An Old and a New Source of Endogeneous Fluctuations

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SAMUELSON'S ORIGINAL MODEL - 1939

- consumption - Keynesian multiplier relating C to national income Y

$$C_t = bY_{t-1} \qquad 0 < b < 1$$

- investment - autonomous and induced through accelerator

$$I_t = I_a + k(C_t - C_{t-1}) \qquad k > 0.$$

- equilibrium in goods market

$$Y_t = C_t + I_t$$

- giving second-order linear difference equation in Y

$$Y_t = I_a + b(1+k)Y_{t-1} - bkY_{t-2}.$$

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LOCAL/GLOBAL DYNAMICS OF SAMUELSON MODEL

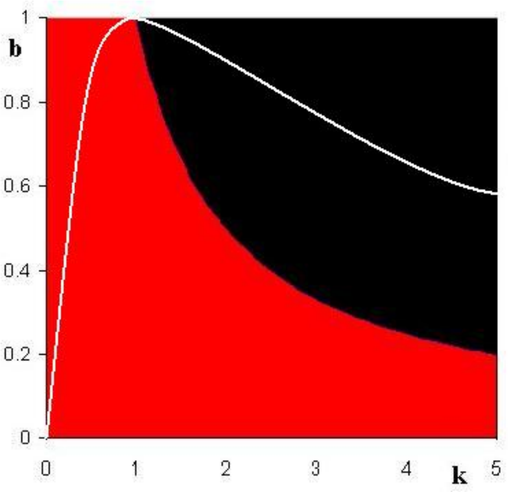
equilibrium $\mathcal{Y} = \frac{1}{1-b} I_a$

stability requres $b < \frac{1}{k}$

oscillations
$b < 4k/(1+k)^2$

damped oscillations
requires both

persistent cycles at
$b = 1/k$
$b < 4k/(1+k)^2$



PERSISTENT FLUCTUATIONS?

criticism:

- investors and consumers expectations are not modeled
- or national income perceived as martingale - best predictor is last value

possible alternatives:

- representative agent - aggregate expectation is the single homogeneous expectation/agent
- rational expectations - aggregate expectation is average over agents, assumed equal to the future state value
- heterogeneous agents - aggregate expectation is weighted average of agents with fixed expectations
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- many-agents - aggregate expectation (if needed) is average of distribution

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HETEROGENEOUS EXPECTATIONS

there is a (small) number of different types of state-dependent expectations circulating in the economy or the market

- for a given current state of the economy agents have differing expectations about the future state
- an agent may change type of expectation for differing states of the economy

for two types the aggregate expectation for national income at time t, given information set available at time $t-1$, with proportion of agents w_t using the expectation formation function of the first type, E^1:

$$E_{t-1}[Y_t] = w_t E^1_{t-1}[Y_t] + (1-w_t)E^2_{t-1}[Y_t] \qquad 0 < w < 1$$

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EXPECTED INCOME

period t income on basis of previous period information: $C_t = bE_{t-1}[Y_t]$

expectations with reference to "long-run" equilibrium - the fixed point of Samuelson's model

- extrapolative expectations (local destabilizing force near normal values)

$$E^1_{t-1}[Y_t] = Y_{t-1} + \mu_1(Y_{t-1} - \mathcal{Y}) \qquad \mu_1 > 0.$$

- equilibrium-reverting expectations (global stabilizing force), with adjustment speed μ_2

$$E^2_{t-1}[Y_t] = Y_{t-1} + \mu_2(\mathcal{Y} - Y_{t-1}) \qquad 0 < \mu_2 < 1$$

expectations depend on the previous value of national income, but so does the proportion of agents using a given type

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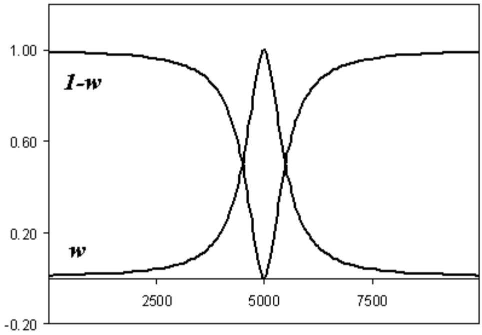
WEIGHTED EXPECTATIONS

more the economy deviates from $\mathcal{Y}$, less weight given to extrapolative expectations

agents believe that extreme economic conditions are not sustainable



formally, with γ a scale factor,

$$w_t = \frac{1}{1 + \left(\gamma\left(\frac{Y_{t-1} - \mathcal{Y}}{\mathcal{Y}}\right)\right)^2} \qquad \gamma > 0$$

