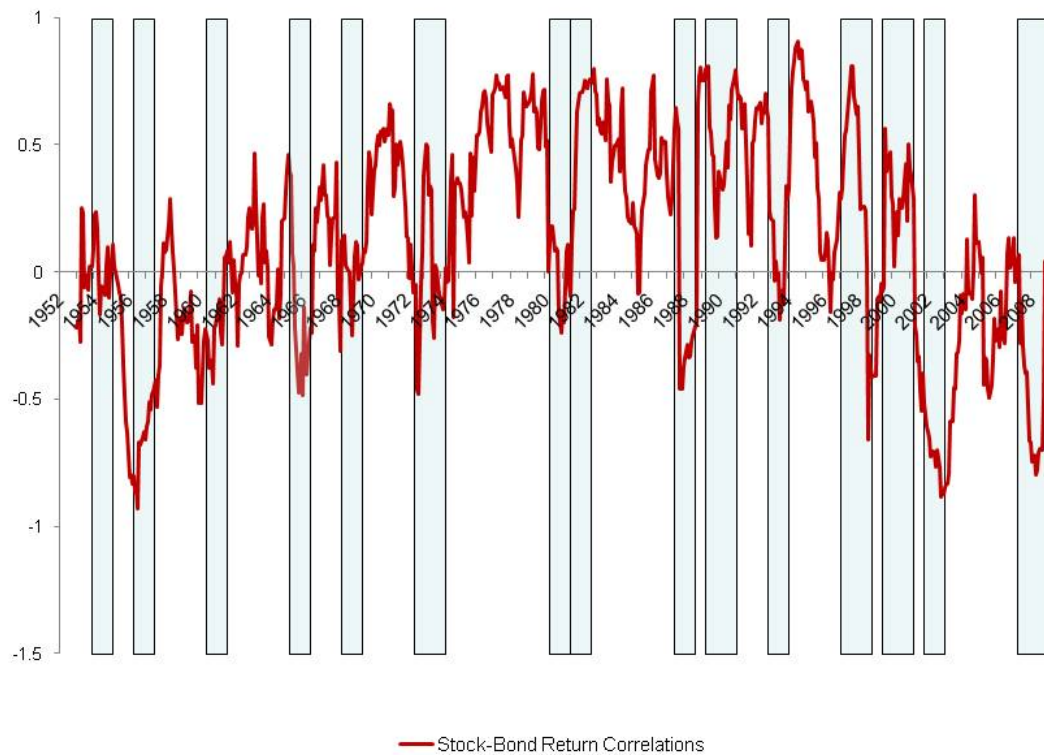
$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{DEF}$	$\hat{\lambda}_{TERM}$	$\hat{\lambda}_{DY}$	$\hat{\lambda}_{Rf}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$R^2_{adj}$
1.87	-1.26		-0.01	1.10	0.06	-0.09			78.08
(6.15)	-(3.75)		-(0.57)	(2.35)	(2.47)	-(4.11)			
[3.15]	[-2.12]		[-0.30]	[1.23]	[1.30]	[-2.12]			
1.87	-1.32	-0.80							77.53
(5.59)	-(3.67)	-(3.19)							
[4.49]	[-3.07]	[-2.59]							
1.46	-0.94						0.13	0.41	70.13
(5.26)	-(2.92)						(1.09)	(3.83)	
[5.09]	[-2.85]						[1.09]	[3.82]	
1.46	-0.92	-1.01					0.17	0.40	77.21
(5.26)	-(2.86)	-(2.99)					(1.51)	(3.74)	
[4.09]	[-2.35]	[-2.33]					[1.51]	[3.66]	



**Figure 1. Realized and Predicted Stock Market Volatilities.** Realized volatility is built on a monthly basis for the period January 1952 to December 2008, using daily market return data from NYSE, AMEX, and NASDAQ maintained by CRSP. It is given by:

