Overbuilding: A Game-Theoretic Approach

Ko Wang* and Yuqing Zhou**

The persistence of excess vacancy has long been documented in the literature. We propose that, because vacant land does not produce income, there is a tendency for developers to build whenever they can identify a development opportunity. Since developers have to compete with each other for the development opportunity, in the aggregate, developers will supply more units than the demand in the market. In the face of an oversupply, we show that, under certain circumstances, developers will not lower the rental rate to eliminate vacancy space. Our model also has implications for investment decisions dealing with projects that could take advantage of existing but not fully utilized assets.

Over the past few decades, an extensive literature on the persistence of excess vacancy of property markets has been developed. Initially, researchers concentrated on the volatile patterns of housing starts, which Maisel (1963, p. 539) characterized as having “earned the dubious distinction of ranking among the most cyclically volatile industries.” Gradually, research interest has shifted to the question of the persistence of excess vacancy in not only the U.S. housing market, but also in international and commercial property markets. However, it is interesting to note that, after a few decades of research and the introduction of numerous theoretical models and empirical evidence, a recent paper of Bar-Ilan and Strange (1996, p. 619) still concludes that the “overbuilding phenomenon is a puzzle.”

In the literature, the explanations for oversupply can be categorized into three groups. The first group attributes the oversupply to agency problems, irrationality, and government policies. They argue that as long as developers are able to raise equity funds from syndications (and/or partnerships) or borrow using non-recourse loans, there is no reason for developers not to continue building, since lenders and equity investors will bear the risk.

*School of Hotel Management, Faculty of Business Administration, The Chinese University of Hong Kong, Shatin, Hong Kong and California State University-Fullerton or kowong@cuhk.edu.hk.
**The Chinese University of Hong Kong, Shatin, Hong Kong.

1 For an example of recent empirical evidence of real estate cycles in international or commercial real estate markets, see Renaud (1997). For a good study on the persistence of vacancy rates among U.S. cities, see Grenadier (1995a).
Indeed, high-income investors in the early 1980’s were eager to invest in real estate for its favorable tax advantages. For them, the fundamentals of a real estate project may not be the only consideration. Before the bailout of the savings and loan associations, less profitable (or perhaps insolvent) lenders were encouraged to lend on risky real estate investments on which they could charge high interest rates while allowing the government to bear the final risk. This type of explanation (relying partially on irrational behavior) has some merits, but because real estate cycles are long-lasting, it is safe to conclude that the agency issues of the 1980’s cannot be the only explanation for the observed oversupply.

The second group of arguments relies on a construction lag and forecasting errors to explain oversupply and cycles. Because construction takes time, developers have to project future demand before they start construction. Although it is possible that projection errors could cause the observed cycles in real estate markets, this explanation has at least two weaknesses. First, although it is reasonable to expect an individual developer to make projection errors, it is difficult to understand why all developers in the market would make systematic errors so that the errors did not offset each other. This is particularly true when each developer is able to observe the moves of other developers. When one developer observes that another developer is building, she/he should take this into consideration when estimating the net demand of the market. Second, for commercial properties the typical construction period ranges from 1 to 3 years (residential properties have even shorter construction periods). For large cities such as Los Angeles and Houston, it is difficult to understand why the errors accumulated in 1 to 3 years could cause an oversupply with a magnitude of more than a 20% vacancy rate. These two arguments indicate that the construction lag alone cannot fully explain the oversupply phenomenon.

The third group of arguments relies on exercise strategies for development options (the value of waiting and the cascade effect) to explain a developer’s optimal investment decision. Because of uncertainty about the future demand, a developer might not want to exercise a development option until she/he feels more comfortable with the demand condition in the market. [See, for example, Titman (1985) for a discussion of this argument.] On the other hand, the opportunity cost of waiting is the forgone income from the project during the waiting period. With construction lags, a firm might hasten, not delay, an investment decision to make sure that the firm will not be out of the market when the economic condition improves. [See, for example, Bar-Ilan and Strange (1996) for a discussion of this argument.] Finally, the strategic exercise of development options among developers could cause a concentration of construction activities in certain periods (the
cascade effect). Under certain circumstances (such as when demand level is dropping), developers in the market might build simultaneously in order to avoid preemption. [See, for example, Grenadier (1996) for this explanation.] While those models shed some light on the optimal decision rules for an individual developer, their implications for overbuilding in real estate markets have yet to be clearly developed.

This paper takes a different approach, which does not rely solely on the uncertainty of future demand, to develop an explanation for the oversupply phenomenon. We model developers’ investment decisions in two stages. In the first stage, developers decide on the number of units to build, with anticipated rental rates to be determined in the second stage of the game. In this stage, developers try to maximize their profits by selecting the optimal supply of units. In the second stage, developers will determine the rental rate based on the supply decisions they made in the first stage. In this stage, since buildings are already completed, developers will maximize their total revenues by selecting the optimal rental rate (or the revenue-maximizing rental rate). In our model we show that if the number of units supplied by developers in the first stage is larger than the demand based on the revenue-maximizing rental rate, developers might not lower the rental rate to eliminate the vacancy and thus cause an oversupply in the market.

The oversupply phenomenon is due to a simple fact: that vacant land does not produce income. To see this, consider a property market in which a real property can be rented at 10% of its land and construction costs. Developers should build as soon as they can identify a demand for the product. By developing the land into a property, developers will begin to receive rental income from the land component, which is not available if the land stays undeveloped.

When there are a number of developers in the market and each holds enough land for development, they will compete for the opportunity to develop once a potential demand for space is identified. Of course, for a given developer, the best scenario is that she/he develops and captures the demand while others do not build. The worst scenario is that all developers build enough units to meet the estimated demand. An acceptable solution seems to be that each developer builds a proportionate share of the demand. However, because there is a profit to share (arising from the rent of the land) and developers can compete for tenants from existing properties, each developer will eventually supply more units to the market than her/his proportional share of the estimated demand. Under certain conditions and when the total supply is higher than the revenue-maximizing demand level, we will observe an overbuilt market.
Our model also carries several testable implications that are supported by the empirical evidence reported in the literature. It predicts that office, hotel and high-end residential property markets have a greater probability of oversupply than multifamily, industrial and low-end residential property markets. It also predicts that suburban areas or newly developed cities have a greater tendency to oversupply than central business districts or well established cities. Our model also has some implications for general investment practices. It predicts that developments are more likely to occur if a project can use some existing, but not fully utilized, resources. It is fair to say that the completion of the fiber-optic cable system encourages the rapid development of the numerous applications in the multi-media and superhighway fields.

Our conclusion that the competition for development opportunities leads to overbuilding is similar in spirit to the arguments presented by several studies addressing issues of excess capacity and inefficient use of resource. For example, Spatt and Sterbenz (1985) indicate that there is an incentive to undertake a project early on just to preempt potential competitors. Mills (1989) argues that if scarce development rights are rationed by local authorities on a first come first-served basis, there will be an incentive for preemptive developments. Eaton and Lipsey (1980) present an industrial organization model to explain wasteful entry. Pindyck (1993) notes that the uncertainty over market demands affects investments through the feedback of industry-wide capacity expansion on the distribution of prices. To a certain extent, we incorporate some of these ideas into the development of our model.