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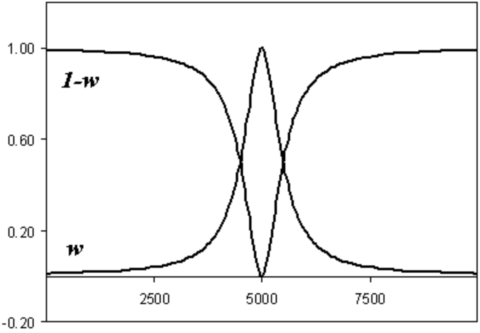

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COMBINED MODEL

$$Y_t = I_a + b(1+k)E_{t-1}[Y_t] - bkE_{t-2}[Z_t]$$
$$Z_t = Y_{t-1}$$

that is

$$\begin{aligned} Y_t &= I_a + b(1+k)\Bigg[\left(\frac{1}{1+\gamma^2\left(\frac{Y_{t-1}-\mathcal{Y}}{\mathcal{Y}}\right)^2}\right)(Y_{t-1}+\mu_1(Y_{t-1}-\mathcal{Y}))+ \\ &\quad + \left(1-\frac{1}{1+\gamma^2\left(\frac{Y_{t-1}-\mathcal{Y}}{\mathcal{Y}}\right)^2}\right)(Y_{t-1}+\mu_2(\mathcal{Y}-Y_{t-1}))\Bigg] \\ &\quad - bk\Bigg[\left(\frac{1}{1+\gamma^2\left(\frac{Z_{t-1}-\mathcal{Y}}{\mathcal{Y}}\right)^2}\right)(Z_{t-1}+\mu_1(Z_{t-1}-\mathcal{Y}))+ \\ &\quad + \left(1-\frac{1}{1+\gamma^2\left(\frac{Z_{t-1}-\mathcal{Y}}{\mathcal{Y}}\right)^2}\right)(Z_{t-1}+\mu_2(\mathcal{Y}-Z_{t-1}))\Bigg] \\ Z_t &= Y_{t-1} \end{aligned}$$

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JACOBIAN MATRICES

$$J(Y,Z) = \begin{pmatrix} b(1+k)\frac{dE_{t-1}[Y_t]}{dY_{t-1}} & -bk\frac{dE_{t-2}[Z_t]}{dZ_{t-1}} \\ 1 & 0 \end{pmatrix}$$

for Samuelson's equilibrium

$$J(\mathcal{Y}) = \begin{pmatrix} b(1+k)(1+\mu_1) & -bk(1+\mu_1) \\ 1 & 0 \end{pmatrix}$$

Conditions for stability of $(\mathcal{Y})$ (eigenvalues $|\lambda_{1,2}| < 1$)

$$\begin{aligned} 1 + \text{ tr } J(\mathcal{Y}) + \text{ det } J(\mathcal{Y}) &> 0 \quad \text{(flip at } \lambda_1 = 1) \\ 1 - \text{ tr } J(\mathcal{Y}) + \text{ det } J(\mathcal{Y}) &> 0 \quad \text{(fold/pitchfork at } \lambda_2 = -1) \\ 1 - \text{ det } J(\mathcal{Y}) &> 0 \quad \text{(Neimark-Sacker at mod=1)} \end{aligned}$$

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BIFURCATIONS

First always true, second and third not necessarily. Parameter assumptions: $\mu_1, k > 0$

Binding inequality:

$$\mu_1 < \frac{1-b}{b} \quad \text{if} \quad k < 1$$

at equality a pitchfork bifurcation, $\mathcal{Y}$ loses stability 2 new (stable) fixed points appear

$$\mu_1 < \frac{1-kb}{kb} \quad \text{if} \quad k > 1$$

at equality a NS bifurcation, $\mathcal{Y}$ loses stability stable invariant curves appear.

