Sale before Completion of Development: Pricing and Strategy

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The paper examines the risk-and-return characteristics of a popular development strategy, the presale system (or sale before completion), used in many Asian cities. We model a presale decision in a real-options framework and suggest that the use of presale is primarily for a risk-sharing purpose. That is, developers can reduce bankruptcy and marketing risks by selling (or leasing) their projects before their completion dates. Our model also indicates that, because of the presale system, there is a barrier for new developers to enter into a market, which helps explain the anecdotal observation that most real estate markets in Asian cities are oligopolistic in nature and dominated by large developers.

Developers have to start construction based on a projected future demand and take the risk that the demand might not be realized when the projects are completed. Developers will be in a difficult financial position (holding vacant buildings and construction loans that are waiting to be paid off) if demand has unexpectedly fallen at the time of completion. Limiting the costs of such an outcome is important to the viability of developers.

An examination of the practices of development communities around the world indicates that there are at least three methods to deal with the risk associated with demand uncertainty. First, developers may delay the property development until they are more confident about the future demand. The seminal work by Titman (1985) explicitly models the value of this “option to wait” in the project investment process. Somerville (2001) also concludes that builders will obtain building permits first, but will only exercise them after the demand conditions become clearer. Quigg (1993), Bulan, Mayer and Somerville (2002) and Schwartz and Torous (2003) all provide empirical evidence that supports the “option to wait” theory.

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While the first method deals with the identification of the demand curve, the second method is a strategy that works with the demand curve. This method is popular when developers have a multiphase project. If the state of the market is unclear when the development starts, a risk-averse developer might reduce the price on the earlier units to ensure that there will be sufficient demand for the product. Developers can increase the price level of the remaining phases if future demand conditions warrant it.

The use of this method has some empirical support. For example, Sirmans, Turnbull and Dombrow (1997) document that developers would be willing to sell the units in the earlier phases of a subdivision at a lower price because of the uncertainty about the future characteristics of the neighborhood. They will sequentially increase the price of the units in the later phases of the subdivision as more (positive) information about the neighborhood is revealed. In other words, when developers are uncertain about the success of their projects, they are willing to trade profit in the earlier stage for more certainty.

The third method is to presell a project. The presale method is to sell (or to lease) a unit before the completion of the project. The presale approach is very popular in many of the cities in Asia (especially in China, Hong Kong, Singapore and Taiwan), and is also used in developments in North America and Europe. Compared with the other two methods, the presale method not only helps developers deal with the uncertainty of future demand (and potential bankruptcy costs), but can also substantially reduce (or eliminate) developers’ inventory costs. Since a developer can have all units sold or leased before the project is completed, this method is particularly useful for large development projects (such as a 50-floor, 500-unit condominium project).

While the presale method has been very popular in Asia, little attention has been paid to the risk-and-return characteristics of this strategy for both developers and buyers of the presale contract. Previous research on presales tends to treat a presale as a forward or futures contract (see, e.g., Shih 1992, Chang and

1 While it can be argued that developers can also enter into pre-lease contracts with potential tenants before projects are completed, such lease contracts are normally not strictly binding, at least to the lessees of the projects. On the other hand, when a presale contract is signed, the buyer will lose all the payments if he/she does not purchase the property at the end.

2 To the best of our knowledge, the first presale contract in the Chinese community was conducted in Taiwan in the late 1960s. A developer presold the units of a condominium and served as the management company for the project. Nearly all residential development projects in Hong Kong and Singapore in recent years have used the presale method to launch the project. For a detailed discussion of the presale system in Hong Kong, see Wang et al. (2000).
Ward 1993, Chang 1994). In other words, once a presale contract is signed, it is implicitly assumed that a buyer will purchase the property when it is completed. However, the buyer may default on the contract if the property value has dropped significantly by the time the property is completed. In fact, some presale contracts explicitly allow buyers to terminate the contract by paying a prespecified forfeiture charge. While developers may be able to go to court to ask the buyers to cover the deficiency when they default, in reality, the high cost of obtaining such a judgment (since buyers can always claim that the default is due to the poor quality of the building) and the low likelihood of collecting the deficiency from the buyer even if a judgment is obtained make it unprofitable for the developers to take the buyers to court when they default.3

Recognizing that buyers have an option to default on a presale contract, this paper examines a developer's presale strategy using a real-options framework.4 We first show how developers maximize their expected payoffs through a presale of residential units rather than selling them upon completion. We then extend the study to situations in which investors are wary about the reputation of the developers. When the reputation of developers is considered, our model is able to explain the existence of entry barriers in property markets where the presale method is popular (and why the markets are oligopolistic).

The organization of the paper is as follows. The next section briefly describes the use and mechanism of a presale system using a typical market (Hong Kong) as an example. The next section shows the optimal strategies a developer can use to launch a presale. Models for contract design and solving for the optimal level of final payments are shown in the fourth section. The last section concludes the paper.

Market Fundamentals

The Presale Mechanism

A presale is normally allowed only at certain periods after construction begins. In most regions, the normal practice is that the developer must complete a certain

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3 We have not heard of a case in which developers successfully recovered the deficiency from the purchasers when they defaulted on presale contracts. A similar situation exists in the mortgage default literature. Wang, Young and Zhou (2002) report that, in the United States, when a borrower defaults on a mortgage, banks rarely go after the borrower even if the bank can obtain a deficient judgment from the court because of the large expenses required in the collection process.

4 The real-options concept has been used to examine development strategies. See, for example, Titman (1985), Williams (1993, 1997), Grenadier (1996), Childs, Riddiough and Triantis (1996), Riddiough (1997) and Wang and Zhou (2003) for theoretical developments in this field.
percentage of the project before the government will release a presale consent. That is, when a developer presells the unit, potential buyers can observe the percentage of the building that was completed at the time of sale (we call this asset in place).

A presale contract gives the buyer the right to buy a property at a preagreed payment schedule (which normally is in accordance with the progress of the construction). At the time the building is completed, the buyer makes a final payment and takes the title of the property. In the case of the Hong Kong market and some other Asian markets, the buyer has the right to terminate the presale contract at any point during the payment period by paying a forfeiture charge.

There are two major benefits for developers to conduct a presale. First, since properties are sold before completion, marketing costs and inventory costs are reduced. Second, the uncertainty about the future demand can also be reduced because projects can be presold (or preleased) at a time close to the date of making the investment decision. The reduction of holding costs alone can provide developers with enough incentive to use the presale system.

Second, without a presale system, there is an expected bankruptcy cost if a developer takes on a large project relative to the value of the developer’s capital base. The concern about bankruptcy costs is a practical one for developers in many Asian cities where typical residential developments are comprised of hundreds to thousands of units and the price movements in property markets are quite volatile. Figures 1, 2 and 3 illustrate the property price movements in selected Asian cities. Figure 1 reports the residential price movement in Shanghai, China, during the 1995–2003 period. As can be seen from the figure, the property price in the city moved downward since the start of the index period (March 1995). The property price started to increase at the end of 1999. However, it took an additional 3 years (until the end of 2002) before the price level reached the 1995 level. If a developer did not presell the units in 1995, it is possible that she/he would face bankruptcy risk when the property is completed later on.