The remaining sections of this paper are organized as follows. In the next section, we will demonstrate that there are incentives for developers in the aggregate to supply more units to the market than the estimated demand. Once the market is overbuilt, we show that it might be optimal for developers not to lower the rental rate to eliminate vacant space. The subsequent section develops testable implications from the model. The model predicts that certain areas or property types will have a larger tendency to oversupply. The next section describes a developer's decision-making process under a monopoly. The next section examines the model implications, assuming that tenants cannot move freely from existing buildings to newly constructed properties. The last section contains our conclusions.

The Model

Our overbuilding model can be considered as a two-stage infinite-horizon non-cooperative game among developers. There are $N$ developers in our
game who hold developable sites that can be developed to meet the demand level. The game is divided into two stages. In stage 1 (at time zero), each developer simultaneously and independently decides to build a certain number of real properties to meet the demand level. In stage 2 (time periods 1 and beyond), given the available supply and demand in the market, developers will select the optimal rental price for their properties.

In order for developers to make the investment decision in stage 1 of the game, they will have to first anticipate the rental price in stage 2. In our model, we demonstrate that in stage 2 of the game it is optimal for developers to collude on a rental price (at a level where the total rental revenue is maximized) when the aggregate market supply is greater than the demand level at the revenue-maximizing rental price. We will then show that developers, realizing that at a certain level the rental price could be sticky at stage 2, will in the aggregate overbuild the market in stage 1.

Our model of overbuilding can be viewed as a special type of mixed game as defined by Brander and Harris (1984) and Davidson and De Necker (1990). This type of game describes settings in which players (in our case developers) collude in some but not in all dimensions (in our case, developers collude on the rental price but not on the investment decision). Our model is developed in three steps. We will first discuss the model framework, which specifies the profit maximization function of developers. We will then address the price-setting decision of developers in the second stage of the game. Given the developers' anticipated outcome at the second stage of the game, we will then discuss their investment decisions in the first stage of the game.

The Framework

Consider a property market with \( M \) units of existing inventory, a vacancy rate \( k \) and a projected incremental demand function \( d(r) \) for the next period, where \( r \) is the rental rate. There are \( N \) developers in the market. Each developer holds a sufficient amount of land, \( T \), that can be built to meet the projected demand. Before all developers make their development decisions,

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2 These types of models have been used in the excess-capacity literature and are motivated by the observation that firms frequently compete in some strategic dimensions, while choosing to cooperate in others (see Scherer 1980, Brander and Harris 1984, and Fershtman and Muller 1986). Firms find it more difficult to coordinate long-run decisions (such as investment, advertising, and research-and-development expenditures) than decisions concerning short-run variables (such as price and rent, among others).
the current market value of each unit of land is \( V \). All developers have similar information about the level of existing inventory \( (M) \) and the projected incremental demand function \( d(r) \). They also know that, besides themselves, there are \( N - 1 \) developers who can choose to build enough units to meet the demand level. Based on this knowledge, each developer will make a decision on how many units \( y_i \) she/he will supply to the market. Let \( \hat{y} = \sum_{i=1}^{N} y_i \) be the total number of units supplied to the market by all the \( N \) developers. Then \( \hat{y} \) can be decomposed into \( y_n \) and \( y_{-n} \), where \( y_{-n} = \sum_{i=1}^{N} y_i - y_n \).

When a developer decides to build, for each unit of the property the developer incurs a constant per-unit construction cost \( c \) and a per-unit opportunity cost \( L(y_{-n}) \) for the use of the land. If land supply is abundant (i.e., if \( T \) and \( N \) are large) and if the reduction of the land inventory is relatively small (i.e., \( y_{-n} \) is small compared to \( T \)), then the opportunity cost of land, \( L(y_{-n}) \), should be the same as its current market value \( V \). However, if a developer decides not to develop the land while others decide to build, the reduction of the potential supply of land could make the developer’s land more valuable. In this case, \( L(y_{-n}) \) can be higher than \( V \). In other words, we model the land cost as a non-decreasing function of \( y_{-n} \) (the total units built by other developers).

The per-unit rent of the real property will be determined by the supply and demand conditions of the market, or by \( R(M, k, d, \hat{y}) \), where \( M \) is the existing inventory, \( k \) is the vacancy rate, \( \hat{y} \) is the new inventory supplied by all developers, and \( d \) is the estimated demand function. The profit function of a developer, \( P_n \), is the difference between the rent and costs, which can be specified as

\[
P_n = R(M, k, d, \hat{y})y_n - cy_n + (T - y_n)L(y_{-n}) - TV. \tag{2.1}
\]

Note that if a developer decides not to build, Equation (2.1) indicates that the profit will be a non-negative quantity \( T[L(y_{-n}) - V] \). This can be zero if the supply decisions of other developers do not affect the land value. If a developer decides to build \( T \) units of the product, the profit will be \( T(R - c - V) \). We assume that, in equilibrium, all the developers will adopt the same strategy. Given this, a symmetric equilibrium result would be that \( \bar{y} = \)

\footnote{We did not make an explicit distinction between rent and price when we defined the profit \( P_n \). To deal with this issue, we can assume that there is a constant multiplier between price and rent. Under this circumstance, there is no need to include the constant in the model. Alternatively, we can view the construction and land costs as the portion of the total costs allocated to each period.}
\((\bar{y}_1, \ldots, \bar{y}_n)\), where \(\bar{y}_1 = \bar{y}_2 = \cdots = \bar{y}_n\). We also assume that each developer is risk-neutral.\(^4\)

Consequently, in equilibrium, the optimal units supplied by developer \(n\), or \(\bar{y}_n\), should satisfy

\[
\bar{y}_n = \arg \max_{y_n=0} E(P_n)
\]  \hspace{1cm} (2.2)

where

\[P_n = R(M, K, d, \bar{y}_{-n}, y_n)\bar{y}_n - cy_n + (T - y_n)L(\bar{y}_{-n}) - TV\]

and

\[\bar{y}_1 = \bar{y}_2 = \cdots = \bar{y}_n\]

**Second Stage: Pricing Decision**

The literature on static or finite-horizon models (in which the component games have a unique Cournot–Nash equilibrium) indicates that collusive outcomes cannot emerge in equilibrium as the result of a non-cooperative game played by profit-maximizing developers. However, the basic insight of repeated games is that if a market situation can be repeated infinitely, the player (in our case, the developers) may settle at a collusive price (in our case, the rental rate) even if they are not explicitly colluding [for example, see Friedman (1971) and Benoit and Krishna (1985)]. Because developers can change the rental rate of the vacant space of their properties at any time, the second stage of our game can be viewed as a price game that can be repeated infinitely. This property will ensure that collusive outcomes may arise when developers decide on the rental rate at the second stage of the game.

However, in order to have collusive outcomes, there must be enforceable “grim trigger strategies.” These strategies specify that developers should remain at the collusion point unless someone can benefit by cheating (deviating from the collusion point). In addition, if at any time any developer is detected cheating, all developers revert to the static Cournot–Nash equilibrium and remain there forever. Given this, developers will cheat if

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\(^4\) Our results are substantially the same if we assume the developer is risk-averse. We will discuss this issue at the end of the subsection “First Stage: Investment Decision.”
and only if their immediate gains from cheating dominate the capitalized value of losses due to retaliation.

To formalize the semi-collusive model, let $\hat{\pi}_n$ denote the per-period profit earned by developer $n$ at a collusive rental price point, $\hat{r}$, let $\pi'_n$ be the per-period profit earned by developer $n$ when cheating optimally by setting the rental rate below $\hat{r}$, and let $\pi''_n$ be the per-period profit earned by developer $n$ in the static Cournot–Nash equilibrium where developers do not collude at a rental price. We define $i$ as the interest rate.

