Derivative securities are financial instruments whose payoffs are derived from those of another financial security, which is often referred to as the “underlying security.”

- A call option on IBM stock: IBM stock is the underlying security. The call option is the derivative.

Examples: Forwards, futures, swaps, options ... on some financial security

Cash markets or spot markets for primary securities

- The sale is made, the payment is remitted, and the good or security is delivered immediately or shortly thereafter.

Examples of primary securities: Stocks, bonds.
Derivatives Markets

- Exchange-traded instruments (Listed products)
  - Exchange traded securities are generally standardized in terms of maturity, underlying notional, settlement procedures ...
  - By the commitment of some market participants to act as market-maker, exchange traded securities are usually very liquid.
    - Market makers are particularly needed in illiquid markets.
  - Many exchange traded derivatives require ”margining” to limit counterparty risk.
  - On some exchanges, the counterparty is the exchange itself yielding the advantage of anonymity.
Derivatives Markets

- Over-the-counter market (OTC)
  - OTC securities are not listed or traded on an organized exchange.
  - An OTC contract is a private transaction between two parties (counterparty risk).
  - A typical deal in the OTC market is conducted through a telephone or other means of private communication.
  - The terms of an OTC contract are usually negotiated on the basis of an ISDA master agreement (International Swaps and Derivatives Association).
Derivatives Products

- Forwards (OTC)
- Futures (exchange listed)
- Swaps (OTC)
- Options (both OTC and exchange listed)
The securities have direct claims to future cash flows.

Valuation is based on forecasts of future cash flows and risk:
- DCF (Discounted Cash Flow Method): Discount forecasted future cash flow with a discount rate that is commensurate with the forecasted risk.

Investment: Buy if market price is lower than model value; sell otherwise.

Both valuation and investment depend crucially on forecasts of future cash flows (growth rates) and risks (beta, credit risk).
Compare: Derivative securities

- Payoffs are linked directly to the price of the *underlying security*.
  - Assumption: The underlying security can be easily bought or sold.

- Valuation is mostly based on replication/hedging arguments.
  - Find a portfolio that includes the underlying security, and possibly other related derivatives, to replicate the payoff of the target derivative security, or to hedge away the risk in the derivative payoff.
  - Since the hedged portfolio is riskfree, the payoff of the portfolio can be discounted by the riskfree rate.
  - Models of this type are called “no-arbitrage” models.

- Key: No forecasts are involved. Valuation is based on cross-sectional comparison.
  - It is not about whether the underlying security price will go up or down (given growth rate or risk forecasts), but about the relative pricing relation between the underlying and the derivatives under all possible scenarios.
Arbitrage in a Micky Mouse Model

- The current prices of asset 1 and asset 2 are 95 and 43, respectively.

- Tomorrow, one of two states will come true
  - A good state where the prices go up or
  - A bad state where the prices go down

\[
\begin{align*}
\text{Asset1} & = 95 \\
\text{Asset2} & = 43 \\
\text{Asset1} & = 100 \\
\text{Asset2} & = 50 \\
\text{Asset1} & = 80 \\
\text{Asset2} & = 40
\end{align*}
\]

Do you see any possibility to make risk-free money out of this situation?
DCF versus No-arbitrage pricing in the Micky Mouse Model

- DCF: Both assets could be over-valued or under-valued, depending on our estimates/forecasts of the probability of the good/bad states, and the discount rate.

- No-arbitrage model: The payoff of asset 1 is twice as much as the payoff of asset 2 in *all states*, then the price of asset 1 should be twice as much as the price of asset 2.
  - The price of asset 1 is too high relative to the price of asset 2.
  - The price of asset 2 is too low relative to the price of asset 1.
  - I do not care whether both prices are too high or low given forecasted cash flows.
  - Sell asset 1 and buy asset 2, you are guaranteed to make money — arbitrage.
  - Selling asset 1 alone or buying asset 2 alone is not enough.

- Again: DCF focuses on time-series forecasts (of future). No-arbitrage model focuses on cross-sectional comparison (no forecasts)!
Keys to understand primary versus derivative securities

- **Primary securities:**
  - Understand the major determinants of the future cash flows and risks

- **Derivative securities:**
  - Understand the **payoffs as a function of the underlying security price**
    - What’s the payoff of (a portfolio of) common derivative contracts?
  - Understand the replication strategy of different payoff structures:
    - How to replicate a certain desired payoff with the underlying security and/or other derivatives?
  - When replication is difficult, what assumptions do you need to make to make it possible? What’s the potential cost/harm in case your assumption is wrong?
A forward contract is an OTC agreement between two parties to exchange an underlying tradeable asset for an agreed upon price (the forward price) at a given point in time in the future (the expiry date).

