Mechanics of Options Markets

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Options Markets
Definitions and terminologies

- An option gives the option holder the right/option, *but no obligation*, to buy or sell a security to the option writer/seller
  - for a pre-specified price (the strike price, \( K \)) (“delivery price” in forward contracts)
  - at (or up to) a given time in the future (the expiry date)

- An option has positive value.

*Comparison: a forward contract can have zero value at inception by setting the strike (delivery) price \( K \) to the forward price \( F_{t,T} \).*

- Option types
  - A call option gives the holder the right to buy a security. The payoff is \((S_T - K)^+\) or \(\max(0, S_T - K)\), when exercised at maturity.
  - A put option gives the holder the right to sell a security. The payoff is \((K - S_T)^+\), \(\max(0, K - S_T)\), when exercised at maturity.
  - American options can be exercised at any time prior to expiry.
  - European options can only be exercised at the expiry.
More terminologies

- **Moneyness**: the strike $K$ relative to the spot $S_t$, or more appropriately for European options, the *forward* level $F_{t,T}$
  - An option is said to be *in-the-money* spot if the option has positive value if exercised right now:
    - $S_t > K$ for call options and $S_t < K$ for put options.
    - The option has positive intrinsic value when in the money. The intrinsic value (in spot) is $(S_t - K)^+$ for call, $(K - S_t)^+$ for put.
  - An option is said to be *in-the-money forward* if the corresponding forward has positive value:
    - $F_{t,T} > K$ for call and $F_{t,T} < K$ for put
    - Intrinsic value in forward is: $e^{-rT}(F_{t,T} - K)^+$ for call, $e^{-rT}(K - F_{t,T})^+$ for put.
  - An option is said to be *out-of-the-money* when it has zero intrinsic value.
    - $S_t < K$ for call options and $S_t > K$ for put options.
    - Out-of-the-money forward: $F_{t,T} < K$ for call and $F_{t,T} > K$ for put.
  - An option is said to be *at-the-money* spot (or forward) when the strike is equal to the spot (or forward).
More terminologies

- The value of an option is determined by:
  - the current spot (or forward) price \( S_t \) or \( F_t \),
  - the strike price \( K \),
  - the time to maturity \( \tau = T - t \),
  - the option type (Call or put, American or European), and
  - the dynamics of the underlying security (e.g., volatility).

- We can decompose the value of each option into two components:
  \[
  \text{option value} = \text{intrinsic value} + \text{time value}. 
  \]

  - \textit{Intrinsic value} is how much one can lock in either by exercising right away (for American) or via a forward contract (for European).
  - \textit{Time value} is determined by time to maturity of the option and the dynamics of the underlying security.

- Out-of-the-money options do not have \textit{intrinsic value}, but only have \textit{time value}.
For European options, the terminal payoff can be written as \((S_T - K)^+\) for calls and \((K - S_T)^+\) for puts at expiry date \(T\).

Since options have positive value, one needs to pay an upfront price (option price) to possess an option.

The P&L from the option investment is the difference between the terminal payoff and the initial price you pay to obtain the option.

Do not confuse the two.

The textbook likes to talk about P&Ls, but I like to talk about payoffs — Different perspectives:

- P&Ls: If I buy/sell an option today, how much money I can make under different scenarios? What’s my return?
- Payoffs: If I desire a certain payoff structure in the future, what types of option positions I need to be in?
Consider a European call option on a stock index. The current index level (spot $S_t$) is 100. The option has a strike ($K$) of $90$ and a time to maturity ($T - t$) of 1 year. The option has a current value ($c_t$) of $14$.

- Is this option in-the-money or out-of-the-money (wrt to spot)?
- What’s intrinsic value for this option? What’s its time value?
- If you hold this option, what’s your terminal payoff?
  - What’s your payoff and P&L if the index level reaches 100, 90, or 80 at the expiry date $T$?
- If you write this option and have sold it to the exchange, what does your terminal payoff look like?
  - What’s your payoff and P&L if the index level reaches 100, 90, or 80 at the expiry date $T$?
Payoffs and P&Ls from long/short a call option

\[(S_t = 100, K = 90, c_t = 14)\]

Long a call pays off, \((S_T - K)^+\), bets on index price going up.
Shorting a call bets on index price going down.
Another example: Put option on an exchange rate

Consider a **European put option** on the dollar price of pound (GBPUSD). The current spot exchange rate \((S_t)\) is $1.6285 per pound. The option has a strike \((K)\) of $1.61 and a time to maturity \((T - t)\) of 1 year. The 1-year forward price \((F_{t,T})\) is $1.61. The dollar continuously compounding interest rate at 1-year maturity \((r_d)\) is 5%. The option \((p_t)\) is priced at $0.0489.