The net gain from cheating is specified as

$$\Pi_n = (\pi'_n - \hat{\pi}_n) - \sum_{i=1}^{n} \frac{1}{(1 + i)^i} (\hat{\pi}_n - \pi''_n)$$

$$= (\pi'_n - \hat{\pi}_n) - \frac{1}{i} (\hat{\pi}_n - \pi''_n) \quad (2.3)$$

Developer $n$ cheats if $\Pi_n > 0$. Let $\Omega$ be the set of prices that can be supported in a semi-collusive agreement,

$$\Omega = \{ r : \Pi_n \leq 0 \ \forall n \} \quad (2.4)$$

Finally, if we let $F(\pi_1, \ldots, \pi_n)$ be the welfare function of developers, then the optimal sustainable rental price $r$ is given by the solution that maximizes

$$\max F(\hat{\pi}_1, \ldots, \hat{\pi}_n) \quad \text{subject to} \quad r \in \Omega \quad (2.5)$$

The solution to Equation (2.5) depends on the number of units supplied by developers in the first stage of the game. Let $\hat{\pi}_n (y_1, \ldots, \bar{y}_n)$ represent the profit for developer $n$ evaluated at the rental price that solves Equation (2.5), and let $r(y_1, \ldots, \bar{y}_n)$ denote the rental price. We are now in a position to define a symmetric semi-collusive equilibrium in the two-stage game.

**Definition** $(\bar{y}, \bar{r})$ is a symmetric semi-collusive equilibrium if

1. $\bar{r} = r(y_1, \ldots, \bar{y}_n)$, where $\bar{r}$ is the optimal rental price determined in stage 2 given the $\bar{y}$ supplied by developers in stage 1 of the game;

2. $\bar{y}_1 = \bar{y}_2 = \cdots = \bar{y}_n = \bar{y}$, where $\bar{y}$ is the solution of Equation (2.2) given the rental price $\bar{r}$ determined in stage 2 of the game.
To solve Equation (2.5), let \( d(r) \) denote a downward-sloping incremental demand curve. This incremental demand curve together with existing tenants can be denoted by \( \hat{d}(r) = (1 - k) M + d(r) \), where \( \hat{d}(r) \) is the total-demand curve, \( k \) is the vacancy rate and \( M \) is the existing supply. To make our model more tractable, we assume that \( d(r) = a - br \). Given the demand function, there exists a unique rent level, \( r'' = (1/2b)a + (1/2b)(1 - k)M \), such that \( r'' \hat{d}(r) = r[(1 - k) M + a - br] \) is maximized.\(^5\) When \( y_1 = y_2 = \cdots = y_n = y \), there exists a unique static Cournot–Nash equilibrium such that the rental price is set at \( r'' = (1/b)[a - kM - Ny] \). It is easy to see that \( r'' \hat{d}(r'') > r'' \hat{d}(r''') \).

**Proposition 1** Given \( y_1 = y_2 = \cdots = y_n = y \), one has \( M + Ny > \hat{d}(r'') \), and for any welfare function \( F \) satisfying the property \( F_n > 0 \ \forall n \), there exists a unique solution to Equation (2.5) with a pro rata sharing rule. This solution is characterized by a critical interest rate \( i^*(y) \) such that:

1. If \( i < i^*(y) \), then \( \hat{r} = r'' = (1/2b) a + (1/2b)(1 - k) M \)

2. If \( i \geq i^*(y) \), then no sustainable price exists that yields profits above what would be earned in the static Cournot–Nash equilibrium.

Furthermore,

\[
i^*(y) = \frac{r'' \hat{d}(r'') - r'' \hat{d}(r''')}{[M + Ny - \hat{d}(r'')]r''} > 0
\]  \hspace{1cm} (2.6)

**Proof** See Appendix.

Proposition 1 indicates that, when \( M + Ny > \hat{d}(r'') \), developers will reach a collusive arrangement if the market interest rate \( i \) is lower than the \( i^*(y) \) specified in Equation (2.6).\(^6\) The collusive agreement will break down if the market interest rate is too high or if \( M + Ny \leq \hat{d}(r'') \). When \( M + Ny \leq \hat{d}(r'') \), developers will not collude at the rental price point where \( \hat{r} = r'' \). Instead, developers will increase the rental price to eliminate the excess demand.

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\(^5\) The reader will find that throughout the paper we only need to assume that \( r\hat{d}(r) \) is concave and therefore has a unique maximum.

\(^6\) We may place an upper bound on \( Ny \) in our model such that \( i^*(y) \) is independent of any endogenous variables.
It is easy to see that in stage 2 of the game, the market interest rate \( i \) will be very low, because at any given time developers can change the rental rate of their property. To use office buildings as an example, developers will have to advertise the rental rates and sign multiple contracts in order to lease out the whole building. When one developer observes that other developers are cheating, this developer can immediately adjust the asking rental rate for all of her/his remaining space. Since the time period required for developers to revert from a collusive state to a non-collusive state is minimal, the market interest rate \( i \) used by developers must be very low.\(^7\) Given this, Proposition 1 indicates that it will be optimal for developers to collude at the revenue-maximizing rental rate even if the actual supply is higher than the demand at that rate.\(^8\) Given this result, the next section will analyze the required conditions under which developers will supply more units than the revenue-maximizing demand and hence create an oversupply in the market.

**First Stage: Investment Decision**

Our analysis of the second stage of the game indicates that when developers supply more units than the revenue-maximizing demand \( \hat{d}(r^m) \) [which is evaluated at the revenue maximizing rental rate \( r^m - (1/2b) a + (1/2b)(1 - k) M \)], developers will not lower the rental rate to eliminate the excess supply. [We will refer to \( \hat{d}(r^m) \) as the projected demand level.] At stage 1 of the game, developers will make an investment decision to maximize their profits by taking this projected demand level into consideration.

In the face of oversupply, using a pro rata sharing rule, the projected rental rate of per unit of real property can be specified as \( R(M, k, d(r_m), \bar{v}, y_n) = r^m \hat{d}(r^m)/(M + \sum y_n) \), where \( \hat{d}(r^m) = \frac{1}{2}a + \frac{1}{2}(1 - k) M \) and is the total demand in the market evaluated at rental rate \( r^m \).\(^9\) The total supply of units

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\(^7\) Although the length of the period will also change the magnitude of per-period cash flows, it will not affect developers' collusion decision. From Equation (2.3), we know that if the period is shorter (or longer), we only need to divide (or multiply) all the cash flows in the equation by a constant number. [For example, change \( \Pi_n = (\pi' - \hat{\pi}_n) - (1/)(\hat{\pi}_n - \pi'_n) \) to \( \Pi_n = (\pi'_n/B - \hat{\pi}_n/B) - (i/B)^{-1} (\hat{\pi}_n/B - \pi'_n/B) \), where \( B \) is a constant.] It should be noted that the later equation can be re-written as \( B\Pi_n = (\pi'_n - \hat{\pi}_n) - (i/B)^{-1} (\hat{\pi}_n - \pi'_n) \). Under this circumstance, since the relative magnitudes of all the cash flows in the equation stay the same, the implications will be the same.

\(^8\) It should be noted that this result is supported by the high vacancy rates observed in the office and other commercial property markets. Developers in the past did not lower the rent to a level that would create enough demand to absorb the oversupplied units.

