Crash-O-Phobia: A Domestic Fear or A Worldwide Concern?

SILVERIO FORESI*

Investment Management Division, Goldman Sachs

LIUREN WU†

Zicklin School of Business, Baruch College

First draft: September 17, 2002

This version: November 25, 2003

*32 Old Slip, New York, NY 10005; tel: (212) 357-3508; silverio.foresi@gs.com.

†One Bernard Baruch Way, Box B10-225, New York, NY 10010; tel: (646) 312-3509; fax: (646) 312-3451; Liuren_Wu@baruch.cuny.edu; http://faculty.baruch.cuny.edu/lwu/.

Crash-O-Phobia: A Domestic Fear or A Worldwide Concern?

ABSTRACT

From an options data set on 12 major equity indexes around the world, we find that worldwide, options on equity indexes exhibit strikingly similar behaviors that present challenges for option modeling. Along the moneyness dimension, implied volatilities underlying all major equity indexes exhibit a heavily skewed average pattern, implying that the out-of-the-money put options are more expensive than the corresponding out-of-the-money call options, and that the risk-neutral distribution for these index returns are heavily negatively skewed.

Along the maturity dimension, the average implied volatility smirk does not flatten out as the option maturity increases from one month up to five years. Instead, the smirk steepens, indicating that the conditional return distribution becomes even more negatively skewed at longer horizons.

Time series analysis of the implied volatility series indicates that the volatility processes are stationary under both the objective measure and the risk-neutral measure. The mean term structure of the implied volatility level is quite flat, and the standard deviation of the volatility level declines readily with increasing maturity.

Finally, principal component analysis on the whole panel of data indicates that there exists one global factor governing the movement of the volatility level, but the variations in the shape of the implied volatility smirk are largely country-specific, even though the average shape of the volatility smirk is strikingly similar across different equity indexes.

JEL CLASSIFICATION CODES: G12, G13.

KEY WORDS: Equity index options; market crashes; volatility smirk; maturity pattern; central limit theorem.

Crash-O-Phobia: A Domestic Fear or A Worldwide Concern?

Since the U.S. stock market crash of 1987, the U.S. equity index options have exhibited a strong regularity along the strike price dimension. Out-of-the-money put options are much more expensive than the corresponding out-of-the-money call options. As a result, when we plot the Black and Scholes (1973) implied volatility on the U.S. equity index options against some measure of moneyness, we observe a "smirk" pattern. This phenomenon has been repeatedly documented in the literature, e.g., Aït-Sahalia and Lo (1998), Jackwerth and Rubinstein (1996), and Rubinstein (1994). In particular, Rubinstein (1994) refers to this phenomenon as "crash-o-phobia," alluding to the strong demand for put options on the S&P 500 index to hedge against market crashes in the United States. More recently, Carr and Wu (2003) analyze the maturity pattern of the implied volatility smirk on S&P 500 index options and find that, when plotted against a standardized measure of moneyness, the smirk remains steep as maturity increases up to the observable horizon of two years. Their evidence suggests that the crash-o-phobia in the U.S. stock market is not only a short-term worry, but also an acute long-term concern.

Is this "crash-o-phobia" phenomenon merely a domestic concern in the United States as a result of that specific stock market crash in 1987? Or is it a global pattern of the worldwide equity markets that is caused by more fundamental reasons? This paper addresses the question by investigating the option price behavior on major worldwide equity indexes.

We obtain from a major investment bank a large data set of daily over-the-counter implied volatility quotes at fixed moneyness and maturities on major equity indexes. At each date and time-to-maturity, the data set contains five fixed moneyness levels, defined in percentages of spot index level at 80, 90, 100, 110, and 120 percent. Each day contains eight fixed time-to-maturities from one month to five years, and each series has about two thousand daily observations from April 1995 to August 2002. Thus, underlying each equity index, we have observations that span the three most important dimensions of option price behavior: the moneyness (strike), the time-to-maturity, and the calendar time. The data set has quotes on options underlying 12 major equity indexes covering the stock market across the world: AEX (Holland), ALO (Australia), CAC (France), DAX (Germany), FTSE (Britain), HSI (Hong

Kong), IBEX (Spain), MIB (Italy), NKY (Japan), OMX (Sweden), SMI (Switzerland), and SPX (the United States).

We analyze the option price behavior along each of three important dimensions. We first document the average shape of the implied volatility curve across moneyness at each fixed maturity. A flat implied volatility line across moneyness would indicate that the index return is normally distributed under the risk-neutral measure. On the other hand, any smile or smirk pattern would be evidence of non-normality. The slope of such a smile or smirk reflects the asymmetry of the return distribution and the curvature captures the fatness of distribution tails. We find that all the 12 equity indexes generate strikingly similar moneyness patterns. The average implied volatility curves across moneyness all exhibit downward sloping "smirk" shapes. Even more striking, at a given maturity, the average slopes of these smirks look close to one another for different equity indexes, indicating that the average magnitude of asymmetry for the risk-neutral return distribution is also close to one another for different equity indexes. If a smirk is caused by concerns for a market crash, this concern is obviously not limited to the U.S. stock market, but ubiquitous around world.

At a fixed maturity, the implied volatility smirk across moneyness presents evidence on the shape of the conditional return distribution over this maturity horizon. Therefore, how these smirks evolve with increasing maturities tells us how the conditional density varies with increasing time horizon. When we plot the implied volatility against a standardized moneyness measure, defined as the log strike over maturity divided by the square root of time-to-maturity, we find that the maturity pattern for the U.S. Index options applies to all the other 11 equity indexes. As the time-to-maturity increases up to five years, the implied volatility smirk does not flatten out, but steepens instead. This maturity pattern implies that the asymmetry of the conditional return distribution does not diminish, but actually increases, as the conditioning horizon increases for as long as five years.

Another important dimension of the option price behavior is the time series property. At each time-to-maturity and calendar time, we summarize the information on the implied volatility smirk with three quantities: the level, the slope, and the curvature of the smirk. We estimate the three quantities via a second order polynomial fit across moneyness. We then document the time series properties of these three quantities. We find that the term structure of the mean volatility level is quite flat for most indexes. The standard deviation of the volatility level declines monotonically with increasing maturity for all indexes. The stable mean term structure on the volatility level and the downward sloping term structure of the standard deviation of the implied volatility are evidence of stationarity for the volatility process. We also measure the autocorrelation of the volatility series and find that the volatility becomes more persistent with increasing maturities, indicating the potential existence of multiple volatility factors and/or nonlinear volatility dynamics.

For all equity indexes, the mean slope of the volatility smirk becomes more negative as maturity increases, confirming our earlier observation that the smirk steepens with increasing maturity. This steepening mainly happens within one year maturity, after which the term structure of the smirk slope flattens out. The curvature of the smirk is usually very small for most indexes. Similar to the volatility level, both the slope and the curvature of the smirk exhibit high persistence that increases with maturity. We also find that the volatility smirk tends to become more negatively skewed when the volatility level is high.

Finally, we perform principal component analysis on the daily changes in the volatility level, slope, and curvature. We find one global factor in the volatility level movement, but the movements of the smirk slopes and curvatures are mainly country-specific. Correlation analysis reveal similar results.

The structure of the paper is as follows. Section I describes the structure of the data. Section II, III, and IV analyze the option price behavior along the three important dimensions: moneyness, maturity, and calendar time, respectively. Section V documents the comovement between the implied volatility smirks of different indexes. Section VI concludes.