The movement of Hong Kong property price indexes seems to be more volatile than that in Shanghai. Figure 2 shows that the Hong Kong residential property price index peaked at 1,050 on September, 1997. The index dropped to around 390 in March 2003. (The office property price index follows a similar pattern.) Without a presale system, most developers in Hong Kong who started projects in 1997 might have faced serious trouble when their projects were completed. We have a long time series of residential prices for Taipei (1974–2003) and, as can be seen from Figure 3, the price level at the end of the index period (March 2003) was still lower than the price level in 1989. This pattern indicates that
Figure 2: Residential and Office Property Price Indexes of Hong Kong, 1980–2003. Source: Hong Kong Housing Bureau and the Hong Kong Monthly Digest of Statistics by the Hong Kong Census and Statistics Department.
Figure 3  ■ Housing Price Index of Taipei, 1974–2003. Source: Taiwan Real Estate Research Center, National Chengchi University, Taiwan.
the volatile price movement presents a real threat to developers if they do not presell their properties.⁵

The presale system also benefits buyers of the properties. Chang and Ward (1993) point out that during periods with a rapid increase in residential property prices, buyers might be willing to pay a premium to buy properties in the presale market because they are afraid that the price of the property might increase so fast in the future that they will not be able to purchase the same property when it is completed. Given that, a presale contract allows buyers to buy insurance which guarantees that their planned savings will be sufficient to buy a home. Consequently, in an inflationary environment, because buyers are willing to pay a premium, the presale system will exist even if it does not offer benefits (the reduction of bankruptcy costs and inventory costs) to the developers.

In our model, we assume that buyers of a property are risk neutral and are thus indifferent between buying a property at a presale and buying the same property when it is completed. In other words, buyers of a property will estimate the expected price to be paid at completion and calculate the present value (or future value) of payments based on a risk-free rate to determine how much they should pay (in terms of down payment and the final payment) for a presale contract. We start our model by assuming that developers are also risk neutral and derive the equilibrium value of a presale contract. We then extend our model to analyze issues related to the optimal time to launch a presale contract and the design of an optimal contract. At this stage, we will only examine a special case where developers are more risk averse than buyers of the projects. Given the bankruptcy and inventory costs that developers have to bear without a presale, the special case that developers are more risk averse than buyers is quite reasonable.⁶

Figure 4 illustrates the typical payment system of a presale contract. Panel A starts with a simple two-period contract. \( Q \) represents the first scheduled payment, \( Q_2 \) is the scheduled payment for period 2, and \( A \) is the forfeiture charge.

⁵ In addition, because of the large investment scale (it could be up to US$2 billion in Hong Kong) and long construction period, developers are generally inhibited from undertaking several different projects simultaneously and, therefore, will not be able to diversify their project risk.

⁶ Conceptually, we can continue to assume that buyers and developers are both risk neutral and derive similar qualitative results. However, by doing so, we will have to formally include bankruptcy and inventory costs into our model. The introduction of two more stochastic variables into the model will make our analysis messy (and will not enhance our understanding about the market). By assuming that developers are risk averse (and do not include the savings on inventory and bankruptcy costs in the cash flows received by developers), we can derive clearer results.
Figure 4: Time line and payments for a presale contract. A: With two payments. B: With $n$ payments. Note: $Q$ represents the first scheduled payment, $Q_i$ is the scheduled payment for period $i$, and $A$ is the forfeiture charge. At the end of each payment period $i$, the buyer will either pay the scheduled payment $Q_i$ or a forfeiture charge $A$. If the buyer pays a forfeiture charge, the contract is terminated immediately. In other words, for the buyer to pay $Q_n$, $Q_{n-1}$ must be paid first. (We term this $Q_n \mid Q_{n-1}$.) When the last payment $Q_n$ is paid, the buyer will receive the property spot price at period $T$, or $S_T$.

Panel A

<table>
<thead>
<tr>
<th>$t = t_0$</th>
<th>$t = t_1$</th>
<th>$t = T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>pay $Q$</td>
<td>pay $Q_n \mid Q_{n-1}$</td>
<td></td>
</tr>
<tr>
<td>or $A$</td>
<td>receive $S_T$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or pay $A$</td>
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</tr>
</tbody>
</table>

Panel B

<table>
<thead>
<tr>
<th>$t = t_0$</th>
<th>$t = t_1$</th>
<th>$t = t_2$</th>
<th>$t = t_3$</th>
<th>$t = t_{n-1}$</th>
<th>$t = T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>pay $Q$</td>
<td>$Q_2$</td>
<td>$Q_3 \mid Q_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>or $A$</td>
<td>or $A$</td>
<td>receive $S_T$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>or pay $A$</td>
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</tbody>
</table>

The buyer signs the presale contract by paying $Q$. At the end of the first period, the buyer pays either the scheduled payment $Q_2$ or pays a forfeiture charge $A$, which terminates the contract. (It should be noted that, before the buyer can pay $Q_2$, she/he must pay $Q$ first.) When the last payment $Q_n$ is paid, the buyer will receive the property spot price at period $T$, or $S_T$. Panel B of Figure 4 extends this concept into a multipayment framework.

Discounted Cash Flow or Forward Contract Approach

Traditionally, a presale contract could be analyzed using the Discounted Cash Flow (DCF) approach or the Forward Contract approach. Figure 5A illustrates the DCF concept using a three-payment case. We assume the scheduled payments $Q$, $Q_2$ and $Q_3$ are made at the end of periods 1, 2 and 3, respectively. If the buyer pays all three scheduled payments, the present value of all cash flows (discounted at $y$) is $Q + Q_2(1 + y)^{-1} + (Q_3 - S_3)(1 + y)^{-2}$, where $S_3$ is the value of the presale unit when completed at the end of period 3. If the buyer defaults at the end of period 3, then the present value of the payments is...
Figure 5: An illustration of DCF and option concepts. A: DCF (or Forward Contract) approach. Note: For simplicity, we assume the scheduled payments $Q_1$, $Q_2$, and $Q_3$ are made at the end of periods 1, 2, and 3. $A$ is the forfeit charge, $p_u$ and $p_d$ are the probabilities of upward and downward movements in price for period 2, so that the buyer will either pay $Q_2$ or $A$, respectively. $p_{au}$ and $p_{ad}$ are the probabilities of upward and downward movements in price for period 3 given that $Q_2$ is paid, so that the buyer will pay either $Q_3$ or $A$. Likewise, $p_{au}$ ($p_{ad}$) is the probability of upward (downward) movement at the end of period 3 given that $A$ is paid at the end of period 2. That is, $p_u + p_d = p_{au}$ $+ p_{ad} = p_{au} + p_{ad} = 1$. $S_1$ is the spot price of the property at the end of period 3, $y$ is the required rate of return demanded by the developer. The DCF approach calculates the present value of all the payments, while the Forward Contract approach calculates the future value of all the payments. Both methods aim to estimate the property value given the payments specified in the contract. Note that neither method offers a closed-form solution for the problem. B: Option approach. Note: $A$ is the forfeit charge. $S_1$ is the spot price of the property at the end of period 3. $Q_2$ and $Q_3$ can be viewed as the exercise prices of $C_1$ and $C_2$, respectively. $C_2$ is an option that allows the buyer to buy the property at an exercise price $Q_2$. $C_1$ is an option that allows the buyer to buy $C_2$ at the exercise price $Q_1$. The first payment $Q$ is the price to purchase $C_1$. Given this, instead of calculating the property value, the real-options approach only calculates the value of the first contract payment given the contractual arrangement. This option concept reflects the fact that the property has not changed hands when the first payment is made and that the first payment can only allow the buyer to advance to the next stage. The method also allows for the possibility that a buyer, after making several payments, willingly cancels the contract. Note that the option approach provides a closed-form solution for the problem.