\(^9\) It should be noted that this specification implies that tenants are able to move freely from existing buildings to the newly constructed properties.
in the market is \( M + \sum_n y_n \), which is the sum of the existing inventory \( M \) and the new inventory supplied by all developers \( \sum_n y_n \). Given the fixed \( r^m \) and \( \hat{d} \), the projected total rent available in the market will be \( \hat{r}\hat{d} = (1/b)[a + \frac{1}{2}(1 - k)M] \). When \( \hat{d}(r_m) \) is held constant, the projected per-unit rent \( R(M, d(r_m), \hat{y}, y) \) is a function of the total units supplied by all developers to the market. When the supply of new inventory \( \hat{y} \) increases, a developer’s per-unit rent \( R \) decreases because of the increase in the vacancy level. This pro rata rule implies that the owners of both the newly constructed and the existing buildings will set their rent simultaneously.

The next proposition details the condition under which developers in the aggregate will supply more units to the market than the demand evaluated at the revenue-maximizing rental rate \( r^m \).

**Proposition 2** If \( L(\hat{y}) = V \) and

\[
\frac{r^m}{c + L} > \frac{N}{M + d(r^m)} - \frac{N}{M + d(r^m)} + \frac{N}{M - 1} \tag{2.7}
\]

then an oversupply (when \( S/\hat{d} > 1 \)) will occur, where \( S = M + N\hat{y} \) and \( \hat{d} = (1 - k) M + d(r^m) \). Furthermore, if

\[
\frac{r^m}{c + L} < \frac{M}{d(r^m) + (1 - k)M} \tag{2.8}
\]

then \( N\hat{y} = 0 \).

**Proof** See Appendix.

This proposition indicates that in the aggregate developers, in general, will supply more units than the demand level as long as \( N \) is sufficiently large or \( d(r^m) \) is small compared with \( M \). It should be noted that the ratio \( r^m/(c + L) \) can be interpreted as a profitability ratio and in general must be greater than one when \( L(\hat{y}) = V \). (This condition implies that the market value of land will not be affected by the supply decisions made by other developers.) On the other hand, as long as \( M/(1 - k)M + d(r^m) \) is not significantly smaller than one, the right side of Equation (2.7) should be only slightly larger than one. This is especially true if \( N \) is large and \( k \) is small. [When \( N \) is sufficiently large and \( k \) is small, the right side of (2.7) approaches one.] It should be noted that \( d(r^m) \) is the incremental demand of one period and \( M \) is the existing inventory. In the real world, it is highly
likely that \( d(r^m) \) will be small compared with \( M \). Given this, it is safe to conclude that, in general, we should observe that developers supply more units to the market than the revenue-maximizing demand level. In other words, we may expect to observe an oversupplied real estate market.

When there is vacant space in the existing inventory, using a similar argument, Equation (2.8) indicates that developers will not supply units \((N\tilde{y} = 0)\) to the market if the existing vacancy rate \( k \) is too high. Since the profitability ratio \( r^m/(c + L) \) should be larger than one, developers will not build when the projected incremental demand is smaller than the existing vacant space \([d(r^m) - kM < 0]\).

It should be noted that Proposition 2 is derived under the condition that \( L(\tilde{y}_0) - V = 0 \). That is, there is no incentive for a developer to wait for another development opportunity. However, a developer might decide to wait for another opportunity if she/he believes that other developers will build and the reduction in the potential supply of land could make her/his land more valuable. In this case, the opportunity cost of land, \( L(\tilde{y}_0) \), could be higher than the current market value \( V \). The next proposition addresses this issue.

**Proposition 3** Let \( \mu - L(\tilde{y}_0) - V \) be positive. In this case, \( M + N\tilde{y} \leq d(r^m) \) if \( \mu > r^m - c - V \).

**Proof** See Appendix.

This proposition states that developers, in the aggregate, will supply fewer units to the market than the projected demand if they expect to make a higher profit \( \mu \) from the land appreciation than from the development of the land \((r^m - c - V)\). In other words, where the rent of property \( r^m \) is not significantly higher than the costs of the development \((c + V)\), it is likely that developers will supply fewer units to the market than the projected demand if land supply is also limited. Under these two conditions (profit is low and land supply is limited), developers could be better off holding the land for a better development opportunity in the future. In other words, developers will not select the rental level that maximizes the total revenues.

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10 Mayer and Somerville (1997) report that a 10% rise in real housing prices leads to a 0.8% increase in housing stock. Given this, it is safe to infer that, with a lower profit, developers will supply fewer properties to the market.

11 The proof of this proposition is available from the authors upon request.
Instead, developers build fewer units so that they can charge a rental rate higher than \( r^* \) for their properties.

However, it should be noted that the conditions for an undersupply are quite restrictive. For most cities in the U.S., it is safe to assume that land supply is not limited. This is especially true for newly established cities and for suburban areas. It is also possible to argue that land supply is not as big a problem for established cities and central business districts if we consider property redevelopment opportunities (to tear down existing old properties to make way for new development). Comparing the conditions required for an oversupply result with those required for a not oversupplied market, it is safe to conclude that in general, it is more likely to observe an over-supplied real estate market than a market where supply just meets the demand.

Propositions 2 and 3 are derived under a risk-neutrality assumption. If we assume that developers are risk-averse, the supply will also depend on the level of risk aversion and the degree of uncertainty about the future demand. Namely, the supply will drop (increase) when developers are less (more) optimistic about the market or have less (more) confidence about the projected demand. Under this circumstance, the degree of oversupply will depend on developers’ attitudes toward the risk and the uncertainty exhibited in the market. These factors (changes in risk tolerance and demand uncertainty) could cause the observed cycles in the real estate market.

Two Intuitive Explanations

Our model indicates that it is possible for us to often observe oversupplied real estate markets. The main reason for the result is that, when the market supply is more than the revenue-maximizing demand, developers believe that it is optimal to stick to the revenue-maximizing rental rate \( r^* \) and allow part of their properties to be vacant. Our model presents a convincing argument as to why developers should stick to a particular rental rate. However, besides the theoretical arguments presented in our model, there are two intuitive arguments that can be developed to explain the stickiness of a high vacancy level.

The first explanation is analogous to the option-to-wait argument developed by McDonald and Siegel (1986). Using the option-to-wait concept, Grenadier (1995b) proposes that because of uncertainty about future demand and the possibility of losing tenants again in the future, owners will be reluctant to lower the rent to attract tenants. By not leasing the property, owners will be able to avoid paying leasing costs again if demand or rent
drops further in the future. Under this argument, property owners will be reluctant to lease a building unless the incremental benefits of leasing are substantially greater than the operating and leasing costs. This argument details the considerations from an owner’s perspective.

While the argument based on the option to wait has a certain appeal, it cannot completely explain why the owner does not use a short-term leasing contract with a leasing structure that will pass all the leasing costs to the tenants. Under this type of arrangement, an owner should be able to receive a certain rent and, in the meantime, minimize the costs associated with the uncertainty about future demand.

We develop a complementary explanation to examine the considerations from a tenant’s perspective. This argument basically proposes that the demand function of a property could be a kinked curve and that the demand curve on the right side of \( r^* \) (the revenue-maximizing rental rate) is nearly vertical. This means that the aggregate demand level will not increase much even if developers set the rent below \( r^* \).

This assumption is quite realistic. As long as the current office space meets a company’s need, the company—realizing that the future rent might be different from the current rent—will be reluctant to increase its space significantly even if the rent level is dropping. In other words, unless a tenant is convinced that the benefits of low rent can be locked in for the long term, a tenant will hesitate to increase her/his consumption beyond the current level. When a reduction of the current rent level will not create more aggregate demand and when one developer’s action (to reduce the rent) will certainly be followed by other developers, there will be no incentive for developers to engage in price competition. Under this circumstance, it is rational to expect a persistence of oversupply.