Sequences on curves represent endogenous fluctuations as a GENERIC dynamical feature.

ADDITIONAL STEADY STATES

The quadratic term in weight leads to existence of 2 steady states ($\bar{Y}_1$, $\bar{Y}_2$) symmetric about $\mathcal{Y}$, the solutions to

$$(\bar{Y} - \mathcal{Y})^2 = \frac{\mathcal{Y}^2\left(b(1+\mu_1) - 1\right)}{\gamma^2\left(b(\mu_2 - 1) + 1\right)}.$$

- complex-valued for $\mu_1 < (1-b)/b$
- real and equal to $\mathcal{Y}$ at critical value pitchfork bifurcation $\mu_1 = (1-b)/b$
- for $\mu_1 > (1-b)/b$ real and positive

$\bar{Y}_1$, $\bar{Y}_2$ also lose stability through NS bifurcations - interesting dynamics result from relations of stable and unstable manifolds of saddle node at $\mathcal{Y}$ and dynamics of $\bar{Y}_1$, $\bar{Y}_2$

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DYNAMICS OVER PARAMETER SPACE

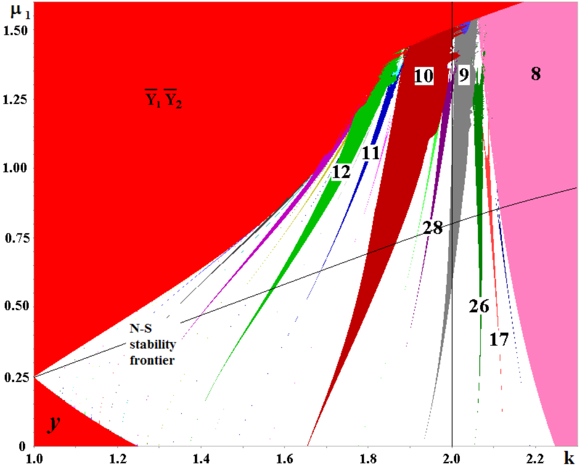
two parameter bifurcation diagram (k, μ_1)

fixed points - red

cycles - other colors (periodicity)

unknown - white: periodic (> 32), quasiperiodic or chaotic attractors

pitchfork $\mu_1 = 0.25$



$b = 0.8$, $I_a = 1000$ ($\mathcal{Y} = 5000$), $\mu_2 = 0.5$, $\gamma = 10$, $(Y_0, Z_0) = (3500, 3500)$
these and all following plots produced with `iDMC` software

BIFURCATION SCENARIO 1

$k = 2$, limit sets as μ_1 varies

$\mathcal{Y}$ undergoes NS for $\mu_1 < 0$ (equilibrium exists but unstable)

μ_1 small - mostly quasiperiodic attractors on invariant curves

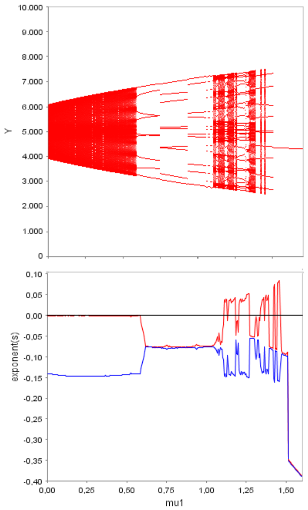
$\mu_1 \approx 0.6$ - period-9 cycles

$\mu_1 \approx 1.15$ - chaotic and periodic windows alternate

$\mu_1 > 1.5$ - fixed point $\bar{Y}_1 < \mathcal{Y}$

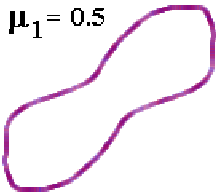
Lyapunov exponents - time ave divergence nearby trajectories

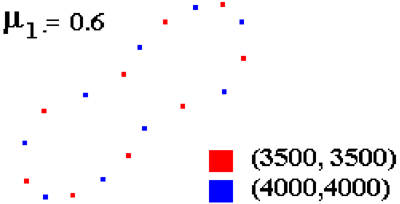
2 negative - periodic cycle;
1 negative 1 zero - quasiperiodic;
1 negative 1 positive - chaotic.