- From the above information, can you infer the continuously compounding interest rate at 1-year maturity on pound \((r_f)\)?
- Is this option in-the-money or out-of-the-money wrt to spot? What’s the moneyness in terms of forward?
- In terms of forward, what’s intrinsic value for this option? What’s its time value?
- If you hold this option, what’s your terminal payoff, if the dollar price of pound reaches 1.41, 1.61, or 1.81 at the expiry date \(T\)?
Another example: Put option on an exchange rate

- Review the forward pricing formula: \( F_{t, T} = S_t e^{(r_d-r_f)(T-t)} \).
- \( r_f = r_d - \frac{1}{T-t} \ln(F_{t, T}/S_t) = 0.05 - \ln(1.61/1.6285)/1 = 6.14\% \).
- Recall **covered interest rate parity**: Annualized forward return \( \left( \frac{1}{T-t} \ln(F_{t, T}/S_t) \right) \) on exchange rates equals interest rate differential \( (r_d - r_f) \) between the two currencies.

- Long a put option pays off, \( (K - S_T)^+ \), and bets on the underlying currency (pound) depreciates.

- Shorting a put option bets on pound appreciates.

- *How does it differ from betting using forwards?*
Payoffs and P&Ls from long/short a put option

\((S_t = 1.6285, F_{t,T} = 1.61, K = 1.61, p_t = 0.0489)\)
What derivative positions generate the following payoff?

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Assets underlying exchanged-traded options

- Stocks
- Stock indices
- Index return variance (new)
- Exchange rate
- Futures
Specification of exchange-traded options

- Expiration date \((T)\)
- Strike price \((K)\)
- European or American
- Call or Put (option class)

OTC options (such as OTC options on currencies) are quoted differently.
Most exchanges use market makers to facilitate options trading.

A market maker is required to provide bid and ask quotes

- with the bid-ask spread within a maximum limit,
- with the size no less than a minimum requirement,
- at no less than a certain percentage of time (lower limit)
- on no less than a certain fraction of securities that they cover.

The benefit of market making is the bid-ask spread;

The risk is market movements.

- The risk and cost of options market making is relatively large.
- The bid-ask is wide (stock options). The tick size is 10 cents on options with prices higher than $3. It is 5 cents otherwise.
Options market making

- Since there can be hundreds of options underlying one stock, when the stock price moves, quotes on the hundreds of options must be updated simultaneously.
  - Quote message volume is dramatically larger than trade message volume.
  - The risk exposure is large compared to the benefit.
    - When a customer who has private information on the underlying stock (say, going up), the customer can buy all the call options and sell all the put options underlying one stock.
    - The market maker’s risk exposure is the sum of all the quote sizes he honors on each contract.
    - Market makers hedge their risk exposures by buying/selling stocks according to their option inventories.
  - Market makers nowadays all have automated systems to update their quotes, and calculate their optimal hedging ratios.
  - Options market makers are no longer individual persons, but are well-capitalized firms.
Margins are required when options are sold/written.

When a naked option is written the margin is the greater of:

- A total of 100% of the proceeds of the sale plus 20% of the underlying share price less the amount (if any) by which the option is out of the money
- A total of 100% of the proceeds of the sale plus 10% of the underlying share price.

For other trading strategies there are special rules.
Dividends and stock splits

- Suppose you own $N$ option contracts with a strike price of $K$:
  - No adjustments are made to the option terms for cash dividends.
  - When there is an $n$-for-$m$ stock split,
    - The strike price is reduced to $mK/n$.
    - The number of options is increased to $nN/m$.
  - Stock dividends are handled in a manner similar to stock splits.

- Example: Consider a call option to buy 100 shares for $20 per share.
  - How should the option contract terms be adjusted:
    - for a 2-for-1 stock split?
    - for a 5% stock dividend?
Other option-type products

- **Warrants**: options that are issued by a corporation
  - When call warrants are issued by a corporation on its own stock, exercise will lead to new stock being issued.

- **Executive stock options**: a form of remuneration issued by a company to its executives
  - usually at the money when issued.
  - When exercised, the company issues more stock and sells it to the option holder for the strike price.
  - They become vested after a period of time (1 to 4 years).
  - They cannot be sold.
Other option-type products

- Convertible bonds: regular bonds that can be exchanged for equity at certain times in the future according to a predetermined exchange ratio.
  - Very often callable, so that the issuer can force conversion at a time earlier than the holder might otherwise choose.

- Stocks: Can be regarded as call options on firm value.
  - The payoff is the difference between firm value and debt liability, \((\text{Firm Value} - \text{Debt})^+\).
  - When firm value is less than debt value, the firm can apply for bankruptcy.
  - Limited liability guarantees that stock price is always positive.
  - When DCF method does not work well, one can value a stock like an option.
Summary

- Basic terminologies: call, put, American, European, in-the-money, out-of-the-money, intrinsic value, time value...
- Basic mechanisms of options trading: market making, margins, exchanges, stock splits, ...
- Inside-out knowledge on payoff structures of different positions (long/short) in different derivatives (call/put, forward, spot), from three perspectives:
  - Contract definition
  - Algebraic payoff function
  - Graphic representation