**Testable Implications**

While Propositions 2 and 3 may be useful in explaining the frequently observed oversupply in the general real estate market, it may not be able to answer completely the question of why the magnitudes of oversupply vary

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1. The rent of this type of leasing structure, holding everything else constant, will be lower than that of a leasing structure where the landlord pays for the leasing costs. The difference between these two types of leasing structures is analogous to the difference between an NNN expense structure (where tenants are responsible for all the expenses related to the operation of the property) and a full-service lease (where the landlord pays all the expenses).
in different submarkets. This section addresses that issue. We will analyze the optimal level of units supplied by developers in relation to factors such as existing inventory, number of developers in the market, and ratio between fixed and variable costs. We also develop testable implications regarding which areas are more likely to have an oversupply problem. The findings are summarized in the next three propositions.

**Proposition 4** If \( L(\bar{y}_n) = V \) and \( \bar{y} \neq 0 \), then \( dS/dN > 0 \).

**Proof** See Appendix.

Proposition 4 indicates that when an individual’s supply decision does not affect the value of land, the total supply to the market increases when there are more developers in the market.\(^{13}\) It should be noted that we are not discussing the level of supply by individual developers. It is expected that, given a fixed level of demand, the supply from each developer will be proportionately reduced if there are more developers in the market. (In other words, when \( S \) is held constant, an increase in \( N \) should be offset by a decrease in \( y \).) Proposition 4 indicates that each developer will supply more units than her/his proportional share of the demand when there are more developers in the market. Consequently, the oversupply will be more severe if there are more developers competing for development opportunities. This makes intuitive sense. If there is only one developer, the problem of oversupply should be the least serious, because the developer will not compete with her/himself. To some extent, our argument in spirit is similar to the common pool resource extraction argument prevailing the literature.

This proposition also carries some testable implications. Holding the amount of existing inventory \( M \) constant, for a given city this proposition predicts that there will be more oversupply in suburban areas (areas assumed to have more developable sites) than in central business districts (areas assumed to have fewer developable sites). Empirical evidence seems to support this conjecture. Voith and Crone (1988) report that the average vacancy rate of suburban office markets in 32 U.S. cities from 1979 to 1987 was about 3.1% higher than that of CBD office markets.\(^{14}\) Using the same line of reasoning, it is also possible that the oversupply problem will be more serious in a residential property market (which is assumed to have more potentially

\(^{13}\) Spatt and Sterbenz (1985) also argue that an increase in the number of rivals increases the incentive to undertake a project early to preempt the rivals.

\(^{14}\) An alternative explanation is that existing buildings (most likely to be located in CBDs) are filled before new buildings (most likely to be located in suburbs) are.
developable sites) than in a commercial property market (which is assumed to have fewer potentially developable sites).

This proposition predicts that, holding the amount of existing inventory constant, a well-established city with less developable land (such as New York) should have a less serious oversupply problem than a newly developed city with abundant developable land (such as Dallas). Table 1 of Wheaton (1987) seems to offer some empirical evidence to support this conjecture. In the table, Wheaton lists the office vacancy rates for six selected years during 1960–1986 for 10 selected cities. The vacancy rates of well-established cities such as Chicago, New York, Washington and Boston are consistently below the all-city average. On the other hand, newer or less well-developed cities such as Denver, Dallas and Kansas City consistently report much higher vacancy rates than the all-city average. The vacancy rates of selected cities in selected years during 1988–1995 reported in exhibit 15 of Rosen (1996) also support our prediction. The table reports that New York and Boston had vacancy rates ranging from 2.8% to 5.5% and from 4.1% to 7.0% during that period, respectively. However, Kansas City and Houston had vacancy rates ranging from 10.2% to 13.5% and from 8.8% to 14% during the same period, respectively.

**Proposition 5** If \( L = V \) and \( \bar{y} \neq 0 \), then \( dN_{\bar{y}}/dk < 0 \). Furthermore, if \( k \) is small enough and the market is oversupplied, then \( dN_{\bar{y}}/dM > 0 \).

**Proof** See Appendix.

This proposition indicates that developers reduce their supply when the vacancy rate increases. This is a very intuitive result. Furthermore, when \( L = V \) (when an individual’s development decision does not affect land value) the magnitude of oversupply is a function of existing inventory when the vacancy rate is very small.\(^{15}\) This suggests that, holding everything else constant, areas with a large existing inventory and a large supply of undeveloped land will have a more serious oversupply problem. It also implies that, in general, special types of properties such as warehouses, hospitals and R&D centers (normally with a smaller existing inventory) will have a less serious oversupply problem than general-purpose properties such as single-family units, office buildings and retail facilities (normally with a larger existing inventory).

\(^{15}\) Otherwise, holding the vacancy rate constant, an increase in the inventory level will also increase vacant space. An increase in vacant space will, in turn, reduce the incentive of developers to build.
This proposition makes intuitive sense. Developers will be more likely to supply more units if there is a larger existing inventory in the market. This means that, when the incremental demand is not large enough to absorb the added supply, developers can compete for existing tenants to minimize their losses. For example, if the projected demand is 10 units and if developers in aggregate supply a total of 20 units of product to the market, the vacancy rate will be 50% if there is no existing inventory. However, if the existing inventory is 80 units, the vacancy rate will only be 10% even if all the developers supply twice as many units as the projected demand in the market. In other words, the consequence of an oversupply will be less severe in an area with a large existing inventory. Under this scenario, developers have more incentive to build.

**Proposition 6** Let \( \alpha = r''/(c + L) \). When \( S \) is specified as a function of \( \alpha \), \( dS/d\alpha > 0 \).

**Proof** See Appendix.

This proposition suggests that both the profit and cost functions of a development affect a developer’s decision to build. From Equation (2.2), we know that a developer’s profit is the difference between rent \( (r'') \) and total costs (construction costs \( c \) and land cost \( L \)). In this regard, \( \alpha \) can be viewed as the profitability ratio of the development. It is reasonable to expect that a high profit ratio will induce developers to build more units.

In our model, a developer’s decision is whether or not to put up the construction cost to build the property. In this regard, land cost \( L \) is fixed and the relevant information for decision making is the magnitudes of rent \( r'' \) and construction costs \( c \). When the rent is held constant, a lower construction cost means a larger value for \( \alpha \). Our proposition indicates that the supply increases when \( \alpha \) increases. In other words, Proposition 6 suggests that developers will supply more units to the market if the construction cost \( c \) is small compared to the land cost \( L \).

The intuition is very simple. Suppose a developer is examining the investment opportunities based on two parcels of land: one with a market value of $90 (the first property) and the other with a market value of $20 (the second property). Both parcels of land are currently vacant and do not produce income. It will take an additional $10 to develop the first property or an additional $80 to develop the second property. Under this circumstance, the total cost (land plus construction) for both developments will be $100. Assuming that both properties can be sold for $120 (with an identical $20 profit), it is more likely the first property will be developed first. The
rationale is simple. For the first property, it will only take an additional $10 investment to realize the $20 profit. For the second property, it requires an additional $80 investment to realize the identical $20 profit. Holding everything else constant, it is more likely that the first project will be developed first because the ratio between the profit and the additional investment is higher.