Example: On June 3, 2003, Party A signs a forward contract with Party B to sell 1 million British pound (GBP) at 1.61 USD per 1 GBP six month later.

- Today (June 3, 2003), sign a contract, shake hands. No money changes hands.
- December 6, 2003 (the expiry date), Party A pays 1 million GBP to Party B, and receives 1.61 million USD from Party B in return.
- Currently (June 3), the spot price for the pound (the spot exchange rate) is 1.6285. Six month later (December 3), the exchange rate can be anything (unknown).
- 1.61 is the forward price.

When thinking of exchange rate, first identify the denominated currency (dollar in this case), and then treat the other side (pound) as a standard financial security, with a price in dollar.
Foreign exchange quotes for GBPUSD June 3, 2003

<table>
<thead>
<tr>
<th>Maturity</th>
<th>bid</th>
<th>offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>spot</td>
<td>1.6281</td>
<td>1.6285</td>
</tr>
<tr>
<td>1-month forward</td>
<td>1.6248</td>
<td>1.6253</td>
</tr>
<tr>
<td>3-month forward</td>
<td>1.6187</td>
<td>1.6192</td>
</tr>
<tr>
<td>6-month forward</td>
<td>1.6094</td>
<td>1.6100</td>
</tr>
</tbody>
</table>

- The forward prices are different at different maturities.
  - Maturity or time-to-maturity refers to the length of time between now and expiry date (1m, 2m, 3m etc).
  - Expiry (date) refers to the date on which the contract expires.
  - Notation: Forward price $F(t, T)$: $t$: today, $T$: expiry, $\tau = T - t$: time to maturity.
  - The spot price $S(t) = F(t, t)$. [or $S_t, F_t(T)$]

- Forward contracts are the most popular in currency and interest rates.
The forward price for a contract is the delivery price ($K$) that would be applicable to the contract if were negotiated today. It is the delivery price that would make the contract worth exactly zero.

Example: Party A agrees to sell to Party B 1 million GBP at the price of 1.3USD per GBP six month later, but with an upfront payment of 0.3 million USD from B to A.

1.3 is NOT the forward price. Why?

If today’s forward price is 1.61, what’s the value of the forward contract with a delivery price ($K$) of 1.3?

The party that has agreed to buy has what is termed a long position. The party that has agreed to sell has what is termed a short position.

In the previous example, Party A entered a short position and Party B entered a long position on GBP.

But since it is on exchange rates, you can also say: Party A entered a long position and Party B entered a short position on USD.
Terminal payoffs from forward investments

- By signing a forward contract, one can lock in a price \textit{ex ante} for buying or selling a security.

- At expiry, whether one makes or loses money from exercising the contract, i.e., one's \textit{payoff from the contract}, depends on the spot price at expiry.

- In the previous example, Party A agrees to sell 1 million pound at $1.61 per GBP at expiry. If the spot price is $1.31 at expiry, what’s the \textit{payoff} or party A?
  - On Dec 3, Party A can buy 1 million pound from the market at the spot price of $1.31 and sell it to Party B per forward contract agreement at $1.61.
  - The net payoff at expiry is the difference between the \textit{strike price} ($K = 1.61$) and the spot price ($S_T = 1.31$), multiplied by the notional (1 million). Hence, 0.3 million.

- If the spot rate is $1.71 on Dec 3, what will be the payoff for Party A? What’s the payoff for Party B?

- How does \textit{payoff} differ from \textit{P&L}?
Terminal payoffs from forward investments

\[(K = 1.61)\]

**long forward: \((S_T - K)\)**

**short forward: \((K - S_T)\)**

- **Counterparty risk:** There is a small possibility that either side can default on the contract. That’s why forward contracts are mainly between big institutions.
  - One of the motivations of the current regulation movement to central clearing is to reduce counterparty risk.

**How to calculate returns on forward investments?**
1. If you are long a share of a stock today \((t)\), what does the payoff function of the stock look like at time \(T\)?