I. Data Description

We obtain over-the-counter implied volatility quotes from a major investment bank on options underlying 12 major equity indexes across the world. The 12 indexes are AEX (Holland), ALO (Australia), CAC (France), DAX (Germany), FTSE (Britain), HSI (Hong Kong), IBEX (Spain), MIB (Italy), NKY

(Japan), OMX (Sweden), SMI (Switzerland), and SPX (the United States). Table I lists these 12 equity indexes.

The quotes are daily on option contracts with fixed time-to-maturity and moneyness. All options are European style. The starting dates vary from index to index. The earliest starts in March, 1994 and the latest starts in April 1995. The ending dates are uniform on August 20th, 2002. The shortest sample has 1,847 daily observations, and the longest has 2037 daily observations. The last column of Table I provides the starting date for each index.

We use the full sample for each series when documenting the properties of individual series. But when we measure the implied volatility comovements between different equity indexes, we use the common sample period of the 12 equity indexes from April 28th, 1995 to August 20th, 2002 (1,847 days of observations).

At each date for each index, the implied volatility quotes are available at eight fixed time-to-maturity levels: one month, three months, six months, one year, two years, three years, four years, and five years. Each maturity level contains five fixed moneyness levels, defined as strike prices in percentages of the spot level at 80, 90, 100, 110, and 120 percent.

Traditionally, options on equities and equity indexes are mostly traded on the options exchanges. In the United States, the S&P 500 index options are listed at the Chicago Board of Options Exchange (CBOE). These exchange-listed option contracts have fixed strike prices and expiry dates (the Saturday following the third Friday of the expiring month). Since early 1990s, an over-the-counter market has developed for equity index options. These over-the-counter contracts are quoted at fixed moneyness, defined as strike price in percentages of spot index levels, and at constant time-to-maturities. The over-the-counter market has developed rapidly over the past decade, mainly for hedging exotic derivative contracts such as variance swaps and long term reinsurance contracts.

Currently, for short-term contracts with time-to-maturities within a year, both the exchanges market and the over-the-counter market are liquid, but the two markets often serve different clienteles. All the small, customer orders go to the CBOE, but many large, institutional orders go through over the counter.

The liquidity of the exchanges market declines rapidly as the options maturity increases. Options with maturities longer than two years are mainly institutional orders and mainly traded over the counter.

The design of the over-the-counter options market differ from the exchanges market in several important ways. First, the quotes are not directly on option prices, but on the Black-Scholes implied volatility. Given the quote on the implied volatility, the invoice price for the option contract is computed based on the Black-Scholes model, with mutually agreed-upon inputs on the underlying spot index level and interest rates. Second, when a transaction takes place, it involves not only the exchange of the option position, but also the corresponding delta hedging position on the underlying index. As a result, the transactions are approximately delta-neutral and hence are not sensitive to the underlying spot movement. Third, for very large trades, the over-the-counter market also has something similar to the “upstairs” market, where the broker dealer try to match buyers directly with sellers so that the broker dealer does not need to take large inventory.

The derivative nature of the options contract creates issues for market making that do not exist in primary security markets. First, the options price is directly linked to the price of the underlying security. Therefore, whenever the underlying spot price changes, the prices of all options underlying this security change with it. On an options exchange, this features forces the market maker to update all the options prices whenever the underlying security price moves. Delays in the updating process can potentially put the market maker in a risky position. Therefore, the market maker in an options market must have advanced technology that can update quotes on hundreds of options underlying one security within a very short interval of time. Second, when a customer has private information on the underlying security, the customer can simultaneously put buy/sell orders on all the options underlying this security. For example, if the customer knows that the price of the underlying security will go up, the customer can send buy orders on all the call options and sell orders on the put options. Thus, even if the market maker provide a small quote size on each option, the aggregate risk exposure to the market maker can still be very large, due to the highly correlated nature of all these options underlying one security. These concerns force the market maker to post wider bid-ask spreads and smaller quote sizes, thus making the options exchange market less liquid and unattractive for institutional players who want to engage options trades of large quantities.

On the other hand, the unique design of the over-the-counter options market addresses these concerns and improves the liquidity and depth of the market. The exchange of the covered position, rather than a naked option position, significantly reduces the broker dealer's exposure to the directional bet on the underlying security. For example, a customer who has private information on the underlying currency going up may not want to trade on the over-the-counter currency market because he has to sell the delta of the currency for either buying a call option or selling a put option. The quotation on the implied volatility rather than the option price itself further reduces the broker dealer's burden in constantly updating the option prices per every move on the underlying currency price. An update is necessary only when the broker dealer thinks that the underlying volatility or other higher moments of the market has changed. The spot price of the underlying currency plays a lesser role given the covered transaction. Finally, for very large transactions, the "upstairs" mechanism further reduces the broker dealer's exposure to large inventory positions and also reduces the transaction costs for both sides. As a result, the over-the-counter market can handle very large trades with small bid-ask spreads and little market impacts, an ideal place for institutional players. For long-dated options (maturities longer than two years), the exchange market is almost non-existent, but the over-the-counter market remains very active. For example, out-of-the-money options on the S&P 500 index with time-to-maturities between three to five years trade almost on a daily basis. Furthermore, one can easily open tight quotes for these options from various brokers. Therefore, the over-the-counter implied volatility quotes we have provide important information that is not readily available for the exchange markets.

For each equity index, we have implied volatility quotes along three important dimensions: moneyness, time-to-maturity, and calendar time. We document the stylized properties along each dimension. Furthermore, with option quotes on 12 major equity indexes, we also investigate whether the options on different indexes share common features or exhibit important differences along the three most important dimensions of the option price. Finally, we study the comovement between the equity indexes around the world.

II. Smile or Smirk? The Moneyness Pattern of Option Implied Volatility

One of the striking features of S&P 500 index options is that the Black-Scholes implied volatility quotes on these options exhibit a heavily skewed line when plotted against moneyness, showing that out-of-the-money put options are more expensively priced relative to their out-of-the-money call option counterparts. These skewed plots are often referred to as the "implied volatility smirk", in contrast to the more symmetric smile shape observed in currency options. The smirk has become a persistent feature of the U.S. equity index market ever since the stock market crash in 1987. Rubinstein (1994) refers to this phenomenon as "crash-o-phobia," alluding to the strong market demand for out-of-the-money put options on the S&P 500 index to hedge against market crashes in the United States.

The implied volatility smirk implies a heavily skewed risk-neutral distribution for the index return that contrasts sharply with the relatively symmetric return distribution estimated from the time series data. In principle, the difference between the two distributions can be attributed to the market price of risk. The asymmetry in the risk-neutral distribution versus the symmetric time-series distribution can be regarded as evidence that the market charges a higher premium on downside moves than on upside moves. Several studies, e.g., Jackwerth (2000), Bates (2001), Engle and Rosenberg (2002), Bliss and Panigirtzoglou (2002, 2003), and Wu (2003), ask whether the much higher premium on the put options is a reasonable compensation for the crash risk, or whether the market participants are so paranoid that they are paying a premium on these options that defies logic. The general consensus is that the premium charged on the downside index movement looks extreme even if it is allowable by no-arbitrage.

Using option quotes from 12 equity indexes around the world, we analyze the extreme behavior of the U.S. equity index options from another perspective. We ask wether the extreme behavior observed in the U.S. equity index options market is merely a local paranoia, or is it a common feature shared by the world market. If it is a unique local phenomenon, we may conclude that the U.S. 1987 stock market crash has indeed scared the U.S. investors into paranoia. If options on the major equity indexes around the world share the same pattern, we would infer that stock market crashes are a worldwide concern. Furthermore, the commonality of the feature may also imply that there exist some fundamental reasons for the large premia charged on out-of-the-money put options.