FROM QUASI TO PERIODIC DYNAMICS

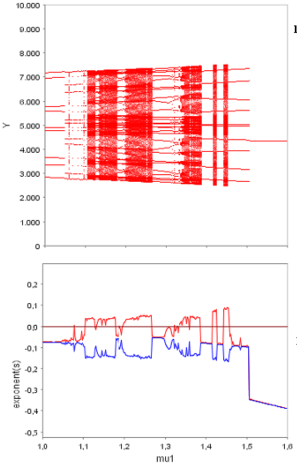
from both initial conditions convergence to invariant circle

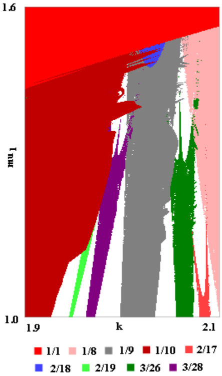




from each initial condition convergence to a different period-9 cycle

FROM PERIODIC TO CHAOTIC DYNAMICS





close ups

bifurcation diagram

Lyapunov exponents

complicated tongue structure near bifurcation

rotation numbers in double bifurcation diagram

MULTI-STABILITY - STATE SPACE

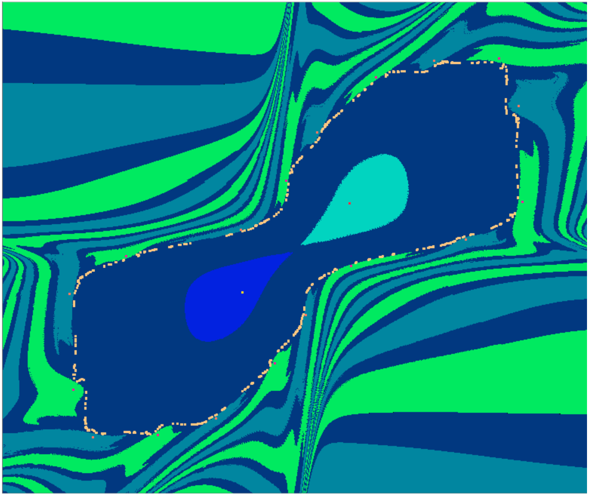
intricate basin structure at $\mu_1 = 1.15$ - axes (2000,8000)

2 stable period-9 cycles, basins in green and cobalt blue

chaotic attractor in pink - basin in dark blue

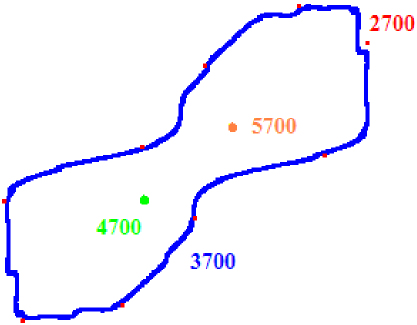
stable $\bar{Y}_1$, $\bar{Y}_2$ surrounded by basins

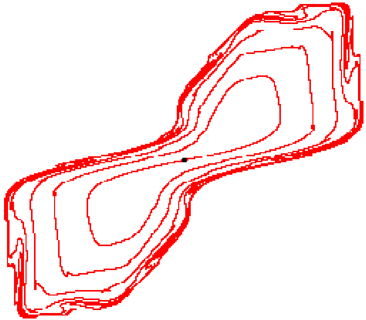
saddle point $\mathcal{Y}$ between them



MULTI-STABILITY - TRAJECTORIES

different i.c. converge to different attractors
periodic points lie close to chaotic attractor