Panel A

\[
\begin{align*}
\text{DCF Approach} & = Q + \frac{p_u Q_2 + p_d A}{(1+y)} + p_{au} p_{au} (Q_3 - S_1) + \frac{p_u A}{(1+y)^2} \\
& = \text{Net present value of the contract} \\
\text{Forward Contract} & = Q(1+y)^2 + (p_u Q_2 + p_d A)(1+y) + [p_{au} p_{au} (Q_3 - S_1) + p_u A] \\
& = \text{Net future value of the payment}
\end{align*}
\]
Figure 5 continued.

Panel B

\[
Q = \begin{cases} 
Q_2 & \text{if } C_2[S_2,2] \geq Q_1 - A \\
Q_3 & \text{if } S_3 \geq Q_2 - A \\
A & \text{Otherwise}
\end{cases}
\]

Option Approach: \( Q = C_1(S_1,1) \) where \( C_1(S_1,2) = \text{Max}(C_2(S_2,2) - Q_2, -A) \), and 
\( C_1(S_1,3) = \text{Max}(S_3 - Q_2, -A) \).

\( Q + Q_2(1 + y)^{-1} + A(1 + y)^{-2} \). Similarly, if the buyer defaults at the end of period 2, her/his aggregate payment will be \( Q + A(1 + y)^{-1} \). (In the last two cases where default occurs, the buyer cannot enjoy the ownership of the unit, and therefore has no inflow of \( S_3 \).) For period 2, we assign the probability that the buyer will pay \( Q_2 \) as \( p_a \) and the probability of paying \( A \) as \( p_d \). (\( p_{aa} \) and \( p_{ad} \) are defined similarly for period 3.) Since buyers make a decision based on the present value (or future value) of all cash flows, a presale transaction can be viewed as a forward contract and the effective presale price (defined as \( F \)) can be modeled as the current market price plus the carrying charge, or

\[
F = P_t \left( 1 + C(r, d, y, t, T) \right).
\]

Note that \( P_t \) is defined as the current house price and \( C(r, d, y, t, T) \) is defined as the carrying charge (which is a function of interest rate, depreciation rate, net rent, date of presale, and date of delivery).\(^7\)

While informative, the use of the DCF approach has four limitations. First, the need to estimate probabilities could be problematic. This is especially true when the number of payments is large. Second, when a presale contract is signed, the only thing at stake is the first payment of the contract. The DCF approach does not estimate the value of the first payment explicitly. Third, there is no explicit guidance in the DCF approach as to whether a buyer should default at each payment stage of the contract. Finally, while the DCF approach can estimate the value of a presale contract given a prespecified payment pattern,

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7 See Chang and Ward (1993) for a detailed description of the model.
it cannot help developers design the optimal payment pattern. In other words, the optimal contract design cannot be determined using the DCF approach.

The Real-Options Approach

Given the limitations of the traditional DCF approach, our paper uses the real-options approach available in the recent finance literature to analyze a presale contract. With the aid of this tool, we are able to provide closed-form solutions for a presale contract, and the default decision of a buyer can be modeled as an exercise of a call option. More importantly, the value of the first payment of a presale contract and the issues on the optimal contract design can now be addressed explicitly.

The idea is simple. In a two-payment (down payment and final payment) presale contract, a developer is actually selling an option to the buyer to purchase a completed project at a fixed exercise price (the final payment). Consequently, the down payment in the presale contract can be viewed as the price of a call option.\(^8\) In this case, the maturity date of the option is the construction completion date. When a presale contract involves multiple payments, the earlier payment can be viewed as the price of an option that entitles the buyer to buy the next option at a predetermined price (the next contract payment). In other words, the buyer of the option can choose either to continue with the contract (by paying the next contract payment) or let the option expire (by paying the forfeiture charge). Since there is no incentive for the buyer to pay earlier than the contract dates of the payments, a presale contract can be considered as a series of European options with no interim profits. In other words, when a buyer decides to make the next payment, she/he is, in fact, buying another option to continue the contract. This process will end with the last payment, where the reward of the payment is a completed property. Figure 5B illustrates this concept using a three-payment contract.

Presale Contract as a Compound Option

Using the real-options framework, we define \(S_t\) as the current spot price of an identical building at time \(t\). A developer who decides on day \(t = t_0\) to launch

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\(^8\) Of course, a presale contract can also be analyzed as a put option. It is well known in the finance literature that a call option is a put option plus the spot market price minus the present value of the exercise price. If a buyer has to buy a property at a given time in the future, a presale contract can put a ceiling on the amount of the final cash payment a buyer has to pay for the property. Given this, a developer's incentive for a presale contract could be to seek additional income by writing a put option. We thank Patric Hendershott, Charles Ward and Bryan MacGreger for a stimulating discussion of this issue.
the presale on day \( t = t_1 \) will demand a contract price of \( V \) so that the buyer will pay the down payment \( Q \) on the day the deal is made (that is, \( t = t_1 \)). On the second payment date, \( t_2 \), the buyer will pay \( Q_2 \). This carries on until the last payment day (usually the day of completion, \( T \)). Thus, the contract price is \( V = Q + \sum_{i=2}^{n} Q_i \). For each payment date, the buyer can choose to continue with the payments or to default by paying a forfeiture charge \( A \) to terminate the presale contract.

To capture its randomness, we let the selling price of the property \( S_t \) follow a geometric Brownian motion, or

\[
dS_t = \mu S_t \, dt + \sigma S_t \, dw_t, \tag{1}
\]

where \( \mu \) is the risk-adjusted expected growth rate of the property price, \( \sigma \) is the instantaneous standard deviation of the return on the asset (\( \mu \) and \( \sigma \) are constant and known) and \( w_t \) is the Wiener process (i.e., a random variable drawn from a normal distribution with \( E(dw_t) = 0 \), and \( Var(dw_t) = dt \)). Equation (1) states that the instantaneous change in property price is its rate of return plus the standard deviation times the instantaneous change in the Wiener process governing its randomness. Given this, \( S_t \) is log-normally distributed such that \( \ln(S_t) \) is normally distributed over a finite time interval \( \bar{t} \) with mean \((\mu - \frac{1}{2}\sigma^2)\bar{t}\) and variance \( \sigma^2\bar{t} \).

In a two-payment case, the buyer pays the down payment at the time of purchase \( (t = t_1) \) upon entering into the presale contract, followed by one payment at the completion date \( (t = T) \). The down payment can be considered as the premium on a presale option that can be exercised by making the second payment. The value of the presale option at time \( t \) is a function of the current property price and the time remaining before the development is completed. Using Ito's Lemma and with appropriate definitions of the boundary conditions, we can see (from Appendix A) that

\[
C(S_t, t) = S_t \{N(d) + \sigma \sqrt{T-t} - (M - \eta)N(d) - \eta)\}, \tag{2}
\]

where

\[
d = \frac{-\ln(M - \eta) - \frac{1}{2}\sigma^2(T-t)}{\sigma \sqrt{T-t}},
\]

\( M \) is the percentage of the expected spot price represented by the final payment, and \( \eta \) is the forfeiture charge reflecting the proportion of the expected spot price.