Figure 1 plots the sample average of the implied volatility against the fixed moneyness levels, defined as the strike price in percentage of the spot index level. For comparison, we group the plots for different equity indexes into the same panel at each time-to-maturity. The eight panels represent eight different maturities from one month to five years.

A striking pattern emerges. For all indexes and at all maturities, the average implied volatility plots against moneyness are all skewed to the left, implying that out-of-the-money put options are more expensive than the corresponding out-of-the-money call options. The main difference among different equity indexes is the level of the implied volatility. The slopes, and hence the degrees of the skew in the volatility smirk plot, look strikingly close from one equity index to another.

These plots indicate that the implied volatility smirk is not a purely domestic concern in the United States, but a worldwide phenomenon apparent in all major equity markets. Therefore, if the implied volatility smirk in the United States is due to local paranoia, the whole world seems to be in a similar paranoid mind set. Investors all over the world equity markets charge a much higher premium for out-of-the-money put options than for the corresponding out-of-the-money call options. This phenomenon is potentially due to the fact that the market takes an aggregate long position on the equity indexes. Downside moves on the equity indexes represent systematic risks that ask for high compensation. On the other hand, the investors view upside movements of the equity indexes not as risks, but as opportunities. Therefore, the investors ask for little compensation for taking on the upside opportunities, thus creating a highly asymmetric risk-neutral distribution for the index returns even though the physical distribution is relatively symmetric. Consistent with this conjecture, based on standard definitions of risk preference, Jackwerth (2000) and Engle and Rosenberg (2002) find that the investors are actually risk-loving rather than risk averse at some sections of the return distribution.

III. The Maturity Pattern of the Implied Volatility Smirk

In plotting the implied volatility smirks in Figure 1, we define moneyness as the strike in percentage of the spot level: $100 \times K/S$, where K denotes the strike price and S denotes the spot level. This choice of moneyness is mainly for convenience since the available implied volatility quotes are at fixed moneyness levels according to this definition.

We obtain a more transparent link between the shape of the implied volatility smirk and the conditional non-normality of the risk-neutral distribution using a standardized measure, d, for moneyness,

$$d \equiv \frac{\ln(K/S)}{\sigma\sqrt{\tau}} \tag{1}$$

where τ denotes the time-to-maturity of the option and σ is some measure of the volatility level. This definition has become an industry standard. The $\sqrt{\tau}$ scaling is important in making the scaling of the moneyness comparable across different maturities. The use of some volatility measure σ in the denominator is to make the moneyness measure also comparable across different securities of different volatility levels. The definition in equation (1) also allows a simple interpretation as roughly the number of standard deviations that the log strike is away from the log spot price in the Black-Scholes model.

With one year worth of exchanged-traded S&P 500 index options data, Carr and Wu (2003) find that when they plot the implied volatility against the standard measure of moneyness as defined in equation (1), the implied volatility smirk shape does not flatten out, but instead steepens slightly, as maturity increases up to the observable horizon of two years. Since the slope of the smirk reflects the asymmetry (skewness) of the risk-neutral distribution for the index return, the fact that the smirk does not flatten out implies that the asymmetry of the conditional distribution persists as the conditioning horizon increases. From an economic perspective, if the highly skewed implied volatility smirk at a certain maturity implies that market participants are phobic of market crashes within that time horizon, the persistence of the smirk over long maturities implies that the phobia stays not only for short term, but also in the long run.

Figure 2 plots the implied volatility smirks at different maturities for all the 12 equity indexes under the standard moneyness measure d in equation (1). To highlight the maturity pattern of the smirk, we plot the implied volatility smirks at different maturities within the same panel for each equity index. The 12 panels represent the 12 equity indexes. Since the implied volatility quotes are at fixed strikes as percentages of the underlying spot level, the range of the standardized moneyness d shrinks as the option maturity increases. Thus, we can easily distinguish the different maturity of each line. In each panel, the longest line represents the smirk of the options at the shortest maturities (one month), and the shortest line represents smirk of the options at the longest maturity (five years).

Figure 2 reveals a strikingly similar maturity pattern across all 12 equity indexes. When plotted against the standard moneyness measure d, the implied volatility smirk does not flatten out, but actually steepens as maturity increases. This finding is similar to the findings in Carr and Wu (2003) for S&P 500 index options, but extends their results in two important dimensions. First, we find that their observed maturity pattern not only holds for the United State, but also extends to all major equity indexes around the world. Second, the time-to-maturities of the exchanged-listed index options data are limited to within two years, but the time-to-maturity for our over-the-counter data extends to five years.

Related to this maturity pattern of implied volatility smirk, the over-the-counter options market also has a popular "collar" contract, under which an out-of-the-money put option is exchanged for an out-of-the-money call option of the same maturity. The put-option receivers are often wealthy individuals or fund managers who have large stakes on the equity market. The call option receivers are mostly the large investment banks. These contracts often possess very long maturities, e.g., ten to 30 years. Anecdotal evidence shows that these contracts are also highly asymmetric. The banks charge a much higher premium on the put options to provide insurance against stock market crashes. The counterparties (put-option receivers) either give up large upside potentials by offering a call option with a low strike price, or by paying cash upfront for the premium differences between the call and put options in the contract. Therefore, the maturity pattern of the implied volatility smirk that we observe from the options data is a robust and persistent feature of the equity index market.

This maturity pattern of the implied volatility smirk has important economic implications for the worldwide equity markets. The demand for out-of-the-money put options is much stronger than the demand for the out-of-the-money call options. This demand difference holds for all major equity indexes and at all quoted maturities. Therefore, the fear for stock market crashes, and the desire to hedge again them, are universally strong at all investment horizons. How to rationalize this fear for the market crashes and explain the persistent excess demand and high premium for put options on equity indexes remains a challenging, but important, topic for future research.

The fact that the observed volatility smirk remains steep when the time-to-maturity extends to as long as five years also presents a challenging task for option pricing modeling. The maturity pattern implies that the conditional risk-neutral distribution for the index return remains heavily non-normal (skewed) as the conditioning horizon increases up to five years. In continuous-time modeling, researchers often incorporate a jump component in the asset return process to generate return non-normality at short conditioning horizons.[1] Return non-normality generated from most of these jump models declines rapidly as the conditioning horizon increases by virtue of the classic central limit theorem (Konikov and Madan (2000) and Backus, Foresi, and Wu (1997)). To generate return non-normality at longer time horizons, researchers often resort to the specification of a persistent stochastic volatility process.[2] However, to maintain the same (or even higher) non-normality for the conditional return distribution for horizons as long as five years, the stochastic volatility process must become near nonstationary. A nonstationary stochastic volatility process generates features along other dimensions that do not match the observed implied volatility behavior. For example, Pan (2002) shows that under a nonstationary stochastic volatility process, the implied volatility level explodes with increasing maturity, a feature not observed in the data.

Another interesting observation from both Figure 1 and Figure 2 is that the implied volatility smirks across all maturities and for all 12 equity indexes are all decidedly one sided, implying that the risk-neutral distribution of the index returns has a very fat left tail, but the right tail of return distribution is thin. Such evidence renders support for the one-sided α-stable specification in Carr and Wu (2003), but not the traditional compound Poisson jump specification in Merton (1976). This observation illustrates

[1] See, for example, Merton (1976), Bates (1991), Madan and Milne (1991), Madan, Carr, and Chang (1998), Heston (1993b), Barndorff-Nielsen (1998), Eberlein, Keller, and Prause (1998), and Carr, Geman, Madan, and Yor (2002).