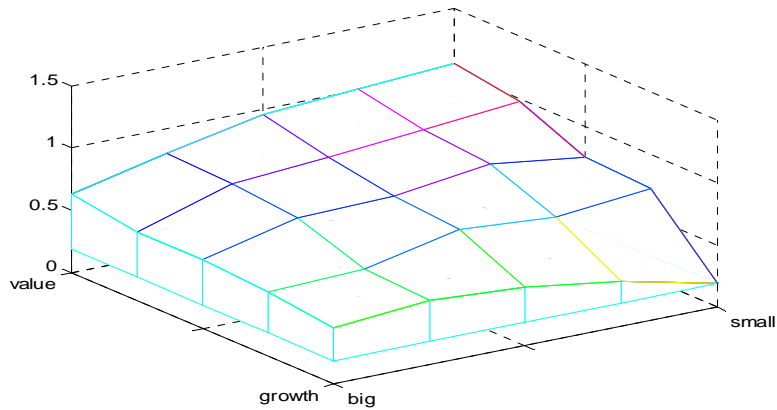
$$v_t = \sqrt{\sum_{i=0}^{\Delta} R_{m,t+i}^2}$$

where  $R_{m,t}$  denotes the daily return on the stock market portfolio, and  $\Delta$  represents the number of trading days in a given month. Predicted volatility is computed based on the estimates of an *Asymmetric-Student GARCH (1,1)* model applied to the monthly stock market return data (see equation (5) in the text). Both series are reported in percentages, at a monthly level. Realized volatility is slightly larger than the predicted volatility. For each year, the tick marks correspond to the month of January. The shaded areas represent NBER recessions or financial crises (see Section 1 for more details).

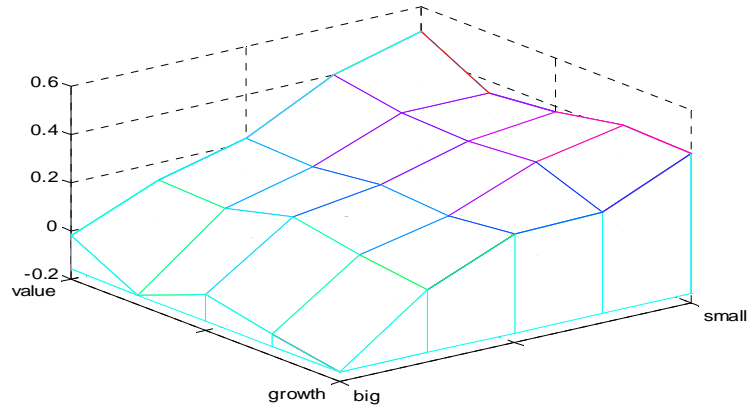


**Figure 2. Realized Stock-Bond Return Correlations.** Rolling 1-year realized correlations between the return on the stock market portfolio and the return on the 10-year Treasury bond are built on a monthly basis, for the period January 1952 to December 2008. For each year, the tick marks correspond to the month of January. The shaded areas represent NBER recessions or financial crises.

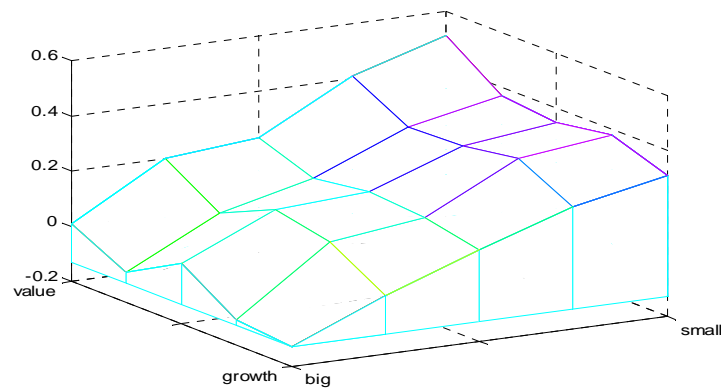
Panel a) Average Excess Returns



Panel b) (-Volatility) Loadings  $\hat{\beta}_i^{UV}$  for the First Volatility Measure



Panel c) (-Volatility) Loadings  $\hat{\beta}_i^{UV}$  for the Second Volatility Measure



**Figure 3. Average Excess Returns and Unexpected Volatility Loadings for Portfolios of Stocks.** Monthly value-weighted portfolio return data collected for the period January 1952 to December 2008 for the 25 Fama-French (1992) portfolios sorted on size and book-to-market equity are obtained from Kenneth French's Web site at Dartmouth. Factors' loadings are estimated using equation (4) in the text for both measures of volatility. Since most volatility loadings are negative, the figures present (-) loadings for better visualization.