Equation (2) shows that the value of the presale option to the buyer is the difference between the expected property market price (the first term on the
right) and the present value of the last payment if paid, minus the possibility of paying a forfeiture charge when the buyer decides not to continue with the contract. This option value, \( C(S_t, t) \), if evaluated at \( t = t_1 \) is essentially the down payment required at the time of sale. This completes the two-payment presale agreement with a simple presale option.

It is important to note that, in this valuation process, we let the amount of the second payment \( ME(S_T) \) be exogenously determined as a percentage \( (M) \) of the expected future spot price at the time of the last payment, \( E(S_T) \). Given this, buyers will determine a corresponding option value \( C(S_t, t) \) for any given level of \( M \) of the second payment determined by the developer. Consequently, the developer can choose the best \( M \) from a menu of presale contracts to form the best presale contract for herself/himself. (We will address the contract design issue in a later section.)

This analysis can be extended to the case where there are three payments: the initial down payment and two more payments. Each payment (until the final) enables the buyer to engage in a further contract. That is, the initial payment enables the buyer to buy the first presale option, which in turn enables the buyer to buy the second presale option that finally results in the decision to make the concluding payment to acquire the property. From Appendix B, we show that the solution to the three-payment case is given by

\[
C_1(S_t, t) = S_t N_2(d_1) + \sigma \sqrt{2 - t} d_2 + \sigma \sqrt{3 - t} \sqrt{2 - t}/(3 - t) - (M - \eta) S_t N_2(d_1, d_2; \sqrt{(2 - t)/(3 - t)}) - (q_2 - \eta) S_t e^{r_T} N_1(d_1) - \eta S_t (1 + e^{r_T})
\]

(3)

where

\[
\tilde{d}_1 = \frac{\ln \left( \frac{S_t}{\bar{S}} \right) + \left( r_f - \frac{1}{2} \sigma^2 \right)(2 - t)}{\sigma \sqrt{2 - t}}, \quad \tilde{d}_2 = \frac{-\ln(M - \eta) - \frac{1}{2} \sigma^2(3 - t)}{\sigma \sqrt{3 - t}}
\]

\( N_2(\cdot) \) is the bivariate cumulative normal distribution, \( N_1(\cdot) \) is the univariate cumulative normal distribution and \( \bar{S} \) is the critical value of \( S \) such that \( C_2(\bar{S}, 2) - q_2 S_t e^{2r_T} = 0 \).

To summarize, in a three-payment case, the down payment is \( Q = C_1(S_1, 1) \). At \( t = 2 \), the buyer will compare the capital gain with a forfeiture charge to decide if she/he should exercise the option. If it is optimal to pay the forfeiture charge, then the presale agreement is no longer valuable and the buyer will instead pay the forfeiture charge to terminate the agreement. Otherwise, the buyer will pay and wait till \( t = 3 \) when she/he once again compares the capital
gain with the forfeiture charge. If the capital gain is greater, the buyer will make the final payment and own the unit. It is clear that a presale contract can be easily extended to incorporate an $n$-payment case with cumulative normal distribution functions.\(^9\)

**Should a Developer Launch a Presale?**

In the previous subsection, we calculated the option value of a presale contract assuming that a developer will launch a presale and receive a down payment at time $t = t_1$. In this section, we will examine whether or not it is optimal for a developer to launch a presale. For simplicity of presentation, we use a two-payment case as the example and will only examine the condition when developers are risk averse.

To do this, we need to specify explicitly the expected payoff to a developer at any time $t$ with or without default. Recall that the call option value to the buyer, $C(S_t, t_1)$, is essentially the down payment required at the time of presale. Therefore, the contract price for a presold property is

$$V = Q + ME(S_T) = C(S_{t_1}, t_1) + MS_{t_1}e^{r(T-t_1)}.$$ (4)

If there is no default, the developer will receive the second payment. Otherwise, she/he will receive a forfeiture charge and then resell the unit at the spot price on date $T$ less the cost of resale. Such a payoff pattern is analogous to having a synthetic option with a payoff pattern shown in Table 1. From Appendix C, it is clear that the expected payoff to a developer from a presale at $t = t_1$ is

$$H(S_t, t) = C(S_t, t) + H_2(S_t, t)$$

$$= S_t\left(p' + e^{-r_0} p_1' - e^{-r_0} p_1\right) - (M - \eta)S_t\left(p - e^{-r_0} p\right)$$

$$- cS_t e^{-r_0} (p_1 - p) - (1 - e^{-r_0})\eta S_t,$$ (5)

where

$p = N(d) \quad \text{and} \quad p' = N(d + \sigma \sqrt{\alpha})$

$p_1 = N(d_1) \quad \text{and} \quad p_1' = N(d_1 + \sigma \sqrt{\alpha})$

$$d = \frac{-\ln(M - \eta) - \frac{1}{2} \sigma^2 \alpha}{\sigma \sqrt{\alpha}} \quad \text{and} \quad d_1 = \frac{-\ln c - \frac{1}{2} \sigma^2 \alpha}{\sigma \sqrt{\alpha}}.$$

Table 1: Payoff pattern of the presale synthetic option of the developer.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Scenarios</th>
<th>Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_r &lt; C$</td>
<td>$C &lt; S_r &lt; (M - \eta)S_r e^{r_f(T-t)}$</td>
<td>$(M - \eta)S_r e^{r_f(T-t)} &lt; S_r$</td>
</tr>
<tr>
<td>Risk-free investment</td>
<td>$\eta S_r e^{r_f(T-t)}$</td>
<td>$\eta S_r e^{r_f(T-t)}$</td>
</tr>
<tr>
<td>Long call with exercise price $C$</td>
<td>$0$</td>
<td>$S_r - C$</td>
</tr>
<tr>
<td>Short call with exercise price $(M - \eta)S_r e^{r_f(T-t)}$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Binary call at exercise price $(M - \eta)S_r e^{r_f(T-t)}$ and payoff $C$</td>
<td>$0$</td>
<td>$C$</td>
</tr>
<tr>
<td>Payoff</td>
<td>$\eta S_r e^{r_f(T-t)}$</td>
<td>$S_r - C + \eta S_r e^{r_f(T-t)}$</td>
</tr>
<tr>
<td>Payoff representation</td>
<td>Not even resale is suitable</td>
<td>Resale when default occurs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Second payment (no default)</td>
</tr>
</tbody>
</table>

$S_r$ is the spot price at time $t$, $S_r$ is the spot price at time $T$, $r_f$ is the risk-free rate, $M$ is the percentage representing the second payment, $\eta$ is the percentage representing the forfeiture charge, $T - t$ is the time interval between two payments and $C$ is the cost of resale.
The effect on the developer’s profit from the presale as a function of the final payment is given by
\[
\frac{\partial H}{\partial M} = -S_t(1 - e^{-\rho t})p - cS_t e^{-\rho t} \frac{N'(d)}{\sigma \sqrt{\alpha(M - \eta)}} < 0. \tag{6}
\]

Examining (6), it can be seen that the developer’s profit is a decreasing function of \( M \). However we can also see that a sale upon completion would be equivalent to the case where \( M \) approaches its maximum value of unity. It therefore follows that a presale is superior to selling upon completion at any time when a developer needs to make a decision.\(^{10} \) This implies that when launching a presale is an option, the developer should launch the presale as soon as possible.