[2] See, for example, Hull and White (1987), Heston (1993a), Bates (1996, 2000), Bakshi, Cao, and Chen (1997), Duffie, Pan, and Singleton (2000), Pan (2002), Andersen, Benzoni, and Lund (2002), Eraker, Johannes, and Polson (2003), Eraker (2003), Carr, Geman, Madan, and Yor (2003), and Huang and Wu (2003).

the extreme asymmetry between the market demand and premium for out-of-the-money put and call options.

IV. The Time Series Properties of Implied Volatility

This section documents the time-series properties of the implied volatility surface. The cross-sectional patterns of the implied volatility surface documented in the previous two sections present important information on the average shape of the conditional return distribution under the risk-neutral measure and on how this conditional distribution evolves with increasing horizon. In contrast, the time series properties of the implied volatility quotes resent evidence on the dynamics of volatility and insights on the dynamic specification of the stochastic volatility process under both the risk-neutral and the objective measures.

At each date, we observe a cross section of 40 implied volatility quotes on eight maturities and five strike levels at each maturity. We summarize the information of each implied volatility smirk by its level, slope, and curvature. To do this, we fit a second order polynomial to the volatility smirk against moneyness,

$$IV(d;t,\tau) = c_0 + c_1 d + c_2 d^2, \tag{2}$$

where $IV(d;t,\tau)$ denotes the implied volatility quote at moneyness d, time t, and time-to-maturity τ. The moneyness d denotes the standard moneyness measure defined in equation (1). We use the at-the-money volatility at each date and maturity to proxy σ in the definition of the moneyness d in equation (1).

With the second-order polynomial fit in equation (2), the regression coefficients, c_0, c_1, c_2, capture the level, slope, and curvature of the implied volatility smirk, respectively. The intercept c_0 represents the fitted value for the at-the-moneyness ($d = 0$) implied volatility. It represents the level of the volatility at that maturity and date. The coefficient c_1 captures the slope of the smirk at $d = 0$ and reflects the asymmetry of the conditional distribution on the index return. The coefficient c_2 measures the global curvature of the smirk and is related to the fat tail of the return distribution under the risk-neutral measure (Backus, Foresi, and Wu (1997)).

Figure 3 plots the sample mean, standard deviation, coefficient of variation, and the first-order autocorrelation of the daily coefficient estimates at each maturity. Each panel represents one property, and each line represents one equity index. The four panels on the left column summarize the time series properties on the volatility level, proxyed by c_0, at different option maturities. The mean term structure of the volatility level c_0 can be either upward sloping or downward sloping, but is on average flat across maturities. The flatness or stability of the mean volatility level across option maturities shows that the instantaneous volatility process is stationary under the risk-neutral measure (Pan (2002)).

For risk-management on derivative positions, it is important to assess the volatility risk, or the volatility of volatility. In Figure 3, we measure the term structure of the volatility risk in terms of both the standard deviation (second row) and the coefficient of variation (third row). For all equity indexes, both the standard deviation and the coefficient of variation for the volatility level c_0 decline with increasing maturity. This feature supports the stationarity conjecture on the volatility process.

The last row of the first column measures the first-order (daily) autocorrelation of the volatility level c_0. The estimates are high, indicating that the volatility process is highly persistent under the objective measure, even though it is stationary. Furthermore, the first-order autocorrelation estimates for the volatility level increase with increasing maturity. Had the return variance followed a one-factor affine process such as the square-root process of Heston (1993a), we would have expected the same autocorrelation estimates for volatilities of different maturities. Therefore, the upward sloping term structure points to the potential existence of the multiple volatility factors, with the less persistent factors(s) dominating the short end of the term structure and the more persistent factor(s) dominating the long end.

The four panels in the middle column of Figure 3 summarize the time series properties of the smirk slope, c_1. First, for all equity indexes, the mean term structure of the smirk slope c_1 is downward sloping. That is, the slope of the implied volatility smirk becomes more negative with increasing maturities, an average maturity pattern that we have documented in the previous section.

The standard deviation of the smirk slope c_1 increases with maturity, but the coefficient of variation declines with maturity, indicating stability on the dynamics of the asymmetry of the conditional return distribution. Finally, the first order autocorrelation of the smirk slope is high, and increasing with maturity, corroborating with the evidence on volatility levels on the existence of multiple factors.

The four panels in the last column of Figure 3 summarize the properties of the curvature c_2 of the implied volatility smirk. The mean values of the curvature estimates are small, except for the Hang Seng index at long maturities. These mean estimates are consistent with our visual observation that the implied volatility smirks look like straight lines. We do not observe strong patterns across maturities for the time series properties of the small curvature estimates.

The fact that the persistence increases with maturity for both the volatility level and the skewness of the conditional density indicates that there are potentially multiple factors governing the dynamics of volatility. Earlier studies, e.g., Buraschi and Jackwerth (2001), and Cont and da Fonseca (2000, 2001) have drawn similar conclusions. What becomes interesting is how these different factors interact with one another to control the evolution of the conditional return density both along the maturity dimension and over time.

The factor dynamics under the objective measure controls the time series variation of the volatility. The risk-neutral dynamics controls the cross-sectional behavior. To obtain more evidence on the volatility dynamics under both measures, we compute the cross-correlation between the daily changes in the volatility level and the daily changes in the smirk slopes. This correlation captures how the volatility level moves with the asymmetry of the return distribution.

Figure 4 presents the correlation estimates at each maturity for all the 12 equity indexes. The left panel measures the correlation based on daily level and slope estimates. The right panel measures correlation based on the first-order differences, or daily changes, of these estimates. The correlation estimated based both on levels and daily changes are predominantly negative. Since the slope of the implied volatility smirk is negative, a negative correlation between the volatility level and the smirk slope implies that the volatility smirk steepens and becomes more negative when the volatility level is high.

Economically, the evidence in Figure 4 indicates that the demand for the out-of-the-money put options relative to the out-of-the-money call options increases with increasing volatility. When the market fear for market crashes increases, the market charges a higher premium on all options, but more so for out-of-the-money put options than for out-of-the-money call options. From a modeling perspective, this negative correlation implies that the jump component becomes more dominant when the overall volatility level is high.

V. Comovements Among Equity Indexes

Options underlying major equity indexes exhibit strikingly similar patterns across the three most important dimensions of the option price behavior: the moneyness, the time-to-maturity, and the calendar time. Along the moneyness dimension, the implied volatilities underlying all major equity indexes exhibit a strong negatively skewed feature, thus the volatility smirk. Along the time-to-maturity dimension, this smirk does not flatten out, but steepens, with increasing maturity. Yet, the term structure and time series properties of the implied volatility series indicate that the instantaneous volatility should be stationary under both the risk-neutral and the objective measures. Finally, we also observe that for all equity indexes, the persistence of the volatility level and smirk slope increases with increasing maturity, indicating the existence of multiple volatility factors.

Economically, these common features imply that for the equity index options markets around the world, a much stronger demand exists for out-of-the-money put options than for out-of-the-money call options. Furthermore, this excess demand on put options remains strong as the option maturity increases. Therefore, crash-o-phobia is not a local phenomenon in the United States, but a global mind set around the world that persists at both short and long horizons. This section tries to identify the source of this commonality.

