Options return risk structure

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Classic option pricing theory has a revolutionary insight

- Black, Merton, and Scholes (BMS): Under certain conditions, one can replicate the payoff of an option by *dynamically trading* the underlying.
  - Options can be perfectly replicated via dynamic trading of the underlying and are hence redundant.

- Business implications:
  - An options market maker/broker dealer can balance the option position with dynamic trading of the underlying — The derivatives market expanded dramatically because of the insight.

- The existential question:
  - Why does the options market exist to begin with if options are redundant?
  - Any risk-return behavior we want to learn from options, we can learn from the underlying market.

- Several finance theories have this paradoxical behavior: They are so strong that they kill the role for their own existence.
The revolutionary insight has practical limits

- Dynamic replication works perfectly only under idealistic conditions:
  - Trading the underlying is free and frictionless.
  - The security price moves continuously with known constant volatility.
- In reality, there are practical limits to the dynamic replication:
  1. Trading the underlying security incurs a cost.
     - Dynamic delta hedge is not free.
  2. Volatility changes randomly over time.
     - Dynamic delta hedge cannot remove the volatility risk.
  3. Underlying security price can jump with random size.
     - Frequent delta rebalancing does not cover flying moves.
- Because of these limits, options are not completely replicable (redundant). They can play primary roles in risk allocation.
  - Investors can take certain types of risks via options that they cannot take via the underlying security.
  - This gives a purpose for their existence
Primary risk taking in derivative securities

What we try to do in this paper (using individual stock options):

1. Examine under practical situations how much of the option investment risk can be removed via delta hedging and how much is left over
   - What can be removed represents *derivative risk*.
     - Their market pricing should be analyzed in the primary (stock) market.

2. What cannot be removed via delta hedging represents *primary risk*.
   - This determines the usefulness of the options market.
   - We try to identify the major sources/types of the primary risk exposures of the derivative investments.
     - Why come to the options market?
   - Examine how these primary risk exposures are compensated in the options market.
     - The risk-return relations of a delta-hedged option investment.
The literature

1. Hedging effectiveness
   - Many papers on hedging, but difficult to find an absolute number on the percentage variance reduction.
     - Figlewski (1989) tries to show why it doesn’t always work...
       Many performance comparisons in RMSE
   - Peter Carr and I did a short exercise on SPX options

2. Risk-return relations

   - The “asset pricing” analysis here is both a first attempt and a synthesis from the perspective of identifying primary risk taking opportunities in option investments and their risk compensation behaviors.
     - Not to identify options trading strategies or “anomalies”
     - Nor on information flow between markets: Link different markets together vs identify the chief *existential purpose* of a specific market
Data and universe selection

- Data source: OptionMetrics, 1996-2019
- Choose one date per month: front-month options have exactly 30 days to expiry.
- Some stock/option filtering criteria to guarantee the option quotes (and sensitivities) are valid and not dominated by bid-ask.
- Choose an option closest to the target delta (e.g., strike closest to spot for atm).

6,299 companies, 368,657 company-date observations.

Average 1,285 companies per month, for 287 months
Write **30-day ATM options** ($O_t$) and hold to maturity, with 3 hedging strategies:

1. **No hedge**: The excess P&L of the naked option investment:
   \[
   \pi_{t,T}^{nh} = O_t e^{r_{t}(\tau)\tau} - O_T
   \]
   Terminal price $O_T$ is replaced with payoff (e.g., $(S_T - K)^+$ for call), assuming the chosen options won’t be exercised early.

2. **One-time hedge at initiation**
   \[
   \pi_{t,T}^{ih} = O_t e^{r_{t}(\tau)\tau} - O_T + \Delta_t \left(S_T - S_t e^{r_{t}(\tau)\tau}\right)
   \]
   This is doable for most names. One can also achieve initial (approximate) delta-neutrality via a call-put straddle combo.

3. **Daily delta rebalancing** whenever possible (traded, with pricing information)
   \[
   \pi_{t,T}^{dh} = O_t - O_T + \sum_{j=1}^{n} \left[\Delta_{t_j} (S_{t_{j+1}} - S_{t_j}) + (O_{t_j} - \Delta_{t_j} S_{t_j}) \left(e^{r_{t_j}(\tau_j)\tau_j} - 1\right)\right]
   \]
   feasible for liquid names.