This proposition also carries a testable implication. We know that different types of properties, in general, have different ratios of construction cost to land cost. For example, offices and hotels typically have a lower ratio than multifamily and industrial properties. Given this, Proposition 6 would predict that, holding the size of the existing inventory and the amount of available sites constant, we are more likely to observe an overbuilt office or hotel market than an overbuilt retail or industrial market. There is some empirical evidence supporting this conjecture. Figure 2B of Grenadier (1996) reports the vacancy rates of office and industrial property markets from 1978 to 1993. The figure shows that the vacancy rates of office property markets are consistently higher than those of industrial property markets. From a casual examination of the figure, it appears that after 1984 the vacancy rate of the office property market should be twice as large as that of the industrial property market. Exhibit 14 of Rosen (1996) reports vacancy rates of multifamily property during 1980–1995. A comparison of the multifamily vacancy rates reported in exhibit 14 of Rosen (1996) and the office vacancy rates reported in Figure 2B of Grenadier (1996) indicates that office properties have much higher vacancy rates during that period. Following this line of thought, our model predicts that, in a given single-family market, oversupply will more likely occur in the high-end market. For a high-end luxury home, it is well known that the most expensive part of the property is its site.

**Oversupply under a Monopoly**

So far, we have demonstrated that developers, when competing for development opportunities, tend to supply more units than demanded by the

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16 Grenadier (1996) offers a different type of explanation for this phenomenon. Using an option-based model, Grenadier proposes that demand volatility will increase the likelihood of construction cascades. The model implies that an office property market should have a higher vacancy rate because the market appears to be more volatile than an industrial property market.

17 Figure 6 of the Real Estate Capital Market Report (November issue, 1997), published by Institutional Real Estate Inc., also reports that office vacancy rates are at least twice as high as the vacancy rates of industrial and residential properties during 1990–1997.
market. The general conditions for this proposition are that there must be a large number of developers in a market and that each developer holds a large quantity of undeveloped land. The next proposition shows that, under certain conditions, oversupply occurs even if there is only one developer with monopoly power in the market.

**Proposition 7** When \( N - 1 \), oversupply occurs if \( r''/(c + L) > [(1 - k)M + d(r'')] / M \).

**Proof** See Appendix.

This proposition suggests that, as long as the profitability ratio \( r''/(c + L) \) is greater than the rate of increase in demand \( ((1 - k)M + d(r'')) / M \), a developer will supply more units to the market than the projected demand \( d(r'') \), even if the developer enjoys monopoly power. This intuition is simple. Instead of competing for the current demand \( d(r'') \), the developer can compete with the existing properties for their existing tenants, \( (1 - k)M \) in number. Because of the ability to compete for existing tenants, the oversupplied units will not be completely vacant, because the overall vacancy rate \( ([1 - k]M + d(r'')) / (M + \bar{y}) \) is determined by the amount of new supply as well as the existing tenants. This indicates that developers could increase their profits by supplying more units to the market than the projected demand if the rent level is high enough and/or if the existing tenant is large enough.

Holding everything else constant, this proposition indicates that the developer, even in a monopoly, will supply more units than the projected demand when the existing inventory \( M \) is large compared to the projected demand \( d(r'') \). With a large base \( M \) and holding the vacancy rate \( k \) constant, a given amount of oversupply \( [N\bar{y} - d(r'')] \) will result in a lower overall vacancy rate for all properties in the market. In other words, existing properties have to share the consequence (and, hence, costs) of the oversupply decision made by the developer.\(^\text{18}\) The same argument holds for a large profitability ratio. Holding everything else constant, a large profit ratio will compensate the developer for taking a lower occupancy rate resulting from the oversupplied units. Because the vacancy will be shared

\(^{18}\) Of course, the decision will be different if the developer also owns the entire existing inventory. Under this circumstance, the developer will not supply more units than the estimated demand to compete with her/himself. [See Coase (1972) for a related discussion of the effect of competition between current and future supply on the monopoly price.]
with the existing inventory, it could be optimal for the developer to build more units in order to maximize her/his profits.

**An Extension**

Our analyses so far rely on one important assumption: that there are no moving costs, so that existing tenants can move freely from existing buildings to newly constructed properties. This assumption ensures that each property can obtain a pro rata share of the total demand (due to existing tenants plus the incremental demand). In this section we will discuss the implications of our model under a scenario where the existing tenants cannot move freely from existing properties to newly constructed buildings.

Under this condition, the newly constructed buildings and the vacant space of existing buildings will compete for the new demand. Consequently, the new demand will be shared proportionately by all the vacant space (new buildings and currently vacant space) in the market. Under this circumstance, $S = kM + N\bar{y}$ (the total vacant space), and $d(r^m)$ is the incremental demand.

**Proposition 8** Oversupply [when $s/d(r^m) > 1$] occurs when $L(\bar{y}_n) = V$ and when $r^m$ is large enough with respect to $c + L(\bar{y}_n)$. In other words,

$$\frac{S}{d(r^m)} > 1 \quad \text{if} \quad \frac{r^m}{c + L} > \frac{N}{kM/d(r^m) + N - 1} \quad (5.1)$$

and

$$N\bar{y} = 0 \quad \text{if} \quad \frac{r^m}{c + L} < \frac{kM}{d(r^m)} \quad (5.2)$$

Furthermore, if $L = V$ and $\bar{y} \neq 0$, then $dS/dN > 0$, $dS/d[r^m/(c + L)] > 0$, and $dN\bar{y}/dk < 0$. In addition, $dN\bar{y}/dM = 0$ if $k = 0$.

**Proof** See Appendix.

It should be noted that $kM$ is the amount of vacant space in the market, while $d(r^m)$ is the projected incremental demand in the market. Thus $kM/d(r^m)$ is the ratio between the currently vacant space and the projected new demand. Since $r^m/(c + L)$ is a profitability ratio and, in general, must be greater than one, Equation (5.1) indicates that developers will supply more units than the projected demand as long as $N$ is sufficiently large or
\( \frac{r^m}{c + L} > \frac{d(r^m)}{kM} \) does not differ significantly from one. From Equation (5.2), it is also clear that when the ratio \( kM/d(r^m) \) is sufficiently large, developers will not add any inventory to the market. In other words, when there is a large amount of vacant space, developers will not build until the demand side catches up.

Proposition 8 provides similar implications to those offered by Propositions 4 and 6. It indicates that there will be a tendency for oversupply when there are many developers in the market or a small ratio of construction cost to land value. However, in contrast with the implication of Proposition 5, the supply decision of developers is not related to the existing inventory if we assume that the current vacancy ratio is zero. This is true because the tenants of currently occupied space will not be shared with the newly constructed buildings. There is no incentive for developers to supply more units to the market when the existing vacancy rate is high.

Under a monopoly, when \( N = 1 \), Equation (5.1) indicates that oversupply occurs when

\[
\frac{r^m}{c + L} > \frac{d(r^m)}{kM} \tag{5.3}
\]

Equation (5.3) indicates that when the profitability ratio \( r^m/(c + L) \) is greater than the ratio between projected new demand and current vacant space, \( d(r^m)/kM \), a developer will supply more units to the market than the projected demand \( d(r^m) \). It should be noted that when there is no vacant space \( (k = 0) \) a developer will never oversupply the market. In other words, a developer will not build more than the projected demand to create vacant units in the market. This makes intuitive sense. If a developer cannot compete with the existing inventory, there is no incentive to overbuild. When \( N = 1 \), Equation (5.1) also indicates that there will be no more new construction \( (\bar{y} = 0) \) if

\[
\frac{r^m}{c + L} < \frac{kM}{d(r^m)} \tag{5.4}
\]

This also makes intuitive sense. If the current vacant space is much larger than the projected new demand and if the new building cannot attract tenants from existing buildings, there is no incentive for developers to construct any more buildings.
Conclusions

Our model demonstrates that it is natural for us to observe the recurrence of oversupply in real estate markets. Because vacant land does not produce income, developers have the incentive to build once they spot a development opportunity. In a world with competition, developers will fight for any development opportunity. As a result, in the aggregate, developers will supply more units to the market than the projected demand. After the market is oversupplied, developers will stop building until the demand absorbs the existing supply. Once the oversupplied units are absorbed by the new demand, overbuilding will occur again. Our model, using a very simple framework, is able to explain the long-lasting oversupply in real estate markets without relying on the assumptions of agency costs, irrational behavior or uncertainty of demand.