To identify comovement among different equity indexes, we first form a consolidated implied volatility sample based on the common sample period. The number of observation for this consolidated sample is 1,847 for each series. We then perform principal component analysis on the daily changes in the volatility levels (c_0) and volatility slopes (c_1).[3]

The principal component analysis is based on the decomposition of the covariance matrix of the sample series. The normalized eigenvalues of the covariance matrix can be interpreted as the percentage variation explained by each principal component. The normalized eigenvector corresponding to each principal component represents the loading of this common component to the sample series.

Figure 5 depicts the percentage of aggregate variation explained by each of the first 30 principal components in each data set. The left panel is for the volatility level c_0. The first principal component in c_0 accounts for 34.5 percent of its variation in the daily changes. The contribution of the second principal component is much lower at 11.5 percent. We also observe an obvious slope change on the plot after the first principal component. The literature has not arrived at a consensus criterion in determining the optimal number of common factors in principal component analysis. Although some researchers have proposed statistical tests, e.g., Connor and Korajczyk (1993), a common practice is to plot the eigenvalues as in Figure 5 and visually inspect for slope changes. Principal components whose eigenvalues fall on different slopes of the plot represent different levels of commonality. Thus, based on plot in the left panel of Figure 5, we conclude that the first principal component of the volatility level represents a different level of commonality from the other principal components.

The right panel of Figure 5 plots the percentage of explained variation for the volatility slope c_1. Unlike the plot for the volatility level c_0, we do not observe a dominant common factor here for the slope c_1. The first principal component in c_1 explains less than 12 percent of the variation. Furthermore, the first eight eigenvalues fall approximately on the same line, indicating that they represent the same level of commonalities.

Figure 6 plots the factor loading (eigenvector) for the first three principal components on different indexes and at different maturities. The first row denotes the factor loadings on the volatility level c_0

---

[3]We have also performed the principal component analysis on the curvature estimates, but the results are noisy due to the small curvature estimates. To save space, we do not report the results on curvatures but will make them available upon request.

and the second row denotes the loading on the slope c_1. In each row, the first column denotes the loading of the first principal component from the respective data series, the second column denotes the loading of the second factor, and the third row for the loading of the third factor. Each line denotes the loading on each index across different maturities.

We first investigate the shape of the loading of the first principal component on the implied volatility level c_0. We find that the loadings of the first principal component on all indexes are all positive and all exhibit similar maturity patterns across all equity indexes. The loading declines with increasing maturity. Therefore, we identify a common movement in the volatility level of index returns across all equity indexes. This comovement explains 34.5 percent of the aggregation variation in all series.

The loading of the second principal component in the volatility level c_0 is quite different. This second component has a large loading on only one index series, the volatility levels on AEX (the bottom line), but its loadings on other indexes are close to zero. Therefore, this second principal component looks more like a country-specific component that belong to AEX. The third factor has significant loadings on only two indexes: The top line corresponds to NKY, and the bottom line corresponds to OMX. The loadings of the subsequent five principal components exhibit similar patterns: They are significant on either one or two index series, but close to zero for all other indexes. Therefore, we conclude that principal components number two to eight are either country specific or shared by only a small number of countries. Only the first principal component of the volatility level represents a global common factor.

The second row of Figure 6 shows a dramatically different picture for the comovement of the smirk slope c_1. The loadings of each of the first three principal components are significant for only one equity index, but close to zero for all other indexes. The first principal component has a large positive loading on OMX (top line), the second principal component (middle panel) has a large positive loading (top line) on HSI, and the third component has a large negative loading on NKY. We observe similar patterns among all of the first 12 principal components, showing that there does not exist an identifiable global component in the movement of the smirk slopes. Instead, the movements of the smirk slope are mainly country specific.

The principal component analysis indicates that the world does share one global component in terms of market risk, as captured by the comovement in index return volatility. Nevertheless, variations on the shapes of the volatility smirks are mainly country specific. If the premium difference between put and call options represents a global phobic mind set against market crashes, our principal component analysis shows that this global mind set is potentially linked to a common source of volatility risk across the world. For international asset pricing, our evidence suggests a global return factor that exhibits stochastic volatility.

VI. Conclusion

Worldwide, options on equity indexes exhibit strikingly similar behaviors that pose challenges for option pricing modeling and reveal needs for fundamental economic interpretation. Along the moneyness dimension, implied volatilities underlying all major equity indexes exhibit a heavily skewed pattern, implying that the out-of-the-money put options are more expensive than the corresponding out-of-the-money call options, and that the implied risk-neutral distribution for these index returns are heavily skewed to the left. This highly asymmetric risk-neutral return distribution forms a sharp contract to the much more symmetric distribution estimated from the time series return data. The discrepancy between the two distributions implies that the market charges an extremely high premium on downside index movements, but asks little for upside index movements.

Along the maturity dimension, the implied volatility smirk does not flatten out with increasing maturity, but exhibits moderate steepening, much to the opposite of the implication from the classic central limit theorem. To account for this feature, the literature either makes the return variance infinite and hence the central limit theorem no longer applies, or incorporates a highly persistent volatility process to slow down the impact of the central limit theorem. In the latter case, the fact that the smirk remains steep after five years of maturity implies that this volatility process has to be borderline nonstationary.

However, the time series properties of the implied volatility series indicate that the volatility process is stationary under both the risk-neutral measure and the objective measure. The mean term structure of the volatility level is rather flat, an evidence contradicting the assumption of a near nonstationary volatility process under the risk-neutral process. The standard deviation of the volatility level declines readily with increasing maturity, an evidence of a stationary volatility process under the objective measure.

Finally, principal component analysis indicates that there is one global factor governing the movement of the volatility level, but the variations in the shape of the implied volatility smirk are largely country-specific, even though the average shape of the volatility smirk is strikingly similar across different equity indexes. Thus, for international asset pricing, we need one global return factor that exhibits stochastic volatility.

References

Aït-Sahalia, Yacine, and Andrew Lo, 1998, Nonparametric estimation of state-price densities implicit in financial asset prices, *Journal of Finance* 53, 499–547.

Andersen, Torben G., Luca Benzoni, and Jesper Lund, 2002, An empirical investigation of continuous-time equity return models, *Journal of Finance* 57, 1239–1284.

Backus, David, Silverio Foresi, and Liuren Wu, 1997, Accounting for biases in Black-Scholes, Working paper, New York University.

Bakshi, Gurdip, Charles Cao, and Zhiwu Chen, 1997, Empirical performance of alternative option pricing models, *Journal of Finance* 52, 2003–2049.

Barndorff-Nielsen, Ole E., 1998, Processes of normal inverse Gaussian type, *Finance and Stochastics* 2, 41–68.

Bates, David, 1991, The crash of 87: Was it expected? The evidence from option markets, *Journal of Finance* 46, 1009–1044.

Bates, David, 1996, Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options, *Review of Financial Studies* 9, 69–107.

Bates, David, 2000, Post-'87 crash fears in the S&P 500 futures option market, *Journal of Econometrics* 94, 181–238.

Bates, David S., 2001, The market for crash risk, Unpublished working paper, University of Iowa IA.

Black, Fisher, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–654.

Bliss, Robert R., and Nikolaos Panigirtzoglou, 2002, Testing the stability of implied probability density functions, *Journal of Banking and Finance* 26, 381–422.

Bliss, Robert R., and Nikolaos Panigirtzoglou, 2003, Option-implied risk aversion estimates, *Journal of Finance* forthcoming.

Buraschi, Andrea, and Jens Carsten Jackwerth, 2001, The price of a smile: Hedging and spanning in option markets, *Review of Financial Studies* 14, 495–527.

Carr, Peter, Hélyette Geman, Dilip Madan, and Marc Yor, 2002, The fine structure of asset returns: An empirical investigation, *Journal of Business* 75, 305–332.