   1. The stock does not pay dividend.
   2. The stock pays dividends that have a present value of \(D_t\).

2. What does the time-\(T\) payoff look like if you are short of a share of the stock at time \(t\)?

3. If you are long (short) 1 million GBP today, what’s your aggregate dollar payoff at time \(T\)?

4. If you are long (short) a \(K\) dollar par zero-coupon bond with an interest rate of \(r\) at time \(t\), how much do you pay (receive) today? How much do you receive (pay) at expiry \(T\)?
Payoff from cash markets: Answers

1. If you long a share of a stock today \((t)\), the time-\(t\) payoff \((\Pi_T)\) is
   - \(S_T\) if the stock does not pay dividend.
   - \(S_T + D_t e^{r(T-t)}\) if the stock pays dividends during the time period \([t, T]\) that has a present value of \(D_t\). In this case, \(D_t e^{r(T-t)}\) represents the value of the dividends at time \(T\).

2. The payoff of a short position is just the negative of the payoff from the long position: \(-S_T\) without dividend and \(-S_T - D_t e^{r(T-t)}\) with dividend.
   - If you borrow stock (chicken) from somebody, you need to return both the stock and the dividends (eggs) you receive in between.

3. If you are long 1 million GBP today, your aggregate dollar payoff at time \(T\) is the selling price \(S_T\) plus the pound interest you make during the time period \([t, T]\): \(S_T e^{rGBP(T-t)}\) million.

4. The zero bond price is the present value of \(K\): \(K e^{-r(T-t)}\). The payoff is \(K\) for long position and \(-K\) for short position.

Plot these payoffs.
Futures versus Forwards

Futures contracts are similar to forwards, but

- Buyer and seller negotiate *indirectly*, through the exchange.
- Default risk is borne by the exchange clearinghouse
- Positions can be easily reversed at any time before expiration
- Value is marked to market daily.
- Standardization: quality; quantity; Time.
  - The short position has often different *delivery options*; good because it reduces the risk of squeezes, bad ... because the contract is more difficult to price (need to price the “cheapest-to-deliver”).

The different execution details also lead to pricing differences, e.g., effect of marking to market on interest calculation.
Futures versus Spot

- Easier to go short: with futures it is equally easy to go short or long.
  A short seller using the spot market must wait for an uptick before initiating a position (the rule is changing...).

- Lower transaction cost.
  - Fund managers who want to reduce or increase market exposure, usually do it by selling the equivalent amount of stock index futures rather than selling stocks.
  - Underwriters of corporate bond issues bear some risk because market interest rates can change the value of the bonds while they remain in inventory prior to final sale: Futures can be used to hedge market interest movements.
  - Fixed income portfolio managers use futures to make duration adjustments without actually buying and selling the bonds.
Futures on what?

- Just about anything. “If you can say it in polite company, there is probably a market for it,” advertises the CME.

- For example, the CME trades futures on agricultural commodities, foreign currencies, interest rates, and stock market indices, including
  - **Agricultural commodities:** Live Cattle, Feeder Cattle, Live Hogs, Pork Bellies, Broiler Chickens, Random-Length Lumber.
  - **Foreign currencies:** Euro, British pound, Canadian dollar, Japanese yen, Swiss franc, Australian dollar, ...
  - **Interest rates:** Eurodollar, Euromark, 90-Day Treasury bill, One-Year Treasury bill, One-Month LIBOR
  - **Stock indices:** S&P 500 Index, S&P MidCap 400 Index, Nikkei 225 Index, Major Market Index, FT-SE 100 Share Index, Russell 2000 Index
How do we determine forward/futures prices?

*Is there an arbitrage opportunity?*

- The spot price of gold is $300.
- The 1-year forward price of gold is $340.
- The 1-year USD interest rate is 5% per annum, continuously compounding.

**Apply the principle of arbitrage:**

- The key idea underlying a forward contract is to lock in a price for a security.
- Another way to lock in a price is to *buy now and carry the security* to the future.
- Since the two strategies have the same effect, they should generate the same P&L. Otherwise, short the expensive strategy and long the cheap strategy.
- The expensive/cheap concept is relative to the two contracts only. Maybe both prices are too high or too low, compared to the fundamental value ...
Pricing forward contracts via replication

Since signing a forward contract is equivalent (in effect) to buying the security and carry it to maturity.