unstable manifold exits saddle $\mathcal{Y}$
converges to chaotic attractor

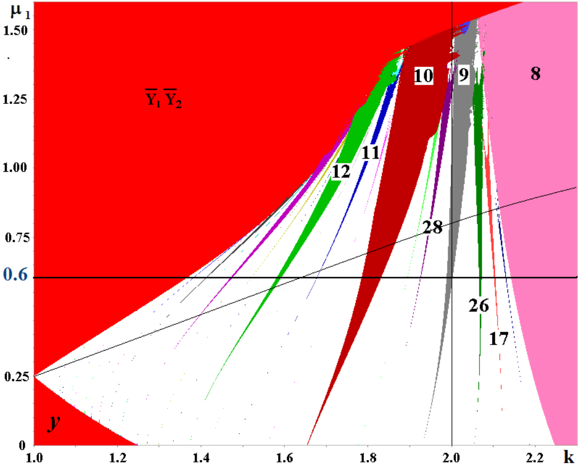
k AS BIFURCATION PARAMETER

setting
$\mu_1 = 0.6$
$1 < k < 2.3$

for all
$\mu_1 > 0.25$ $\mathcal{Y}$
is unstable

as k
increases:
fixed points,
quasi and
periodic
attractors,
period-8 cycle

with
multi-stability
at transition



BIFURCATION SCENARIO 2

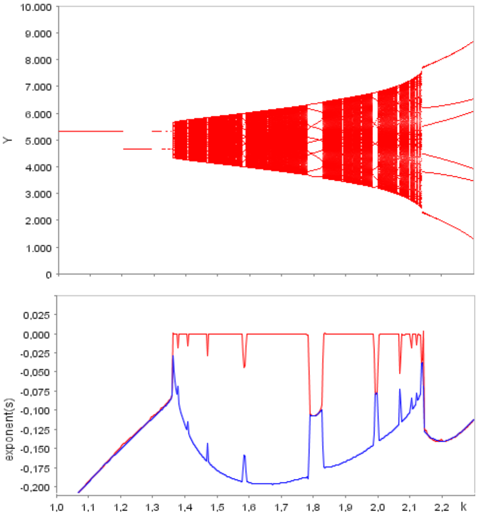
from (4000,4000)

for small k ($\mu_1 = 0.6$) $\mathcal{Y}$ unstable

$\bar{Y}_1 = 4658$, $\bar{Y}_2 = 5342$ stable with respective basins which become more entwined as k increased

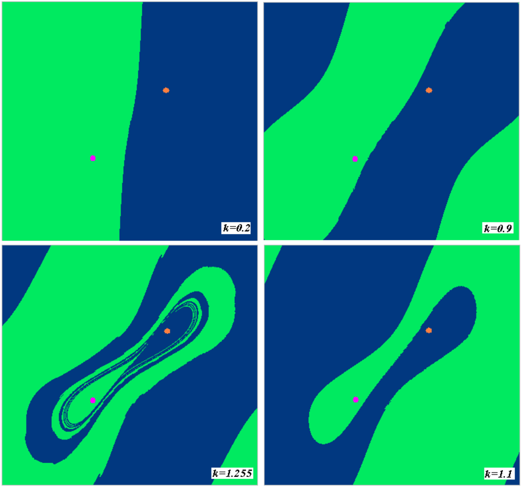
larger fluctuations alternating on invariant curve between quasi and periodic attractors

period-8 cycle (outside invariant curve) first stable then loses stability



STEADY STATE BASINS ENTWINE

distinct basins for $\bar{Y}_1$, $\bar{Y}_2$, separated by stable manifold $\mathcal{Y}$



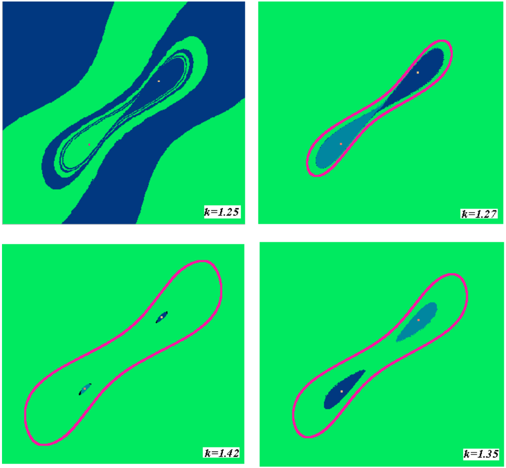
eigenvalues complex, basins begin to distort