Carr, Peter, Hélyette Geman, Dilip Madan, and Marc Yor, 2003, Stochastic volatility for Lévy processes, *Mathematical Finance* 13, 345–382.

Carr, Peter, and Liuren Wu, 2003, Finite moment log stable process and option pricing, *Journal of Finance* 58, 753–777.

Connor, Gregory, and Robert A. Korajczyk, 1993, A test for the number of factors in an approximate factor model, *Journal of Finance* 48, 1263–1291.

Cont, Rama, and Jose da Fonseca, 2000, Deformation of implied volatility surfaces: An empirical approach, in Takayasu, eds.: *Empirical Science of Financial Fluctuations* (Springer, Tokyo).

Cont, Rama, and Jose da Fonseca, 2001, Dynamic modeling of implied volatilities: Quantifying and hedging volatility risk, Unpublished working paper, Centre de Mathématiques Appliquées, Ecole Polytechnique France.

Duffie, Darrell, Jun Pan, and Kenneth Singleton, 2000, Transform analysis and asset pricing for affine jump diffusions, *Econometrica* 68, 1343–1376.

Eberlein, Ernst, Ulrich Keller, and Karsten Prause, 1998, New insights into smile, mispricing, and value at risk: The hyperbolic model, *Journal of Business* 71, 371–406.

Engle, Robert F., and Joshua V. Rosenberg, 2002, Empirical pricing kernels, *Journal of Financial Economics* 64, 341–372.

Eraker, Bjørn, 2003, Do stock prices and volatility jump? Reconciling evidence from spot and option prices, *Journal of Finance* forthcoming.

Eraker, Bjørn, Michael Johannes, and Nicholas Polson, 2003, The impact of jumps in equity index volatility and returns, *Journal of Finance* 58, 1269–1300.

Heston, Stephen, 1993a, Closed-form solution for options with stochastic volatility, with application to bond and currency options, *Review of Financial Studies* 6, 327–343.

Heston, Stephen, 1993b, Invisible parameters in option prices, *Journal of Finance* 48, 933–947.

Huang, Jingzhi, and Liuren Wu, 2003, Specification analysis of option pricing models based on time-changed Lévy processes, *Journal of Finance* forthcoming.

Hull, John, and Alan White, 1987, The pricing of options on assets with stochastic volatilities, *Journal of Finance* 42, 281–300.

Jackwerth, Jens Carsten, 2000, Recovering risk aversion from option prices and realized returns, *Review of Financial Studies* 13, 433–451.

Jackwerth, Jens Carsten, and Mark Rubinstein, 1996, Recovering probability distributions from contemporary security prices, *Journal of Finance* 51, 347–369.

Konikov, Mikhail, and Dilip Madan, 2000, Pricing options of all strikes and maturities using a generalization of the VG model, Working paper, University of Maryland.

Madan, Dilip, and Frank Milne, 1991, Option pricing with VG martingale components, *Mathematical Finance* 1, 39–56.

Madan, Dilip B., Peter P. Carr, and Eric C. Chang, 1998, The variance gamma process and option pricing, *European Finance Review* 2, 79–105.

Merton, Robert C., 1976, Option pricing when underlying stock returns are discontinuous, *Journal of Financial Economics* 3, 125–144.

Pan, Jun, 2002, The jump-risk premia implicit in options: Evidence from an integrated time-series study, *Journal of Financial Economics* 63, 3–50.

Rubinstein, Mark, 1994, Implied binomial trees, *Journal of Finance* 49, 771–818.

Wu, Liuren, 2003, Dampened power law: Reconciling the tail behavior of finanical asset returns, Unpublished working paper, Fordham University.

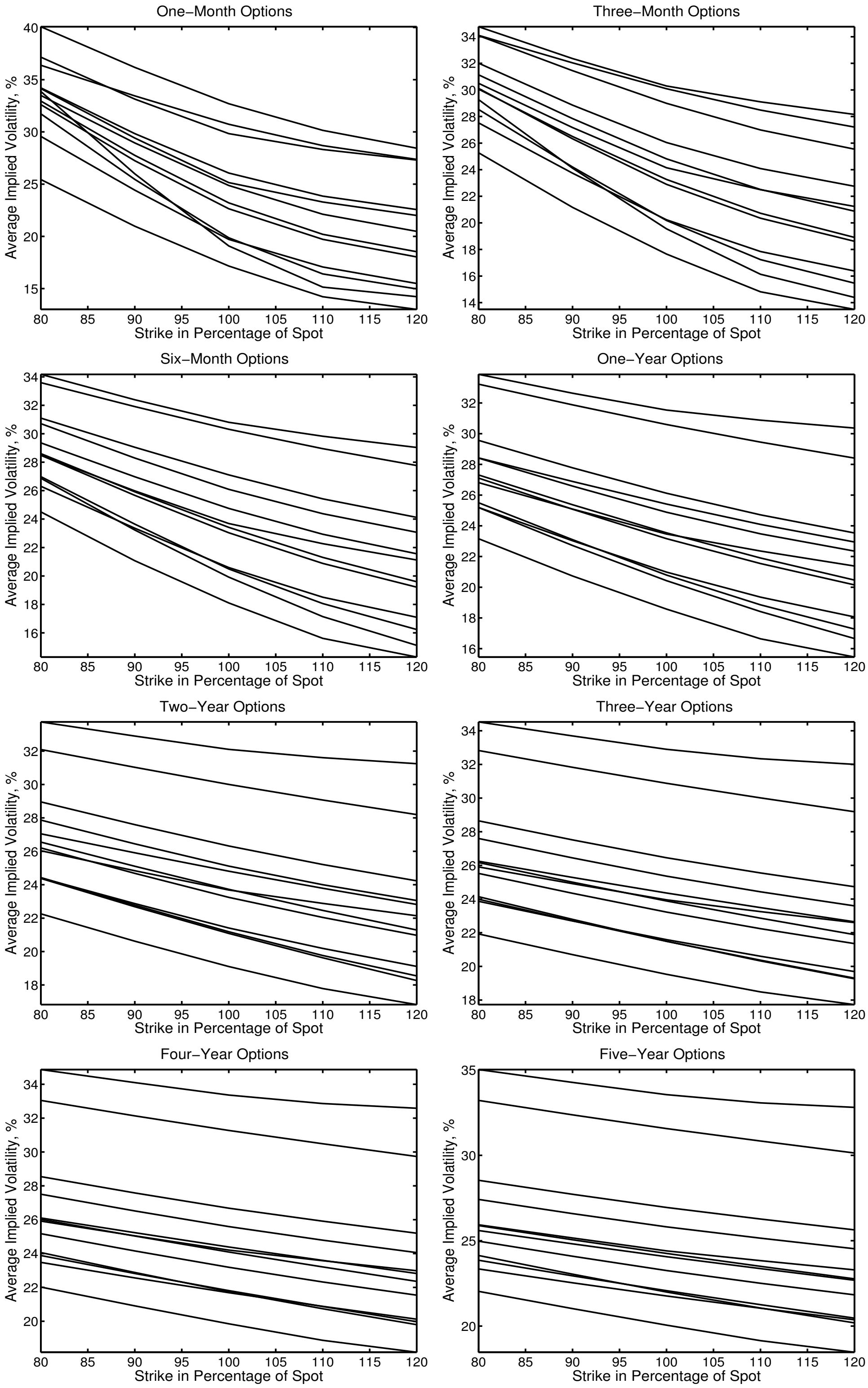

Figure 1. The universal implied volatility smirk on major equity indices across the world. Lines are the sample averages of the implied volatility quotes plotted against the fixed moneyness levels (strike prices in percentages of the spot level). Different panels denote options at different maturities. The 12 lines in each panel represent the 12 equity indexes listed in Table I.