The implications of our model for general investment practices are worth discussing. The unique aspect of the model is that land (a special type of asset) alone does not produce income. In this regard, land can be viewed as excess capacity or as public infrastructure that can be utilized cheaply.\textsuperscript{19} Given this, it is reasonable to observe projects or investments with similar characteristics experiencing similar boom-and-bust market cycles. As an example, the establishment of the Hoover Dam provided cheap excess electricity, which is partially responsible for the establishment of the city of Las Vegas. (The development of Las Vegas was not successful in the initial years.) Another example could be the recent establishment of the fiber-optic cable system. With a tremendous increase in capacity, we have seen numerous developments and creations in the multimedia and Internet fields to take advantage of the capacity. It would not be surprising if we find out later that a significant number of the newly developed firms do not survive for long.

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\textsuperscript{19} Jensen (1986) proposes that when a firm has a large amount of free cash flow, there will be a tendency for the firm to overinvest. This implication is similar to the one implied by our model. Our model proposes that, once there are excess capacities or cheap resources, there will be a tendency for excess development.
References


Appendix

Proof of Proposition 1

We specify \( \hat{\pi}_n \) (the revenue earned by developer \( n \) at a collusive rental price point \( \hat{r} \)) as

\[
\hat{\pi}_n = \frac{\hat{r}d(\hat{r})}{M + Nv} y.
\]

This means that developers will share the total revenue available in the market proportionately. Without loss of generality, we assume that when a developer is cheating optimally against \( \hat{r} \), the developer can reap the maximum possible profit \( \pi'_n \). That is, by slightly reducing the rental price, the developer will be able to rent out all her/his units. Therefore, \( \pi'_n = \hat{r}y \).

We specify \( \pi''_n \) (the profit earned by developer \( n \) in the static Cournot–Nash equilibrium where developers do not collude at a price) as \( \pi''_n = r''y \).

Given these conditions, Equation (2.3) can be re-written as

\[
\Pi_n(\hat{r}) = (\pi'_n - \hat{\pi}_n) - \frac{1}{i} (\hat{\pi}_n - \pi''_n)
\]

\[
= \left(1 - \frac{\hat{d}(\hat{r})}{M + Nv}\right) \hat{r}y - \frac{1}{i} \left(\frac{\hat{r}\hat{d}(\hat{r}) - r''\hat{d}(r'')}{M + Nv}\right)y. \tag{A.1}
\]

Define

\[
i^* = \sup \{i : \Pi_n(r^{''}) \leq 0 \ \forall n\} \tag{A.2}
\]

To prove that \( i^* \) exists, observe that as \( i \to 0 \), \( \Pi_n (r^{''}) < 0 \) for all \( n \). As \( i \to \infty \), \( \Pi_n (r^{''}) > 0 \). The continuity of \( \Pi_n (r) \), therefore, guarantees the existence of \( i^* \). Since \( \Pi_n \) is monotonic increasing in \( i \), it immediately follows that \( r^{''} \) is sustainable for all \( i \geq i^* \). For \( i \geq i^* \), developers revert to the static Cournot–Nash equilibrium. We note from Equation (A.1) that

\[
i^* = \frac{r''\hat{d}(r'') - r''\hat{d}(r'')} \left|M + Nv - \hat{d}(r'')r''\right|
\]

It is easy to see that, when \( M + Nv > \hat{d}(r'') \), \( i^* \) must be a positive number.
because \( r^n \hat{d}(r^n) \) is the maximum total revenue, which must be greater than the total revenue evaluated at other demand points such as \( r^n \hat{d}(r^n) \).

**Proof of Proposition 2**

First, we note that when

\[
M + \bar{y}_{-n} + y_n > (1 - k)M + d(r^n)
\]

the expected value of \( P \) is\(^{20}\)

\[
E(P) = \frac{r^n \hat{d}(r^n)}{M + \bar{y}_{-n} + y_n} \cdot y_n - c y_n + (T - y_n) L(\bar{y}_{-n}) - TV
\]  

(A.3)

Taking the derivative of \( E(P) \) with respect to \( y_n \), the first-order condition of the optimal solution implies that

\[
\frac{r^n \hat{d}(r^n)}{(M + N \bar{y})^2} [M + (N - 1) \bar{y}] - c - L(\bar{y}_{-n}) = 0
\]  

(A.4)

When we solve this equation for \( M + N \bar{y} \), we obtain

\[
M + N \bar{y} = -\frac{N - 1}{N} \frac{r^n \hat{d}(r^n)}{N} + \sqrt{\left(\frac{N - 1}{N} r^n \hat{d}(r^n)\right)^2 + \frac{4[c + L(\bar{y}_{-n})] r^n \hat{d}(r^n) M}{N}} \\
\frac{2[c + L(\bar{y}_{-n})]}{N}
\]  

(A.5)

Equation (A.5) explains how the optimal supply \( \bar{y} \) of each developer is derived. It should be noted that \( M + N \bar{y} \) will be the total supply in the market under this optimal condition.

Dividing Equation (A.5) by \( \hat{d} \), we obtain

\[^{20}\text{To keep the notation simple, we write } P_n \text{ as } P \text{ hereafter.}\]
\[
\frac{N - 1}{N} r^m + \sqrt{\left(\frac{N - 1}{N} r^m \right)^2 + \frac{4[c + L(\bar{y}_{-n})] r^m M}{Nd}} \bigg/ 2[c + L(\bar{y}_{-n})] > 1
\]

(A.6)

Set
\[
\frac{N - 1}{N} r^m + \sqrt{\left(\frac{N - 1}{N} r^m \right)^2 + \frac{4[c + L(\bar{y}_{-n})] r^m M}{Nd}} > 1
\]

(A.7)

and solve the inequality. We obtain
\[
\sqrt{\left(\frac{N - 1}{N} r^m \right)^2 + \frac{4[c + L(\bar{y}_{-n})] r^m M}{Nd}} > 2[c + L(\bar{y}_{-n})] \frac{N - 1}{N} r^m
\]

(A.8)

We note that \( \hat{d} = (1 - k) M + d(r^m) \). By simplifying Equation (A.8) further we obtain
\[
\frac{r^m}{c + l} > \frac{N}{M \hat{d} + N - 1}
\]

which is a necessary and sufficient condition for oversupply. A necessary and sufficient condition for \( N\bar{y} = 0 \) is
\[
\frac{r^m \hat{d}(r^m)[M + (N - 1) \bar{y}]}{(M + N\bar{y})^2} - c - L(\bar{y}_{-n}) \leq 0
\]

(A.9)

Putting \( N\bar{y} = 0 \) into Equation (A.9), we obtain Equation (2.8).