Factors Affecting a Presale Decision

In the previous subsection, we showed that it is optimal for a developer to launch a presale when permitted. We are now in a position to examine the impact of other factors on the decision of a developer to launch a presale. Since the impact of \( c \) (the cost of presale) on a developer’s decision is relatively small, without loss of generality, we set \( c = 0 \) to highlight the impact of volatility \( \sigma \), risk-free rate \( r_F \), spot market price \( S_t \), risk premium of the developer \( r \) and the length of time \( \alpha \) on the expected payoff of a presale decision. Under such a condition, \( p'_i = 1 \). Thus, we can rewrite a developer’s expected payoff as
\[
H(S_t, t) = C(S_t, t) + e^{-r\alpha} \left( [S_t - C(S_t, t)] \right). \tag{7}
\]

From Equation (7), we obtain
\[
\frac{\partial H}{\partial \sigma} = (1 - e^{-\rho t}) \frac{\partial C}{\partial \sigma} > 0,
\]
\[
\frac{\partial H}{\partial r_F} = (1 - e^{-\rho t}) \frac{\partial C(S_t, t)}{\partial r_F} + \alpha e^{-\rho t}(S_t - C(S_t, t)) > 0,
\]
\[
\frac{\partial H}{\partial S_t} = (1 - e^{-\rho t}) \frac{\partial C}{\partial S_t} + e^{-\rho t} > 0,
\]
\[
\frac{\partial H}{\partial r} = -\alpha e^{-\rho t}(S_t - C(S_t, t)) < 0,
\]
and
\[
\frac{\partial H}{\partial \alpha} = (1 - e^{-\rho t}) \frac{\partial C(S_t, t)}{\partial \alpha} - r e^{-\rho t}(S_t - C(S_t, t)).
\]

\(^{10} \) An alternative way to derive the same conclusion is to compare the profit of the developer using the presale method with that of using the sale-upon-completion method. The profit to the developer is always higher with the presale method.
From the derivative $\partial H/\partial \sigma = 0$, we know that the profit of a presale contract increases with an increase in demand volatility. This is true because, from the finance literature, we know that the value of a call option increases with an increase in demand volatility (or $\partial C/\partial \sigma > 0$). Since the purpose of a presale contract is to share risk with buyers by securing at least a portion of the sale proceeds of the property in the form of a down payment, a presale contract must be of more value to a developer if demand volatility is high. The profit from a sale upon completion is $e^{-r\sigma}S_t > 0$, which is independent of the demand volatility if the risk premium $r$ is constant. Another way to think about this proposition is to set $\sigma = 0$. If there is no uncertainty about the demand level (hence $r = 0$), it makes no difference if a developer sells the property now or at completion. Indeed, under this circumstance, there is no need for developers to presell their projects. Given this, it is more likely for a developer to conduct a presale when demand uncertainty is high.

Since $\partial H/\partial r_F > 0$, an increase in the risk-free rate implies that a presale contract is more valuable to the developer. However, holding risk aversion $r'$ constant and since an increase in the risk-free rate will increase the profit from a sale upon completion, the incremental benefit from a sale upon completion (when compared with a presale) does not necessarily increase as the risk-free rate increases. This can be seen from the second derivative $\partial^2 H/(\partial r_F \partial M) = \alpha S_t e^{-r\sigma} p > 0$. The derivative means that, as the risk-free rate increases, the incremental benefits from a sale upon completion (when compared with a presale) actually decrease. Therefore, holding everything else constant, a developer is less likely to launch a presale as the risk-free rate increases.

It is obvious that $\partial H/\partial S_t > 0$. This indicates that the profit of a presale contract is an increasing function of the current spot market price. This proposition is reasonable because a buyer should be willing to pay more for a presale contract if the current spot market price of the same product commands a high price. Furthermore, since the profit from a sale upon completion is $e^{-r\sigma}S_t$, the incremental benefits from a sale upon completion (when compared with a presale) are $(1 - e^{-r\sigma})C(S_t, t) > 0$. Therefore, holding everything else constant, a developer is more likely to launch a presale as the current spot market price increases.

We also note that $\partial H/\partial r < 0$. This implies that the profit for launching a presale decreases when a developer is more risk averse. This is true because, holding everything else constant, an increase in the risk-aversion level decreases the present value of the property upon completion. Holding everything else constant, the profit of a developer must be reduced because the present value of the second payment is reduced. However, higher risk aversion also leads to a lower benefit from a sale upon completion. This is true because
\[ \partial^2 H(\partial r \partial M) = -\alpha S_t e^{-\alpha r} p < 0. \] This derivative implies that the incremental benefits from a sale upon completion (when compared with a presale contract) increase as the risk aversion increases. Given this, a developer is more likely to launch a presale when the risk-aversion level increases.

The impact of the time to maturity on a developer’s profits is ambiguous because the sign of \( \partial H / \partial \alpha \) is unknown. Although \( [\partial C(S_t, t)] / \partial \alpha > 0 \) and \( S_t - C(S_t, t) > 0 \), we need to know the magnitudes of both terms in order to make a judgment. In other words, while a longer time period makes a call option more valuable, it also makes the expected present value of the second payment less valuable. Given this, whether the length of the maturity period has a positive or negative effect on a developer’s profits depends on which force dominates the other. However, a longer time period makes a sale upon completion less valuable because the profit from a sale upon completion is \( e^{-\alpha r} S_t \), which is a decreasing function of \( \alpha \). Given this, the incremental benefits from a sale upon completion (when compared with a presale) are \((1 - e^{-\alpha r}) C(S_t, t) > 0\). Therefore, holding everything else constant, a developer is more likely to launch a presale as the time to maturity increases.

**Contract Design: Optimal Exercise Price**

When a developer makes a presale decision, she/he will have to decide on the magnitudes of the last payment (the exercise price of the option) and the down payment (the call option). Clearly, there are many combinations of down payments and last payments that developers can choose from. However, we can now attempt to identify the combination that will be optimal for the developer.

Before signing a presale contract, buyers would like to make sure that a developer will deliver the product at the option exercise date. Given this, they will demand that the down payment be not more than the asset in place (the construction cost already sunk into the project) so that the developer will not be better off by taking the down payment and defaulting on the presale option contract. Buyers also realize that developers might have an incentive to avoid delivering the product (or deliver an inferior product) if there is a negative demand shock in the market (the future spot price drops significantly) at the time of delivery. Given this, to ensure that developers will complete the project, buyers would like to ensure that the exercise price is greater than the remaining construction costs required to complete the project.

With this in mind and holding everything else constant, buyers will be willing to pay a higher price (the present value of both the down payment and the last payment) for the presale contract if the exercise price is high or if the developer is reputable (so that the buyers know that the developer will honor...
the contract when the market condition is bad). In other words, the value of a presale contract will be positively correlated with the exercise price of the option, regardless of the reputation of the developer. In addition, holding the level of the second payment constant, the market will penalize less reputable developers more than those who are more reputable when a low down-payment contract is used.

Given this, it is reasonable to argue that for any given exercise price a developer chooses, buyers will respond with a down payment level they are willing to pay based on the magnitude of the exercise price and the reputation of the developer. Consequently, when designing presale contracts, developers are in fact choosing from a menu of exercise prices, with the expected future spot prices and value of the asset in place serving as two possible boundary conditions. Clearly, developers will select a high down payment if their borrowing rate is high. Less reputable developers will also be obliged to select a high exercise price in order to attract buyers.