The forward price should equal to the cost of buying the security and carrying it over to maturity:

\[ F(t, T) = S(t) + \text{cost of carry} - \text{benefits of carry}. \]

Apply the principle of arbitrage: Buy low, sell high.

- The 1-year later (at expiry) cost of signing the forward contract now for gold is $340.
- The cost of buying the gold now at the spot ($300) and carrying it over to maturity (interest rate cost because we spend the money now instead of one year later) is:

\[ S_t e^{r(T-t)} = 300e^{0.05 \times 1} = 315.38. \]

(The future value of the money spent today)

- Arbitrage: Buy gold is cheaper than signing the contract, so buy gold today and short the forward contract.
Carrying costs

- **Interest rate cost:** If we buy today instead of at expiry, we endure interest rate cost — in principle, we can save the money in the bank today and earn interests if we can buy it later.
  
  - This amounts to calculating the future value of today’s cash at the current interest rate level.
  - If 5% is the annual compounding rate, the future value of the money spent today becomes, \( S_t(1 + r)^1 = 300 \times (1 + .05) = 315 \).

- **Storage cost:** We assume zero storage cost for gold, but it could be positive...
  
  - Think of the forward price of live hogs, chicken, ...
  - Think of the forward price of electricity, or weather ...
Carrying benefits

- **Interest rate benefit**: If you buy pound (GBP) using dollar today instead of later, it costs you interest on dollar, but you can save the pound in the bank and make interest on pound. In this case, what matters is the interest rate difference:

  \[ F(t, T)[GBPUSD] = S_t e^{(r^{USD} - r^{GBP})(T-t)} \]

- In discrete (say annual) compounding, you have something like:

  \[ F(t, T)[GBPUSD] = S_t \frac{(1 + r^{USD})^{(T-t)}}{(1 + r^{GBP})^{(T-t)}} \]

- **Dividend benefit**: similar to interests on pound
  - Let \( q \) be the continuously compounded dividend yield on a stock, its forward price becomes, \( F(t, T) = S_t e^{(r-q)(T-t)} \).
  - The effect of discrete dividends: \( F(t, T) = S_t e^{r(T-t)} - \text{Time-}T \text{ Value of all dividends received between time } t \text{ and } T \)
  - Also think of piglets, eggs, ...
Another example of arbitrage

Is there an arbitrage opportunity?

- The spot price of gold is $300.
- The 1-year forward price of gold is $300.
- The 1-year USD interest rate is 5% per annum, continuously compounding.
Another example of arbitrage

Is there an arbitrage opportunity?

- The spot price of oil is $19
- The quoted 1-year futures price of oil is $25
- The 1-year USD interest rate is 5%, continuously compounding.
- The annualized storage cost of oil is 2%, continuously compounding.
Another example of arbitrage

Is there an arbitrage opportunity?

- The spot price of oil is $19
- The quoted 1-year futures price of oil is $16
- The 1-year USD interest rate is 5%, continuously compounding.
- The annualized storage cost of oil is 2%, continuously compounding.

Think of an investor who has oil at storage to begin with.
Another example of arbitrage?

**Is there an arbitrage opportunity?**

- The spot price of electricity is $100 (per some unit...)
- The quoted 3-month futures price on electricity is $110
- The 1-year USD interest rate is 5%, continuously compounding.
- Electricity cannot be effectively stored

**How about the case where the storage cost is enormously high?**
The cleanest pricing relation is on currencies:

\[ F(t, T) = S_t e^{(r_d - r_f)(T - t)}. \]

Taking natural logs on both sides, we have the covered interest rate parity:

\[ f_{t, T} - s_t = (r_d - r_f)(T - t). \]

The log difference between forward and spot exchange rate equals the interest rate difference.

Notation: \((f, s)\) are natural logs of \((F, S)\): \(s = \ln S, f = \ln F\).
Understand the general idea of derivatives (products, markets).

Understand the general idea of arbitrage
Can execute one when see one.

The characteristics of forwards/futures

Payoff under different scenarios, mathematical representation:
\((S_T - K)\) for long, \((K - S_T)\) for short
Understand graphical representation.

Pricing: \(F(t, T) = S_t + \text{cost of carry}\). Know how to calculate carry cost/benefit under continuously/discrete compounding.

Combine cash and forward market for arbitrage trading