Normalize the P&L by the stock price and call it the **excess return** on notional:
\[
 r_{t,T} = \frac{\pi_{t,T}}{S} — “Return” definition matters but is not straightforward.
Writing options naked can be very risky

Summary stats of pooled option investment excess returns without delta hedge:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Stdev</th>
<th>Percentiles</th>
<th>Tails</th>
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<tbody>
<tr>
<td></td>
<td>%</td>
<td>%</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Call</td>
<td>0.02</td>
<td>8.71</td>
<td>-9.41</td>
<td>-2.74</td>
</tr>
<tr>
<td>Put</td>
<td>0.52</td>
<td>7.72</td>
<td>-8.71</td>
<td>-1.35</td>
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</tbody>
</table>

- Mean return dominated by delta exposure, and average positive stock return.
- 30-day atm option price averages at 5% of spot level. Risk can much higher than the premium.
  - About 20% chance of collecting the premium without paying anything.
  - About 20% chance of losing more than the size of the premium, loss can be larger for writing calls.
- Return has large negative tails, typical for writing insurance-like contracts.
Neutralizing delta at initiation can remove 70% of risk

Summary stats of option excess returns with one-time delta hedge:

<table>
<thead>
<tr>
<th></th>
<th>Mean %</th>
<th>Stdev %</th>
<th>Percentiles 10</th>
<th>50</th>
<th>90</th>
<th>Tails Skew</th>
<th>Kurt</th>
<th>VR %</th>
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<tr>
<td><strong>A. Writing options without delta hedge</strong></td>
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<td><strong>B. One-time delta hedge at initiation</strong></td>
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<tr>
<td>Call</td>
<td>0.23</td>
<td>4.34</td>
<td>-4.36</td>
<td>0.80</td>
<td>4.34</td>
<td>-2.36</td>
<td>21.94</td>
<td>75.19</td>
</tr>
<tr>
<td>Put</td>
<td>0.32</td>
<td>4.33</td>
<td>-4.25</td>
<td>0.86</td>
<td>4.44</td>
<td>-2.30</td>
<td>21.78</td>
<td>68.59</td>
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- Delta-neutral option investments make 23-32 basis points per month.
  — Investments in put and call at the same strike are ~ identical
- Return volatility is reduced by half, from 7.72-8.71% to 4.33-4.34%, leading to about 70% variance reduction.
- 70% is not 100%, but still a lot; and one-time delta hedge at initiation (or just straddle combination) is easy (low cost) to implement.
Daily delta rebalancing removes 90% of risk

Summary stats of option excess returns with one-time delta hedge:

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<td><strong>C. Daily rebalancing on delta hedge</strong></td>
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</tr>
<tr>
<td>Call</td>
<td>0.17</td>
<td>2.59</td>
<td>-2.04</td>
<td>0.34</td>
<td>2.41</td>
<td>-4.24</td>
<td>89.02</td>
<td>91.15</td>
</tr>
<tr>
<td>Put</td>
<td>0.24</td>
<td>2.63</td>
<td>-1.96</td>
<td>0.40</td>
<td>2.53</td>
<td>-3.78</td>
<td>67.11</td>
<td>88.41</td>
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- When the stock price moves a lot over the investment horizon, the initially delta-neutral investment can become highly delta-exposed again.
- Rebalancing the delta hedge whenever the delta becomes large can keep the exposure low throughout the investment.
- Performing this rebalancing daily can remove 90% of the risk
  - Size-based updating (whenever delta becomes large) is similarly effective. In practice, it is probably a combination of both.
  - Daily rebalancing v. one-time hedge creates a stylized reference range on how effective the hedging can be.
Caveat: The risk of delta-hedged positions is very stable over time, but the risk of naked option investment is not.

- Writing put options naked can show little risk for a whole year (or longer) if the market keeps going up.

- Overall, delta hedging is effective through all/most market conditions ...
90% risk reduction is quite remarkable, highlighting the economic significance of the BMS insight.

- This allows the market maker to make ∼ 3 times as large a market.
- Further lowering the risk can allow the market maker to have an even larger book, offer more attractive spreads to attract more flows ...
- Despite many hedging papers, it remains a practically important question: How to construct/update delta hedge to generate the most cost-effective hedge for a large options book.

Researchers have used the idea of option pricing in other areas (such as “real options” in investments) ...

The insight of dynamic hedging (using fewer instruments to cover a wider space via movement) also have important implications in many other areas: portfolio insurance, capital structure, ...
90% risk reduction is quite remarkable, and economically significant, but ...

The remaining 10% risk is also quite significant. (2.59-2.63)% per month translates into 9% annualized volatility on the daily-delta hedged option excess return (on notional).

for $\sim 3\%$ per annum mean excess return

If these risks cannot be effectively hedged with the stock, it also means that investors cannot take on these risks easily in the stock market.

If investors want to take these risks, they can only do so with options. The options market plays a primary role in allocation of these risks.

What exactly are these risks (that one can take through options, but not through the underlying stock)? (risk identification & measurement)

How are investors compensated for taking these primary risks? (risk-return relation estimation)
What are the primary risks in option investments?