We are now in a position to examine a more complicated (and more realistic) scenario. That is, we will take the developer’s reputation and financial strength into consideration. So far in the model, we have assumed that the developer will deliver the prespecified product, and that the proportion $M$ is exogenously determined. We now revise the model to allow buyers to assign a probability that they will receive an inferior product when exercising the option. The probability of receiving a product that is exactly specified in the contract is higher with a more reputable developer and if the exercise price is high. In addition, there will be a joint effect between reputation and exercise price. When a reputable developer sets a low exercise price, the buyer will worry less than when a less reputable developer sets an identical exercise price. This is true because a developer with an excellent reputation will still deliver the product (as specified in the contract) in order to sustain future development opportunities. When a developer does not deliver the product as specified, we assume that there is a cost to the developer. We term this a reputation cost.

To formalize the idea, let $\theta$ be the probability that a developer will deliver the product as specified in the contract. We use an index $K > 0$ to represent a developer’s reputation cost when the developer chooses to deliver an inferior product. We assume that the information on $K$ is readily available to all players in the market.\footnote{We can also treat $K$ as private information possessed by a developer. Under this circumstance, buyers will have to infer the reputation of a developer from her/his actions and our model will be able to include the signaling aspect of a developer’s actions. Work is currently underway to address this issue. We thank Charles Ward for providing a stimulating discussion on this issue.} A more reputable developer will have a higher reputation cost.
when she/he delivers an inferior product. The developer’s reputation cost is normalized to be zero if he/she delivers the product as prespecified \((i.e., \theta = 1)\). Thus, the expected reputation cost for a developer is \(K(1 - \theta)\) for a given probability \(\theta\) estimated by both the developer and the buyers.

Let the remaining construction cost of the building at the time of initiating the presale be \(I\). The magnitude of the remaining construction cost \(I\) can be a function of the developer’s cost of capital, but may be independent of the developer’s reputation. In other words, holding the completion level of the building constant, if the borrowing cost of a developer is high, then the remaining construction cost for this particular developer could be higher than another developer who has a lower borrowing cost.

Again, to simplify the presentation, when a developer decides to deliver an inferior product, we normalize the remaining construction cost to zero. Under this circumstance, we model the remaining expected construction cost as \(I \theta\). Given a developer’s reputation, the call option premium that a rational buyer is willing to pay in a two-payment case has to be adjusted to reflect the likelihood that a developer will deliver a prespecified product. To do this, we rewrite Equation (2) as

\[
C(S_i, t) = \theta S_i N(d + \sigma \sqrt{\alpha}) - X(1 - \eta) e^{-r_i \alpha} N(d) - \eta X e^{-r_i \alpha},
\]

(8)

where

\[
d = \frac{\ln(\theta S_i) - \ln[X(1 - \eta)] + \left( r_F - \frac{1}{2} \sigma^2 \right) \alpha}{\sigma \sqrt{\alpha}}.
\]

In Equation (8), we have used \(\theta S_i\) to highlight the idea that the quality of the final product is affected by the likelihood a developer will deliver the prespecified product (based on the reputation cost) and the magnitude of the second payment. Note that Equation (8) differs from Equation (2) not only because the quality factor is introduced, but also because the exercise price now affects the potential quality of the final product via \(\theta S_i\), thus resulting in the call option priced in a different way. With the call premium given in Equation (8), the developer’s expected payoff function can be revised as

\[
H(S_i, t) = C(S_i, t) + \theta S_i e^{-r \alpha} (p'_1 - p') + X(1 - \eta) e^{-r \alpha} p - c \theta S_i e^{-r \alpha} (p_1 - p) + \eta X e^{-r \alpha} - I \theta - K (1 - \theta) = \theta S_i (p'_1 + e^{-r \alpha} p'_1 - e^{-r \alpha} p') - X(1 - \eta) e^{-r \alpha} (p - e^{-r \alpha} p) - c \theta S_i e^{-r \alpha} (p_1 - p) - \eta X e^{-r \alpha} (1 - e^{-r \alpha}) - I \theta - K (1 - \theta),
\]

(9)
where
\[ p = N(d) \quad \text{and} \quad p' = N(d + \sigma \sqrt{\alpha}), \]
\[ p_1 = N(d_1) \quad \text{and} \quad p'_1 = N(d_1 + \sigma \sqrt{\alpha}), \]
\[ d = \frac{\ln(\theta S_t) - \ln[X(1 - \eta)] + \left( r_F - \frac{1}{2}\sigma^2 \right) \alpha}{\sigma \sqrt{\alpha}} \quad \text{and} \quad d_1 = \frac{-\ln c - \frac{1}{2}\sigma^2 \alpha}{\sigma \sqrt{\alpha}}. \]

Let \( H_2(S_t, t) = H(S_t, t) - C(S_t, t) \) be the developer’s expected payoff, which excludes the option premium. The optimal \( \theta \) can be solved by setting \( \frac{\partial H_2}{\partial \theta} = 0 \). Thus, we have
\[
\frac{\partial H_2}{\partial \theta} = S_t e^{-r \alpha}(p_1' - p') - c S_t e^{-r \alpha}(p_1 - p) + ce^{-r \alpha} \frac{S_t}{\sigma \sqrt{\alpha}} N'(d) - I + K = 0.
\]

Note that \( \frac{\partial H_2}{\partial \theta} \) is a homogeneous function of \( X \) and \( \theta \). Therefore, the optimal solution \( \theta \) must be a linear function of \( X \) and can be written in a form such as \( \theta = \lambda(K, I, c, r')X \). Note that \( r' = r + r_F \) and is the discount rate of a risk-averse developer (\( r \) is the risk premium above the risk-free rate). Putting \( \theta = \lambda(K, I, c, r')X \) back into Equation (10), we can obtain
\[
\frac{\partial \lambda}{\partial K} = \frac{\sigma \sqrt{\alpha} \lambda}{S_t e^{-r \alpha} N'(d + \sigma \sqrt{\alpha})} > 0,
\]
\[
\frac{\partial \lambda}{\partial I} = \frac{-\sigma \sqrt{\alpha} \lambda}{S_t e^{-r \alpha} N'(d + \sigma \sqrt{\alpha})} < 0,
\]
and
\[
\frac{\partial \lambda}{\partial r'} = \frac{-\sigma \sqrt{\alpha} \lambda (1 - p')}{N'(d + \sigma \sqrt{\alpha})} > 0.
\]

Note that, in the above calculation, we set \( c = 0 \) to highlight the effect of \( K, I \) and \( r' \) on \( \lambda \). However, the result will not change if \( c \) is relatively small.

