Options Trading Strategies

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Options Markets

(Hull chapter: 10)
Objectives

A strategy is a set of options positions to achieve a particular risk/return profile.

For simplicity, we focus on strategies that involve positions in only European options on the same underlying and at the same expiration.

The zero-coupon bond and the underlying forward of the same maturity are always assumed available.

We hope to achieve three objectives:

1. Given a strategy (a list of derivative positions), we can figure out its risk profile, i.e., the payoff of the strategy at expiry under different market conditions (different underlying security price levels).

2. Given a targeted risk profile at a certain maturity (i.e., a certain payoff structure), we can design a strategy using bonds, forwards, and options to achieve this profile.

3. Be familiar with (the risk profile, the objective, and the composition of) the most commonly used, simple option strategies, e.g., straddles, strangles, butterfly spreads, risk reversals, bull/bear spreads.
Put-call conversions

*Plot the payoff function of the following combinations of calls/puts and forwards at the same strike $K$ and maturity $T$.*

1. Long a call, short a forward.
   - Compare the payoff to long a put.

2. Short a call, long a forward.
   - Compare the payoff to short a put.

3. Long a put, long a forward.
   - Compare the payoff to long a call.

4. Short a put, short a forward.
   - Compare the payoff to short a call.

5. Long a call, short a put.
   - Compare the payoff to long a forward.

6. Short a call, long a put.
   - Compare the payoff to short a forward.
The dash and dotted lines are payoffs for the two composition instruments. The solid lines are payoffs of the target.
The linkage between put, call, and forward

• The above conversions reveal the following parity condition in payoffs of put, call, and forward at the same strike and maturity:

\[
\text{Payoff from a call} - \text{Payoff from a forward} = \text{Payoff from a put} \\
\text{Payoff from a put} + \text{Payoff from a forward} = \text{Payoff from a call} \\
\text{Payoff from a call} - \text{Payoff from a put} = \text{Payoff from a forward}
\]

• If the payoff is the same, the present value should be the same, too (\textit{put-call parity}): 

\[
c_t - p_t = e^{-r(T-t)}(F_{t,T} - K).
\]

• At a fixed strike ($K$) and maturity $T$, we only need to know the two prices of the following three: $(c_t, p_t, F_{t,T})$. One of the three contracts is redundant.
In the absence of forward, use spot and bond:

- Can you use a spot and bond to replicate a forward payoff?
- What’s the payoff function of a zero bond?
Can you generate the above payoff structure (solid blue line) using (in addition to cash/bond):

- two calls
- two puts
- a call, a put, and a stock/forward

Who wants this type of payoff structure?
Generating a bull spread

- **Two calls**: Long call at $K_1 = $90, short call at $K_2 = $110, short a bond with $10 par.

- **Two puts**: Long a put at $K_1 = $90, short put at $K_2 = $110, long a bond with $10 par.

- **A call, a put, and a stock/forward**: Long a put at $K_1 = $90, short a call at $K_2 = $110, long a forward at $K = 100$ (or long a stock, short a bond at $100$ par).
Pointers in replicating payoffs

- Each kinky point corresponds to a strike price of an option contract.
- Given put-call party, you can use either a call or a put at each strike point.
- Use bonds for parallel shifts.
- A general procedure using calls, forwards, and bonds
  - Starting from the left side of the payoff graph at $S_T = 0$ and progress to each kinky point sequentially to the right.
  - If the payoff at $S_T = 0$ is $x$ dollars, long a zero-coupon bond with an $x$-dollar par value. [Short if $x$ is negative].
  - If the slope of the payoff at $S_T = 0$ is $s_0$, long $s_0$ shares of a call/forward with a zero strike — A call at zero strike is the same as a forward at zero strike. [Short if $s_0$ is negative.]
  - Go to the next kinky point $K_1$. If the next slope (to the right of $K_1$ is $s_1$, long $(s_1 - S_0)$ shares of call at strike $K_1$. Short when the slope change is negative.
  - Go to the next kinky point $K_2$ with a new slope $s_2$, and long $(s_2 - s_1)$ shares of calls at strike $K_2$. Short when the slope change is negative.
  - Keep going until there are no more slope changes.
Pointers in replicating payoffs, continued

- A general procedure using puts, forwards, and bonds
  - Starting from the right side of the payoff graph at the highest strike under which there is a slope change. Let this strike be $K_1$.
  - If the payoff at $K_1$ is $x$ dollars, long a zero-coupon bond with an $x$-dollar par value. [Short if $x$ is negative].
  - If the slope to the right of $K_1$ is positive at $s_0$, long $s_0$ of a forward at $K_1$. Short the forward if $s_0$ is negative.
  - If the slope to the left of $K_1$ is $s_1$, short $(s_1 - s_0)$ shares of a put at $K_1$. Long if $(s_1 - s_0)$ is negative.
  - Go to the next kinky point $K_2$. If the slope to the left of $K_2$ is $s_2$, short $(s_2 - s_1)$ put with strike $K_2$.
  - Keep going until there are no more slope changes.
Example: Bear spread

- How many (at minimum) options do you need to replicate the bear spread?
- Do the exercise, get familiar with the replication.
- Who wants a bear spread?
Example: Straddle

- How many (at minimum) options do you need to replicate the straddle?
- Do the exercise, get familiar with the replication.
- Who wants long/short a straddle?
Example: Strangle

- How many (at minimum) options do you need to replicate the strangle?
- Do the exercise, get familiar with the replication.
- Who wants long/short a strangle?
Example: Butterfly spread

- How many (at minimum) options do you need to replicate the butterfly spread?
- Do the exercise, get familiar with the replication.
- Who wants long/short a butterfly spread?
Example: Risk Reversal

- How many (at minimum) options do you need to replicate the risk reversal?
- Do the exercise, get familiar with the replication.
- Who wants long/short a risk reversal?
Smooth out the kinks: Can you replicate this?

How many options do you need to replicate this quadratic payoff?
- You need a continuum of options to replicate this payoff.
- The weight on each strike $K$ is $2dK$.

Who wants long/short this payoff?
- The variance of the stock price is $\mathbb{E}[(S_T - F_{t,T})^2]$.
- **Variance swap** contracts on major stock indexes are actively traded.
Replicate any terminal payoff with options and forwards

\[ f(S_T) = f(F_t) + f'(F_t)(S_T - F_t) + \left\{ \begin{array}{l}
\int_0^{F_t} f''(K)(K - S_T)^+ dK \\
\int_{F_t}^{\infty} f''(K)(S_T - K)^+ dK
\end{array} \right\} \]

What does this formula tell you?

▶ With bonds, forwards, and European options, we can replicate any terminal payoff structures.
▶ More exotic options deal with path dependence, correlations, etc.

You do not need to memorize the formula.

Replication applications

- Replicate the return variance swap using options and futures.

- Based on the replication idea, think of ways to summarizing the information in the options market.
  - Information about the directional movement of the underlying.
  - Information about return variance.
  - Information about large movements of a certain direction.
  - Information about large movements of either direction.


- Caveat: Far out-of-the-money options may not be actively traded. Quotes may not be reliable.
  Example: ATM volatility versus synthetic variance swap.