**Proof of Proposition 3**

From Equation (A.4), we know that the optimal supply \( \bar{y} \) must satisfy
\[
\frac{r^m \hat{d}[M + (N - 1) \bar{y}]}{(M + N\bar{y})^2} - c - L(\bar{y}_{-n}) = 0
\]

Let \( \mu = L(\bar{y}_{-n}) - V \) be a non-negative number. If
\[ \mu \geq r'' - (c + V) \]
\[ \geq r'' \left( M + \frac{N - 1}{N} \left[ d(r'') - kM \right] \right) \]
\[ \geq \frac{M}{M} + \frac{N}{[d(r'')] - kM} \]
\[ - (c + V) \quad \text{(A.10)} \]

then \( \bar{y} \leq (d(r'')) - kM)/N \).

**Proof of Proposition 4**

From Equation (A.4), we know

\[ \frac{(N - 1)\hat{r}''}{N} S + \frac{Mr''}{N} = [c + L(\bar{y}_{-n})]S^2 \quad \text{(A.11)} \]

which implies

\[ \left( \frac{(N - 1)\hat{r}''}{N} - 2[c + L(\bar{y}_{-n})]S \right) \frac{dS}{dN} = \frac{(M - S)r''}{N^2} + \frac{dL}{dN} S^2 < 0 \quad \text{(A.12)} \]

From Equation (A.4), we know

\[ \frac{(N - 1)\hat{r}''}{N} - 2[c + L(\bar{y}_{-n})]S < 0 \quad \text{(A.13)} \]

which implies \( dS/dN > 0 \).

**Proof of Proposition 5**

From Equation (A.4), we know

\[ \frac{[a/2 + \frac{1}{2}(1 + k)M]^2}{c + L} [M + (N - 1)\bar{y}] = b(M + N\bar{y})^2 \quad \text{(A.14)} \]

Taking the derivative of both the right and left sides of Equation (A.14) with respect to \( k \), we have
\[
\frac{d\bar{y}}{dk} = \frac{-M[a/2 + \frac{1}{2}(1 - k)M][M + (N - 1)\bar{y}]}{2b(c + L)(M + N\bar{y})N - [a/2 + \frac{1}{2}(1 - k)M]^2(N - 1)}
\]

\[
\leq 0
\]

Taking the derivative of both the right and left sides of Equation (A.14) with respect to \(M\), we obtain

\[
\frac{[a/2 + \frac{1}{2}(1 - k)M](1 - k)}{c + L} [M + (N - 1)\bar{y}]
\]

\[
\frac{1}{1 + (N - 1) \frac{d\bar{y}}{dM}}
\]

\[
= 2b(M + N\bar{y}) + 2b(M + N\bar{y})N \frac{d\bar{y}}{dM}
\]

which implies

\[
\frac{d\bar{y}}{dM} = \frac{\frac{1}{c + L} \bar{d}(1 - k)[M + (N - 1)\bar{y}] + \frac{1}{c + L} \bar{d}^2 - 2b(M + N\bar{y})}{2b(M + N\bar{y})N - \frac{1}{c + L} \bar{d}^2(N - 1)}
\]

\[
\frac{(M + N\bar{y})^2}{a/2 + \frac{1}{2}(1 - k)M} \frac{(1 - k)}{M + (N - 1)\bar{y}} \frac{(M + N\bar{y})^2}{2(M + N\bar{y})} 2(M + N\bar{y})
\]

\[
= \frac{1}{c + L} \frac{1}{b} \left( \frac{a}{2} + \frac{1}{2} (1 - k)M \right)^2 (N - 1)
\]

(A.17)

where

\[
\bar{d} - \frac{a}{2} + \frac{1}{2} (1 - k)M
\]

From Equation (A.4) and the fact that

\[
M + N\bar{y} > \frac{a}{2} + \frac{1}{2} (1 - k)M
\]

we know that both the numerator and denominator of Equation (A.17) are positive if \(k\) is small enough. This implies that \(d\bar{y}/dM\) must be positive.
Proof of Proposition 6

From Equation (A.5), we know

\[
S = \frac{N - 1}{N} \frac{r^m d + \sqrt{\left(\frac{N - 1}{N} r^m d\right)^2 + \frac{4[\bar{c} + L(\bar{y}_{-n})]r^m d M}{N}}}{2[\bar{c} + L(\bar{y}_{-n})]}
\]

\[
= \frac{\frac{N - 1}{N} \alpha d + \sqrt{\left(\frac{N - 1}{N} \alpha d\right)^2 + \frac{4\alpha d M}{N}}}{2}
\]  \hspace{1cm} (A.18)

A simple calculation implies that \(dS/d(r^m/c + L) > 0\).

Proof of Proposition 7

When \(N = 1\), Equation (A.5) can be written as

\[
M + \bar{y} = \frac{\sqrt{r^m d M}}{\sqrt{c + L}}
\]  \hspace{1cm} (A.19)

Consequently,

\[
\frac{M + \bar{y}}{\bar{d}} = \sqrt{\frac{r^m M}{c + L}} = \sqrt{\frac{r^m M}{c + L}[M + d(r^m)]}
\]  \hspace{1cm} (A.20)

Under this circumstance,

\[
\frac{M + \bar{y}}{\bar{d}} > 1 \quad \text{if} \quad r^m > \frac{(c + L)[M + d(r^m)]}{M}
\]  \hspace{1cm} (A.21)

Proof of Proposition 8

When the new demand is shared proportionately with the vacant space, Equation (A3) can be written as

\[
E(P) = \frac{r^m d(r^m)}{k M + \bar{y}_{-n} + y_n \bar{y}_n} c y_n + (T - y_n) L - TV
\]  \hspace{1cm} (A.22)

The first-order condition of Equation (A.22) is
\[
\frac{r''d(r'')}{(kM + N\tilde{y})^2} = c + L \quad (A.23)
\]

To demonstrate the oversupply result, we only need to check that

\[
\frac{\partial E(P)}{\partial \tilde{y}} \bigg|_{y = 0} \leq 0 \quad (A.24)
\]

Equation (A.24) implies that Equations (5.1) and (5.2) will hold. The proof of the first two inequalities is similar to that of Propositions 4 and 6. To be more specific, we note that from Equation (A.11)

\[
dS/dN = \frac{-\tilde{y}r''d(r'')}{(N - 1)r''d(r'') - 2N(c + L)S} < 0 \quad (A.25)
\]

Let \( \alpha = r''/(c + L) \). From Equations (A.23) and (A.11), we know

\[
S(\alpha) = \frac{N - 1}{N} \alpha d(r'') + \alpha d(r'') \sqrt{\left(\frac{N - 1}{N} \alpha d(r'') \right)^2 + \frac{4\alpha d(r'')kM}{N}} \quad (A.26)
\]

Obviously, \( S \) is an increasing function of \( \alpha \). From Equation (A.23), we know

\[
\frac{dy}{dk} = \frac{Mr''d(r'') - 2M(c + L)(kM + N\tilde{y})}{2(c + L)(kM + N\tilde{y})N - r''d(r'')(N-1)} < 0 \quad (A.27)
\]

From Equation (A.23), we also know

\[
\frac{d\tilde{y}}{dM} = \frac{\frac{r''}{c + L}k \alpha d(r'') - 2(kM + N\tilde{y})}{2(kM + N\tilde{y})N - \frac{r''}{c + L}[kM + d(r'')] \cdot (N-1)} \leq 0 \quad (A.28)
\]

We note that from Equation (A.23) the denominators of Equations (A.27) and (A.28) are always positive.