We now know that \( \lambda(K, I, c, r') \) is an increasing function of \( K \), a decreasing function of \( I \) and an increasing function of \( r' \) and \( c \) (if \( c \) is relatively small). Replacing \( \theta \) by \( \lambda(K, I, c, r')X \) in Equation (8) we have \( H(S_t, t) \) as a linear function of \( X \). Thus, the optimal solution \( X \) for the developer must satisfy
\[
\lambda(K, I, c, r')X = 1.
\]

In other words, given our model setup, in equilibrium a developer will choose an exercise \( X \) at which she/he will provide a prespecified product with probability
one, regardless of the developer’s reputation cost $K$. Of course, in equilibrium the level of the second payment $X$ chosen by a developer is a function of $K$ and other factors, which can be seen from Equation (11). Intuitively, the optimal solution $X$ selected by a developer should be the minimal one above which the developer has no incentive to deliver an inferior product. Given this, the nominal contract price of a presale contract can be specified as the sum of $C(S_t, t)$ (see Equation (7)) and the optimal solution of $X$ (see Equation (11)), or

$$V = C(S_t, t) + \frac{1}{\lambda(K, I, c, r')}.$$  

Given that the optimal level of second payment selected by a developer is determined by setting

$$X = \frac{1}{\lambda(K, I, c, r')}$$

we can now discuss the change in $X$ in relation to changes in the remaining construction cost $I$, the developer’s reputation cost $K$ and the developer’s risk aversion factor $r'$. From our discussion about $\lambda(K, I, c, r')$, it is straightforward to obtain

$$\frac{\partial X}{\partial I} > 0, \quad \frac{\partial X}{\partial K} < 0, \quad \text{and} \quad \frac{\partial X}{\partial r'} < 0.$$  

Holding everything else constant, $\partial X / \partial I > 0$ implies that the optimal exercise price $X$ increases as the developer’s remaining construction cost $I$ increases. This makes intuitive sense. Keeping everything else constant, a high remaining construction cost increases a developer’s incentive to deliver an inferior good. A rational buyer will anticipate this effect, and thus will reduce the option premium she/he is willing to pay for the product. Given this, to obtain a high call premium and a high expected payoff, the developer will have to offer a high exercise price $X$ to signal her/his commitment to offer the prespecified product in the contract. Holding building quality and quantity constant, the remaining construction costs could be higher for developers with a higher borrowing cost. Consequently, a developer with a high borrowing cost might be forced to select a presale contract with a low down payment, while a developer with a lower borrowing cost can have the freedom to select a presale contract with a higher down payment.

When $\partial X / \partial K < 0$, holding everything else constant, the optimal exercise price $X$ will decrease as a developer’s reputation cost $K$ increases. Intuitively, a highly reputable developer will have less incentive to deliver an inferior product. Therefore, holding the exercise price constant, a rational buyer will be willing to pay a high option premium for a presale contract offered by a developer with a high reputation cost.
Finally, $\frac{\partial X}{\partial r'} < 0$ implies that the more risk averse a developer is, the lower the exercise price will be; risk-averse developers will choose a low exercise price. This confirms the intuition from the DCF approach that risk-neutral buyers can benefit the most from a presale when the developer is more risk averse. Similarly, for developers with high potential bankruptcy costs, the use of a presale method reduces their risk exposure.

It should be noted that the model results in a situation where less reputable developers will be forced to select presale contracts with low down payments and high exercise prices. This has important implications for the market structure in real estate markets. Clearly, we can safely assume that small and new developers will find it difficult to create a reputation for quality and reliability. Given this, the presale method undoubtedly establishes a barrier for small developers to enter into a new market. Such is the case with the Hong Kong (and Taiwan) residential market, where a small number of developers dominate the market. On the other hand, our model also indicates that a presale market could be suboptimal for buyers if they cannot distinguish between the quality of developers. In that situation, all developers will choose similar types of presale contracts and similar levels of down payments regardless of their inherent quality. Consequently, we would expect to observe more noncompleted projects in this type of immature market. The current circumstance of residential markets in some of the smaller cities in mainland China may reflect this situation.

Conclusion

Given its importance in (and impact on) the societies of many Asian countries, the presale system has received too little attention. In this paper, we analyze the rationale for the existence of the presale method and develop strategies to design a presale contract using a real-options framework. Our model provides three important results. First, it establishes that developers should optimally presell whenever they are allowed to. Second, the presale system exists because it allows the developer to share risk (and profit) with buyers. In this regard, it creates benefits for the society. Finally, different developers will choose different types of contracts, depending on their reputations and financial strengths. Given this, new developers face an entry barrier into existing markets.

In this paper we have used the sales of property units as the central focus of the model and have employed mainly examples of Asian cities as potential applications of the result of the model. However, there is no reason why the presale system cannot be applied to large development projects in other parts of the world.

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References


Appendix A: The Derivation of the Initial Call Option of the Presale Buyer

Let the value of the presale option of the buyer at time \( t \), or \( C(S_t, t) \), be a function of property price \( S_t \) and time \( t \). From Ito’s Lemma and Equation (1), the instantaneous change of the call option \( dC \) is

\[
dC = \sigma \frac{\partial C}{\partial S_t} dS_t + \left( \mu S_t \frac{\partial C}{\partial S_t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2} + \frac{\partial C}{\partial t} \right) dt. \tag{A1}
\]

As there is no interim income, the only return from the purchase of the unit is its capital gain. Since buyers are risk neutral, all assets can be evaluated at the risk-free rate \( r_F \), and \( r_F = \mu \). Likewise, the expected gain from holding the option, \( E(dC/C) \), is also the risk-free rate. Hence, substituting \( r_F \) for \( \mu \) and taking expectation on both sides of Equation (A1) and rearranging the terms, we obtain

\[
r_F S_t \frac{\partial C}{\partial S_t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2} + \frac{\partial C}{\partial t} - r_F C = 0. \tag{A2}
\]

Three conditions are required to solve for \( C \). They are:

\[
C(0, t) = 0, \quad \text{if } S_t \to 0,
\]

\[
C(S_T, t) = S_T, \quad \text{if } S_t \to \infty.
\]

\[
C(S_T, T) = \operatorname{Max} \left( S_T - M S_T e^{r_F(T-t)}, -A \right)
= \operatorname{Max} \left( S_T - M S_T e^{r_F(T-t)}, -\eta S_T e^{r_F(T-t)} \right)
= \operatorname{Max} \left( S_T - (M - \eta) S_T e^{r_F(T-t)}, 0 \right) - \eta S_T e^{r_F(T-t)}. \tag{A3}
\]

\( MS_T e^{r_F(T-t)} \) is the amount of the last payment and is a function of \( M \) (a percentage) and \( S_T e^{r_F(T-t)} \) (the expected property spot price at the end of the last period).

The forfeiture charge \( A \) is specified as a percentage \( (\eta) \) of the expected spot market price \( S_T e^{r_F(T-t)} \). The first condition indicates that when the property price goes to zero, the option has no value. The second condition states that if the price at any time \( t \) rises to infinity, the option will be as valuable as the
underlying asset. The last condition represents the profit on the exercise date. Note that the last condition also ensures that there will be no early exercise. We define that call option as having an exercise price equal to the second payment minus the forfeiture charge. From this, it follows that buying a presale unit is in fact buying the call option and taking out a loan equal to the forfeiture charge. If the call is exercised, the buyer pays the second payment (which is the repayment of the loan plus the exercise price). If the call is not exercised, the buyer will merely repay the loan by paying the forfeiture charge. The combined value of this synthetic option is the solution to the partial differential Equation (A2) subject to the boundary conditions in Equation (A3), or
\[ C(S_t, t) = S_t[N(d + \sigma \sqrt{T - t}) - (M - \eta)N(d) - \eta], \]  
(A4)

where
\[ d = \frac{-\ln(M - \eta) - \frac{1}{2} \sigma^2 (T - t)}{\sigma \sqrt{T - t}}. \]