dense convolutions form ring

convolutions around 3 fixed points

FROM FOCI TO INVARIANT CURVE

dense convolutions near where attracting curve, Γ_s will appear



enclosed is repelling Γ_u, forming a separatrix

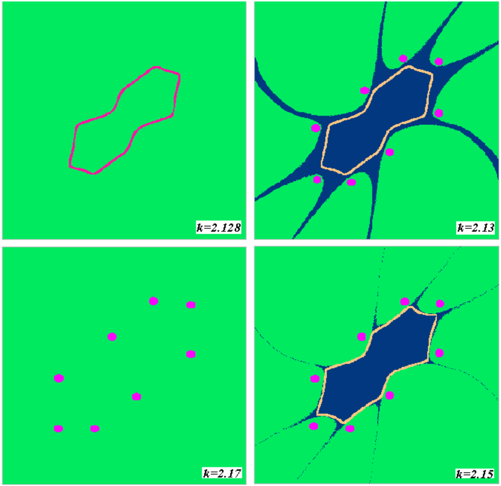
homoclinic connection

foci lose stability through Neimark-Sacker bifurcation

double homoclinic loop, basins disjoint

INVARIANT CURVE TO PERIOD-8 CYCLE

invariant curve only attractor



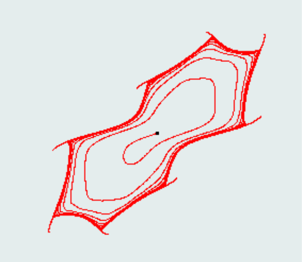
quasiperiodic attractor (blue basin) stable focus period-8 cycle (green basin), invisible saddle

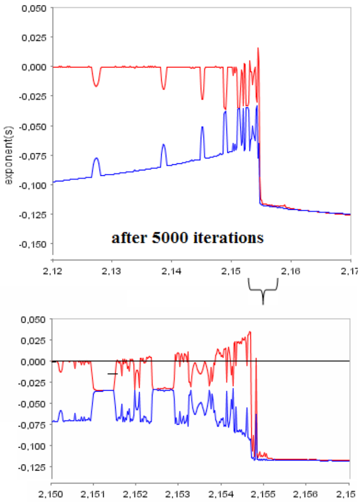
Γ_s destroyed by heteroclinic loop sequence

heteroclinic tangency, tangle, tangency

CHAOTIC TRANSITIONS

transversal crossing of loop sequence associated with chaotic repeller and long chaotic transients - below unstable manifold from Samuelson equilibrium near transition at $k = 2.155$ leads to chaotic frills at boundary of invariant curve and period-8 cycle