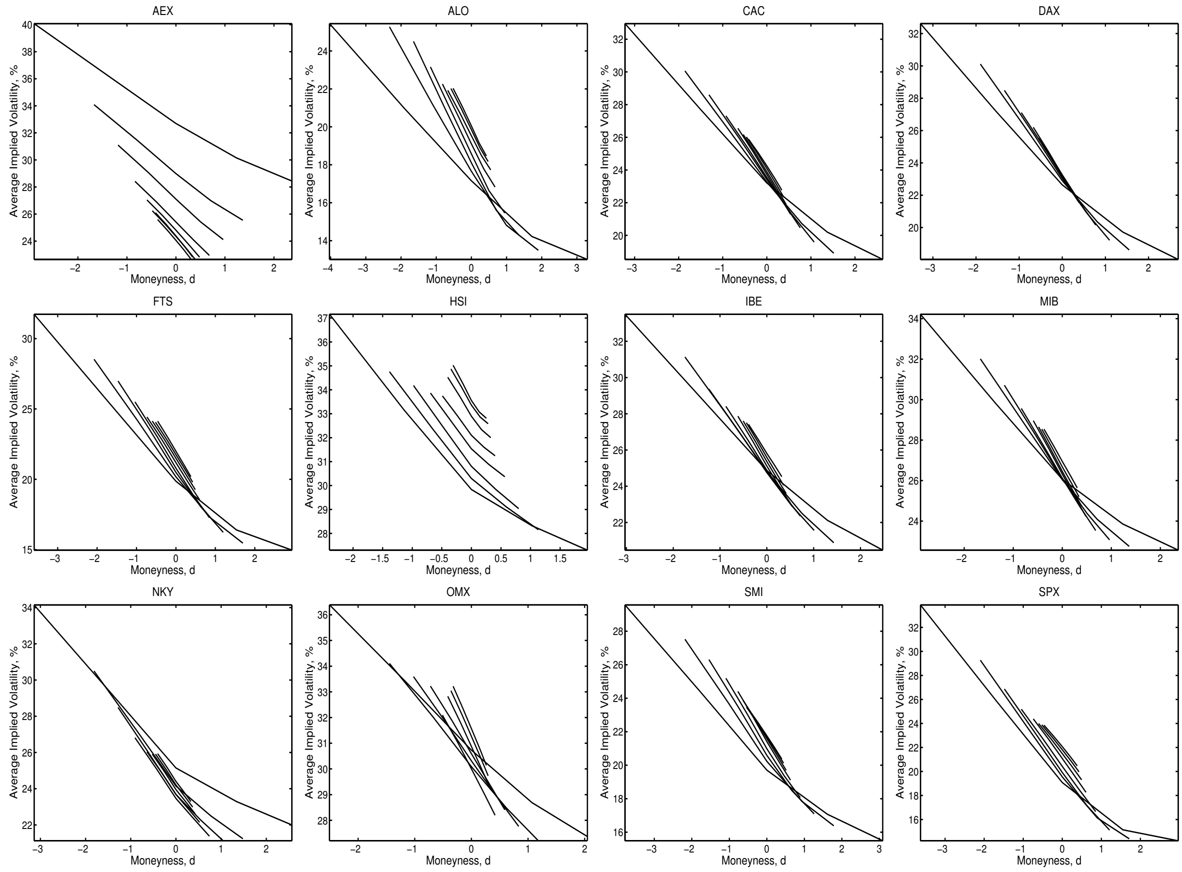

Figure 2. The universal steepening of the implied volatility smirk with increasing maturity. Lines denote the sample average of the implied volatility quotes, plotted against a standard measure of moneyness $d = \ln(K/S)/\sigma\sqrt{\tau}$, where K, S, and τ denote the strike price, the spot index level, and the time-to-maturity in years, respectively. The term σ represents a mean volatility level for each equity index, proxyed by the sample average of the implied volatility quotes underlying each equity index. For each equity index, we plot the implied volatility smirks at the eight different maturities in the same panel. The maturities for each line are one month, three months, six months, one year, two years, three years, four years, and five years, respectively. The length of the line shrinks with increasing maturity, with the longest line representing the shortest maturity (one month). The 12 panels denote the 12 equity indexes.

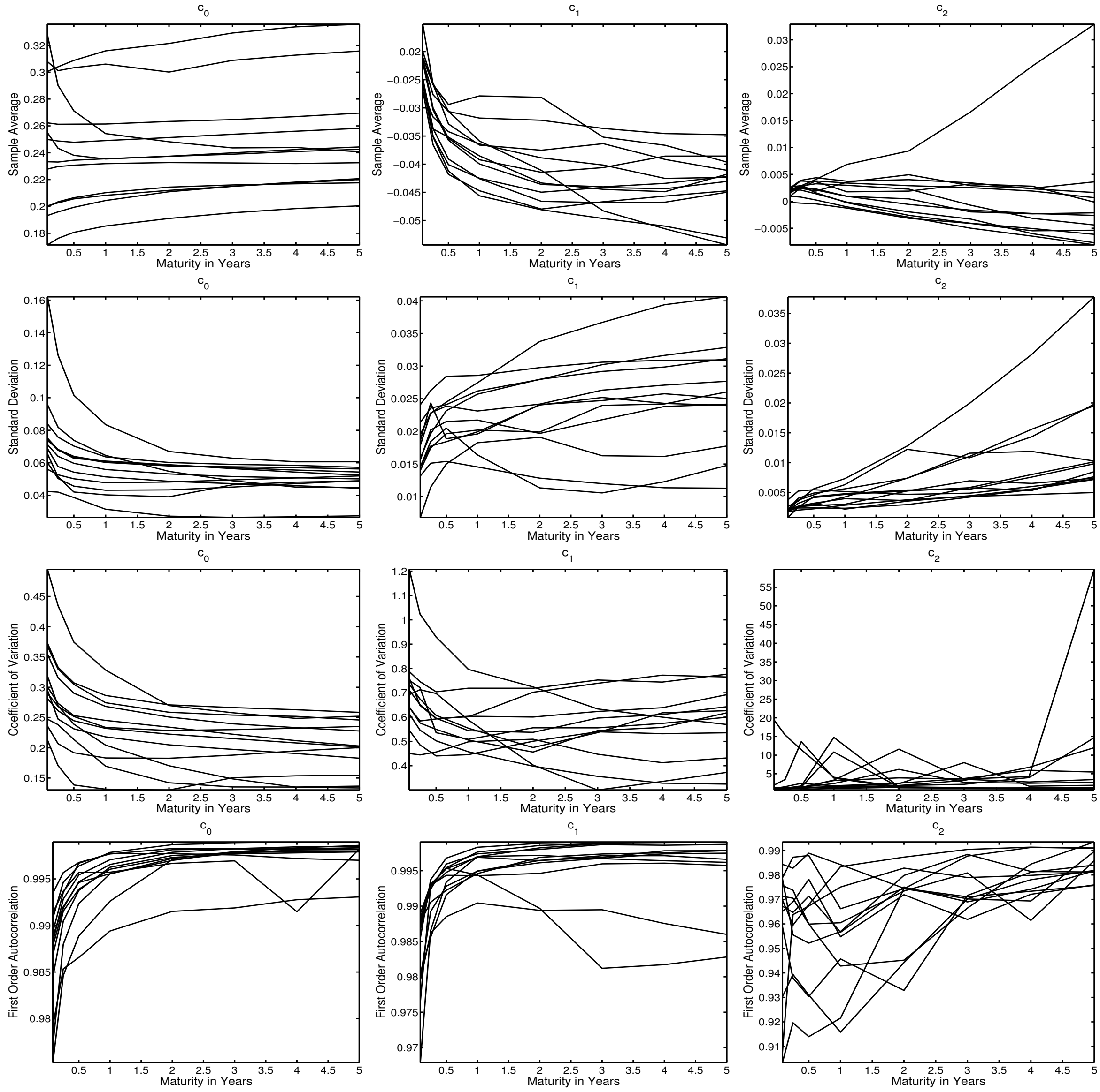

Figure 3. The time series property of the implied volatility smirks. Lines denote the sample mean (first row), the standard deviation (second row), the coefficient of variation (third row), and the first-order autocorrelation (last row) of the daily coefficients estimates, $[c_0, c_1, c_2]$ from a second polynomial fit of the implied volatility smirk at each maturity and date,