We come up with 3 main primary risk sources:

1. **Delta hedging cost (HC)**
   - One usually thinks of delta exposure as “derivative risk” as it reflects the variation of the underlying security price ...
   - But delta risk can become primary risk of the option investment when one cannot perform delta hedge trade with the underlying.
     - The delta exposure of electricity futures?
     - More generally, if investors cannot trade the underlying, the options market can become the primary place to gain the exposure (e.g., gaining short exposure when one is not allowed to short)
   - **Measurement:** Trading cost increases with return volatility and trading size as a proportion of average daily trading volume:
     \[
     HC_{t,i} = \sigma_{t,i}^2 (1 - \rho_{t,i}^2) / DV_{t,i}
     \] (1)
   - When the underlying is highly correlated with a liquid index/ETF, investors can perform “beta” hedge with index futures/ETF to remove the exposure with little cost.
   - Similar measures: bid-ask, Amihud, some other combinations of (vol/volume)
What are the primary risks in option investments?

We come up with 3 main primary risk sources:

1. **Delta hedging cost (HC)**
2. **Stochastic volatility risk (VR)**

   - Delta hedge removes the exposure to underlying price variation, but not (the independent component of the) volatility variation.
     - Stochastic volatility also increases the uncertainty about the current volatility level and the correct delta.
   - Options market is the natural market for volatility trading: ATM option’s (price/return) exposure to volatility/variance is almost linear.
     - One cannot (easily) gain vol risk exposure via stock investment.
   - **Measurement**: We take 3-month 50-delta as the “pivot” point of the vol surface and use the interpolated implied vol at that point as a volatility proxy.
     - Use the one-month historical volatility estimator on the log changes in the implied volatility as the VR measure.
   - Variations/measurement issues: Different vol proxies, different historical horizons, or conditional forecasting specifications (vol is not observable, neither is vol of vol)
What are the primary risks in option investments?

We come up with 3 main primary risk sources:

1. *Delta hedging cost (HC)*
2. *Stochastic volatility risk (VR)*
3. *Random jump risk (JR)*

- Delta hedge works perfectly well if jump size is known (as in binomial), it is the randomness of the jump size that breaks down the hedge.
- Far OTM options are designed to target the occurrence/protection of rare/large events
  - Stock investment targets the average
- *Measurement*: We use daily return kurtosis over the past month as a measure of the intensity of rare events. We scale it by the historical volatility estimator to match the option return exposure.
- Variations/measurement issues: Kurtosis at different frequencies and over different horizons may capture different things...
  - Options can also price in jumps that can happen in the future but have not happened in the past month/year (e.g., crash, default)
What are the primary risks in option investments?

We come up with 3 main primary risk sources:

1. \textit{Delta hedging cost (HC)}
2. \textit{Stochastic volatility risk (VR)}
3. \textit{Random jump risk (JR)}
4. Other types of option investment risks?

- \textit{Option bid-ask} (volume, open interest...): This reflects the same/similar set of risk of what we want to capture for an options market maker

- \textit{Diversification effect}: We want to know the risk type before we examine how much is diversifiable.

- \textit{Firm fundamental characteristics}: The stock risk-return literature seems to find that historical risk exposure estimates (e.g. beta on factor portfolios) are not as stable as firm characteristics as proxies
  - HC/VR/JR are all characteristics built on historical behaviors.
  - What fundamental characteristics capture option investment risks? Size (HC), operational liquidity (VR), and leverage/credit (JR)?
What are the primary risks in option investments?

We come up with 3 main primary risk sources:

1. **Delta hedging cost (HC)**
2. **Stochastic volatility risk (VR)**
3. **Random jump risk (JR)**
4. Controls (to capture things missed by the above risk measures)
   - **Historical risk premium (HRP):** average excess returns over the past year excluding last month.
     - If risk premiums are persistent (say driven by stable structural firm characteristics), momentum effect can be strong.
   - **Volatility risk premium (VRP):** Difference between implied volatility and breakeven volatility forecast \(\sim\) expected return
     - Doesn’t tell the source, can be either risk compensation or mispricing.
     - The measure is as good (bad) as the breakeven vol forecast...
     - We forecast future 1m variance with historical variance estimators at 1m, 1q, and 1y via cross-sectional regressions over the past year...
     - Stock return variance prediction can be a project of its own ...

It is important to distinguish risks and controls...
How to identify and construct primary risk measures?

- All 3 risk exposures that we identify are subject to many variations in measurements...

- There are definitely other primary risk exposures in option contracts across different strikes/maturities...

- We use two risk premium measures (HRP, VRP) as control variables to absorb the remaining risk exposures.