Appendix B: The Derivation of the Presale Option in a Three-Payment Contract

In a three-payment contract (see Figure 5B), after the down payment at \( t = 1 \), there will be two more payments at \( t = 2 \) and \( t = 3 \). As defined before, \( Q, Q_2 \) and \( Q_3 \) are the payments at the end of periods 1, 2 and 3, respectively. Notice that \( Q + Q_2 + Q_3 = V \) (the contract price). In this case, the buyer purchases the first presale option, \( Q = C_1(S_t, 1) \), at \( t = 1 \) by making the down payment. This option gives him the right to buy the second option, \( C_2(S_2, 2) \), at \( t = 2 \) by making the second payment. If \( C_2 \) is exercised at the end of period 3 (by making the last payment), the buyer will possess ownership of the property (valued at \( S_3 \)). At the same time, \( C_1 \) gives the right to default on or before the second payment (i.e., \( t \leq 2 \)); whereas \( C_2 \) gives the buyer the right to default on or before the final payment (i.e., \( t \leq 3 \)). The first option \( C_1 \) is, hence, a compound option on a simple option \( C_2 \). The partial differential equations of the two presale options, \( C_1 \) and \( C_2 \), are
\[ r_F S_t \frac{\partial C_1}{\partial S_t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 C_1}{\partial S_t^2} + \frac{\partial C_1}{\partial t} - r_F C_1 = 0 \]  
(A5)

and
\[ r_F S_t \frac{\partial C_2}{\partial S_t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 C_2}{\partial S_t^2} + \frac{\partial C_2}{\partial t} - r_F C_2 = 0. \]  
(A6)

Assume further that \( Q_3 = MS_t e^{2rt} \). Since the presale option at \( t \geq 2 \) is a simple option, it can be valued as a two-payment case discussed earlier, or
\[ C_2(S_t, t) = S_t[N(d + \sigma \sqrt{T - t}) - (M - \eta)N(d) - \eta]. \]  
(A7)
where

\[
d = \frac{-\ln(M - \eta) - \frac{1}{2} \sigma^2 (3 - t)}{\sigma \sqrt{3 - t}}
\]

and Equation (A7) is the solution to Equation (A6).

To evaluate the solution for Equation (A5), the boundary conditions of both \(C_1\) and \(C_2\) must be satisfied simultaneously. In addition to the three conditions stated in Equation (A3) (with \(T = 3\), and the payments become \(M S_1 e^{2r_1}\) and \(\eta S_1 e^{2r_1}\)), we also have

\[
C_1(S_2, 2) = \max \left( C_2(S_2, 2) - Q_2, -\eta S_1 e^{2r_1} \right)
\]

and

\[
C_2(S_3, 3) = \max \left( S_3 - M S_1 e^{2r_1}, -\eta S_1 e^{2r_1} \right).
\]

The two conditions in Equation (A8) are the payoffs of the two options at the expiry dates, \(t = 2\) and \(t = 3\), respectively. Similar to the assumption we made for the last payment in the two-payment case, we assume that the second payment in the three-payment case is \(Q_2 = q_2 E(S_T) = q_2 S_1 e^{2r_1}\), where \(q_2 + M \leq 1\). (It should be noted that, at this moment, the level of the second payment is still exogenously determined in the model.) Following Geske (1979), the solution to the partial differential Equation (A6) subject to the boundary conditions (A3) and (A8) is

\[
C_1(S_t, t) = S_t N_2(\tilde{d}_1 + \sigma \sqrt{2 - t}, \tilde{d}_2 + \sigma \sqrt{3 - t}; \sqrt{(2 - t)}/(3 - t))
\]

\[
- (M - \eta) S_t N_2(\tilde{d}_1, \tilde{d}_2; \sqrt{(2 - t)}/(3 - t))
\]

\[
- (q_2 - \eta) S_t e^{r_1} N_1(\tilde{d}_1) - \eta S_t (1 + e^{r_1})
\]

where

\[
\tilde{d}_1 = \frac{\ln \frac{S_t}{S} + \left( r_F - \frac{1}{2} \sigma^2 \right) (2 - t)}{\sigma \sqrt{2 - t}},
\]

\[
\tilde{d}_2 = \frac{-\ln(M - \eta) - \frac{1}{2} \sigma^2 (3 - t)}{\sigma \sqrt{3 - t}}.
\]

\(N_2(\cdot)\) is the bivariate cumulative normal distribution, \(N_1(\cdot)\) is the univariate cumulative normal distribution and \(S\) is the critical value of \(S\) such that \(C_2(S, 2) - q_2 S_1 e^{2r_1} = 0\). As Equation (A9) shows, the compound option value at the time of signing the presale agreement consists of the expected value of
the property price, the net of the two subsequent payments and any forfeiture charge should a default arise. The difference between Equation (A9) and Equation (A4) is caused by the option value at $t = 2$ (i.e., $C_2(S_2, 2)$, or the right to default at $t = 3$).

Appendix C: The Derivation of the Value of the Payoff to the Developer

Evaluated at the first payment date, $t_1$, the value of the synthetic option that expires on the second payment date is the sum of four cash flows and options evaluated by the Black–Scholes formula. Those four items are:

1. $\eta S_t e^{-r\alpha}$,

2. $e^{-r\alpha} S_t N(d_1 + \sigma \sqrt{\alpha}) - e^{-r\alpha} C N(d_1)$,

3. $(M - \eta) S_t e^{-r\alpha} N(d) - e^{-r\alpha} S_t N(d + \sigma \sqrt{\alpha})$, and

4. $e^{-r\alpha} C N(d)$,

where $\alpha = T - t_1$ is the time interval between the two payments, $r = r' - r_F$ ($r'$ is the discount rate of a risk-averse developer and $r$ is the risk premium above the risk-free rate), and

$$d = \frac{-\ln(M - \eta) - \frac{1}{2} \sigma^2 \alpha}{\sigma \sqrt{\alpha}}, \quad d_1 = \frac{\ln S_t - \ln C + \left(r_F - \frac{1}{2} \sigma^2\right) \alpha}{\sigma \sqrt{\alpha}}.$$

If we further simplify the situation by assuming the cost of resale is dependent upon the expected future property price (i.e., $C = c E(S_T) = c S_t e^{r(T-t_1)}$), then the expected payoff of a developer from a presale at $t = t_1$ is

$$H(S_t, t) = C(S_t, t) + H_2(S_t, t)$$

$$= S_t \left[p' + e^{-r\alpha} p'_1 - e^{-r\alpha} p'\right] - (M - \eta) S_t(p - e^{-r\alpha} p) - c S_t e^{-r\alpha} (p_1 - p) - (1 - e^{-r\alpha}) \eta S_t,$$

where $H_2(S_t, t) = S_t e^{-r\alpha} [p'_1 - p'] + (M - \eta) p - c(p_1 - p) + \eta$ represents the expected present value of the second payment, and

$p = N(d)$ and $p' = N(d + \sigma \sqrt{\alpha})$

$p_1 = N(d_1)$ and $p'_1 = N(d_1 + \sigma \sqrt{\alpha})$

$$d = \frac{-\ln(M - \eta) - \frac{1}{2} \sigma^2 \alpha}{\sigma \sqrt{\alpha}}$$

and $d_1 = \frac{-\ln c - \frac{1}{2} \sigma^2 \alpha}{\sigma \sqrt{\alpha}}$. 

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