$$IV(d;t,\tau) = c_0 + c_1 d + c_2 d^2.$$

The properties are computed based on the full daily sample for each index, with the starting dates for each index given in Table I and ending date uniform on August 20th, 2002. Each line in each panel represents one equity index.

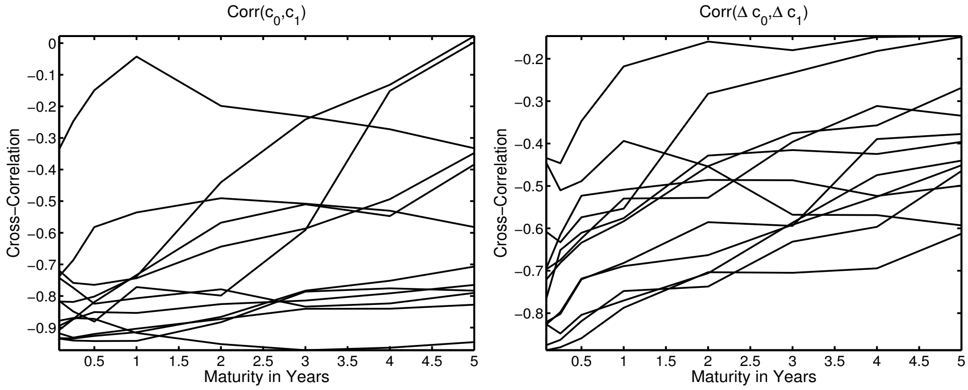

Figure 4. Cross-correlations between the volatility level and smirk slope. Lines denote the cross-correlation estimates between the volatility level and the volatility smirk slope. The left panel measures the correlation based on daily estimates, the right panel measures the correlation based on daily changes of the estimates.

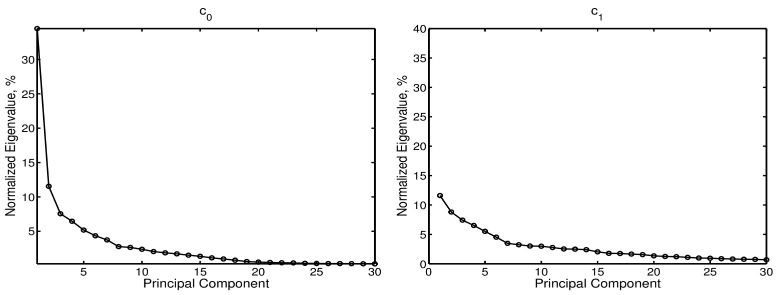

Figure 5. Percentage variance explained by each principal component. The plot reports the percentage explained variation of each of the first 30 principal components on the implied volatility level c_0 (the left panel) and the implied volatility slope c_1 (the right panel). The sample starts on April 28th, 1995 and ends on August 20th, 2002, with 1,847 daily observations. We compute the percentage variance based on the eigenvalues of the covariance matrix of the c_0 and c_1 series, respectively.

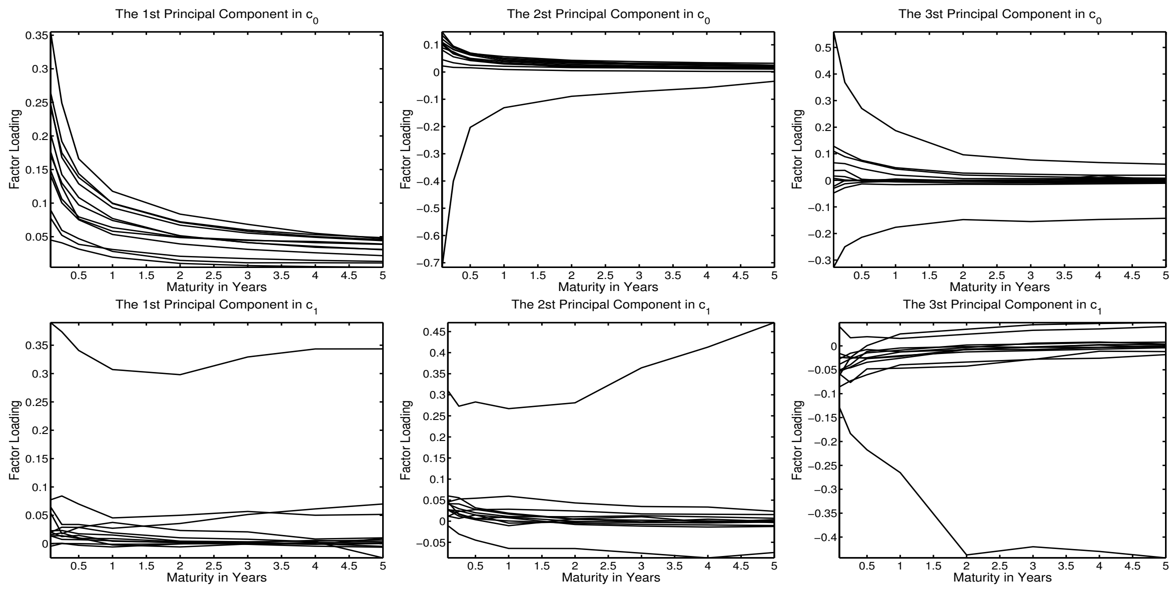

Figure 6. Factor loading. Lines represent the value of the eigenvector of the covariance matrix, corresponding to the first three principal components (from left to right). We perform principal component analysis on the implied volatility level estimates c_0 (the first row) and the implied volatility slope estimates c_1 (the second row), separately. The sample starts on April 28th, 1995 and ends on August 20th, 2002, with 1,847 daily observations. Each line represents one equity index.

Table I
Index List

Entries report the list of stocks that are used for the empirical analysis in this paper.

Symbol	Name	Country	Starting Date
AEX	Amsterdam Exchanges Index	Holland	1994/08/03
ALO	The Australian All Ordinaries Index	Australia	1995/03/16
CAC	The CAC-40 Index	France	1994/08/09
DAX	The German Stock Index	Germany	1994/08/02
FTS	The FTSE 100 Index	UK	1994/08/02
HSI	The Hang Seng Index	Hong Kong	1995/03/16
IBE	The IBEX 35 Index	Spain	1994/08/02
MIB	The Italian MIB Index	Italy	1995/04/28
NKY	The Nikkei-225 Stock Average	Japan	1995/03/30
OMX	The Swedish Market Index	Sweden	1994/08/02
SMI	The Swiss Market Index	Switzerland	1994/08/02
SPX	The S&P 500 Index	US	1994/08/02