- Future research:
  - Theoretical/structural analysis that provides insights on the underlying primary risk sources
  - Econometric techniques that enhance the accuracy/timeliness of the risk exposures given data/information constraints
Cross-sectional asset pricing in option investments

\[ r_{t,T,i} = \zeta_{t,T} + \eta_{t,T,1} HC_{t,i} + \eta_{t,T,2} VR_{t,i} + \eta_{t,T,3} JR_{t,i} + \phi_{t,T,1} HRP_{t,i} + \phi_{t,T,2} VRP_{t,i} + \varepsilon \]

- \( r_{t,T,i} \) — daily delta hedged straddle excess return in percentages
- Control variables are cs standardized: deduct mean, scale by std
- Risk exposures (HC, VR, JR) are scaled by cs std.
  - All 3 risk exposures are positive. 0 means “no” risk.
  - Intercept \( \zeta_{t,T} \): the excess return on an option portfolio with zero exposures to the 3 primary risks.

Hypotheses:

1. **No risk exposure, no risk premium**: The average excess return on the no-risk option portfolio \((\zeta_{t,T})\) is not significantly different from zero.
2. **Risk premium increases with all 3 risks**: Average \((\eta_{t,T})\) are positive.
### Risks and risk premiums of option investments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Stdev</th>
<th>t-value</th>
<th>IR</th>
<th>Auto</th>
<th>Skew</th>
<th>Kurt</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW portfolio</td>
<td>0.22</td>
<td>0.71</td>
<td>4.40</td>
<td>1.08</td>
<td>0.22</td>
<td>-1.48</td>
<td>8.67</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.02</td>
<td>0.75</td>
<td>0.36</td>
<td>0.09</td>
<td>0.20</td>
<td>-1.05</td>
<td>9.51</td>
</tr>
<tr>
<td>Hedging cost</td>
<td>0.12</td>
<td>0.16</td>
<td>9.99</td>
<td>2.55</td>
<td>0.17</td>
<td>-0.02</td>
<td>0.51</td>
</tr>
<tr>
<td>Volatility risk</td>
<td>0.06</td>
<td>0.11</td>
<td>7.79</td>
<td>1.68</td>
<td>0.07</td>
<td>-0.35</td>
<td>2.33</td>
</tr>
<tr>
<td>Jump risk</td>
<td>0.18</td>
<td>0.14</td>
<td>22.11</td>
<td>4.61</td>
<td>0.01</td>
<td>-0.11</td>
<td>3.78</td>
</tr>
</tbody>
</table>

1. **No risk exposure, no risk premium**: Average intercept $(\zeta_{t,T})$ is not significantly different from zero.

2. **All 3 risks are highly priced**: Average $(\eta_{t,T})$ are highly significantly positive.
   - Jump risk the most highly priced; pricing of vol risk is less than half.

3. **Control variables**:
   - Remaining momentum effect is small, after controlling 3 risk exposures.
   - VRP captures some mispricing effect, independent of risk exposures.
Intertemporal asset pricing on option factor portfolios

\[ r_{t,T} = \alpha + \zeta_1 \overline{HC}_t + \zeta_2 \overline{VR}_t + \zeta_3 \overline{JR}_t + \cdots + e_{t,T} \]

- \( r_{t,T} \) — excess return time series on option factor portfolios.
- \( \overline{HC}_t, \overline{VR}_t, \overline{JR}_t \) — the average risk levels of the 3 risk factors

<table>
<thead>
<tr>
<th></th>
<th>( \overline{HC} )</th>
<th>( \overline{VR} )</th>
<th>( \overline{JR} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC</td>
<td>0.04 ( (3.92) )</td>
<td>-0.01 ( (1.14) )</td>
<td>0.02 ( (2.12) )</td>
</tr>
<tr>
<td>VR</td>
<td>-0.01 ( (0.77) )</td>
<td>0.02 ( (2.46) )</td>
<td>-0.01 ( (0.91) )</td>
</tr>
<tr>
<td>JR</td>
<td>0.03 ( (2.67) )</td>
<td>0.00 ( (0.24) )</td>
<td>0.03 ( (3.48) )</td>
</tr>
</tbody>
</table>

- The expected excess return on each factor portfolio increases intertemporally with the magnitude of the corresponding factor risk level.
- As each factor portfolio is constructed to have no exposure to other risks factors, the excess returns show little dependence on the average levels of other risk factors.
Extensions and future research

- Our analysis focuses on 30-day returns on 30-day atm options
  - We have identified 3 sources of primary risk in option investments that are positively priced both cross-sectionally and over time.

- Extensions: 30-day investment returns on options across delta and maturity

- Unresolved questions for future research
  - Other types of primary risks in option investments? Measurements?
  - The role of diversification (in the presence of high trading cost)
  - Do options across delta-maturity buckets serve similar/different risk-taking purposes?

  - *An integrated market place*: How to control/measure the different risk exposures across delta and maturity?

  - *Different risk taking across different buckets*: How (should) we form option combinations to highlight different risks? e.g., straddle, strangle, risk reversal, butterfly, calendar spread ...