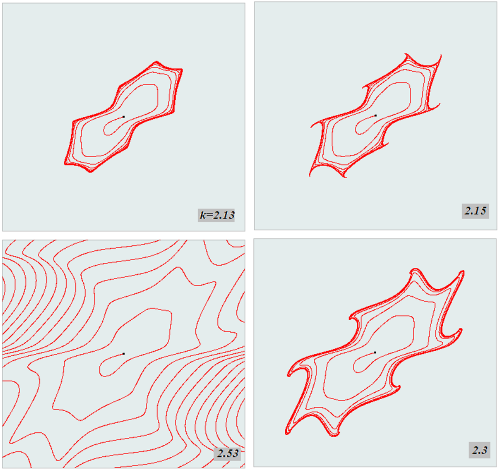




UNSTABLE MANIFOLDS OF $\mathcal{Y}$

invariant curve

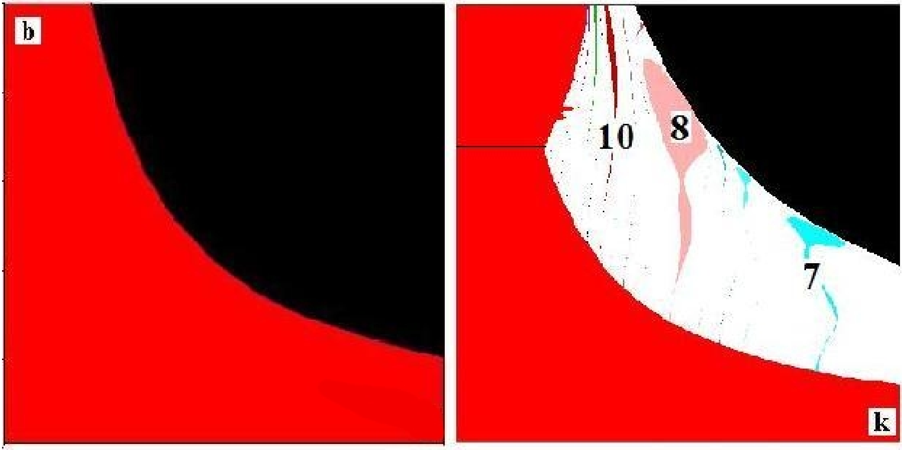
close to chaotic attractor



period-8 cycle loses stability, no attractors

unstable manifold converges to period-8 cycle

COMPARISON OF LIMIT SETS IN ORIGINAL PARAMETER SPACE



$\mu_1 = \mu_2 = 0.5$, $I_a = 1000$, $\gamma = 10$, $(Y_0, Z_0) = (4000, 4000)$, $\infty = 10^{10}$, transients are 5000, max period 32, precision 0.01

SAMUELSON'S MULTIPLIER-ACCELERATOR WITH HETEROGENEOUS EXPECTATIONS

- conditions for local stability of Samuelson's equilibrium are more restrictive
- but global stability continues as trajectories converge to
  - either of two co-existing stable steady states
  - closed curves with periodic or quasiperiodic sequences
  - chaotic attractors
- more of the original parameter space (k, b) leads to persistent oscillations

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GENERAL ISSUES in DYNAMICAL ANALYSIS

economic models are nonlinear and the study of linearized versions around a steady state is only the beginning of dynamical analysis

which, given

- widespread availability of cheap, powerful computers
- open-source software
- developments in dynamical systems theory

should be accompanied by the analysis of global dynamics

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- and to the effects of disturbance processes on dynamics by studying models as stochastic processes

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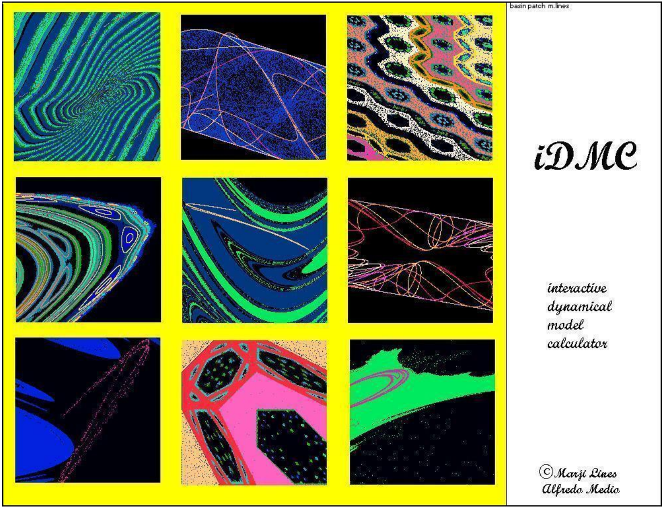
REFERENCES

Lines, M., Westerhoff, F., 2006. Expectations and the multiplier-accelerator model, in: Puu, T., Sushko, I. (Eds.), *Business Cycle Dynamics. Models and Tools*, Springer-Verlag, Berlin, pp. 255-276.

Lines, M., 2007a. Bifurcation scenarios in a heterogeneous agent, multiplier-accelerator model. *Pure Mathematics and Applications* 16, 429-442

see these for more references on heterogeneous expectations, multi-stability and global dynamics

SOFTWARE FOR ANALYSIS OF DYNAMICAL MODELS
http://code.google.com/p/idmc/



iDMC windows, linux, source code

RiDMC, package for running iDMC algorithms in R, forthcoming

WORKSHOP
INTERACTING AGENTS AND NONLINEAR DYNAMICS IN MACROECONOMICS

UDINE, ITALY JUNE 9-11 2010

Invited speakers
Carl Chiarella, Rainer Franke, Paul de Grauwe, Cars Hommes, Tom Lux, Jan Tunistra

Open Sessions are being organized by Sebastiano Manzan