Options Trading Strategies

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Options Markets

Objectives

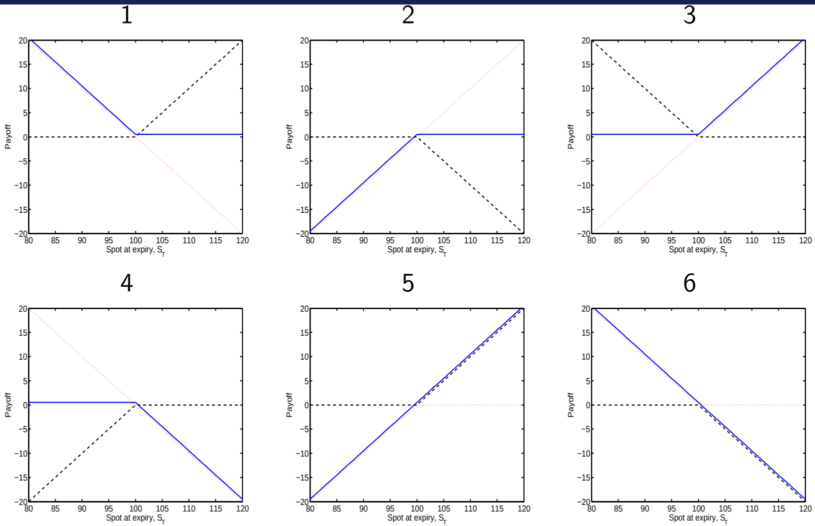
- A strategy is a set of options positions to achieve a particular risk/return profile.
- For simplicity, we focus on strategies that involve positions in only European options on the same underlying and at the same expiration.
- The zero-coupon bond and the underlying forward of the same maturity are always assumed available.
- We hope to achieve three objectives:
  1. Given a strategy (a list of derivative positions), we can figure out its risk profile, i.e., the payoff of the strategy at expiry under different market conditions (different underlying security price levels).
  2. Given a targeted risk profile at a certain maturity (i.e., a certain payoff structure), we can design a strategy using bonds, forwards, and options to achieve this profile.
  3. Be familiar with (the risk profile, the objective, and the composition of) the most commonly used, simple option strategies, e.g., straddles, strangles, butterfly spreads, risk reversals, bull/bear spreads.

Put-call conversions

Plot the payoff function of the following combinations of calls/puts and forwards at the same strike K and maturity T.

1. Long a call, short a forward.
   - Compare the payoff to long a put.
2. Short a call, long a forward.
   - Compare the payoff to short a put.
3. Long a put, long a forward.
   - Compare the payoff to long a call.
4. Short a put, short a forward.
   - Compare the payoff to short a call.
5. Long a call, short a put.
   - Compare the payoff to long a forward.
6. Short a call, long a put.
   - Compare the payoff to short a forward.

Put-call conversions: Payoff comparison ($K = 100$)



The dash and dotted lines are payoffs for the two composition instruments.
The solid lines are payoffs of the target.

The linkage between put, call, and forward

- The above conversions reveal the following parity condition in payoffs of put, call, and forward at the same strike and maturity:

  Payoff from a call − Payoff from a forward = Payoff from a put
  Payoff from a put + Payoff from a forward = Payoff from a call
  Payoff from a call − Payoff from a put = Payoff from a forward

- If the payoff is the same, the present value should be the same, too (*put-call parity*):

$$c_t - p_t = e^{-r(T-t)}(F_{t,T} - K).$$

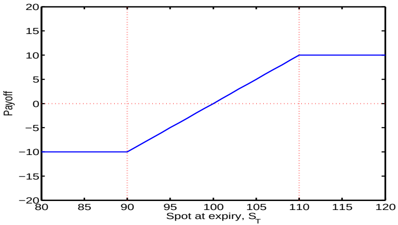
- At a fixed strike (K) and maturity T, we only need to know the two prices of the following three: $(c_t, p_t, F_{t,T})$.
  One of the three contracts is redundant.

Review: Create forward using spot and bond

In the absence of forward, use spot and bond:

- Can you use a spot and bond to replicate a forward payoff?
- What's the payoff function of a zero bond?

Popular payoff I: Bull spread



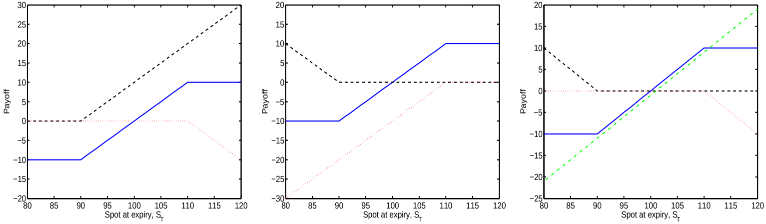
Can you generate the above payoff structure (solid blue line)
using (in addition to cash/bond):

- two calls
- two puts
- a call, a put, and a stock/forward

Who wants this type of payoff structure?

Generating a bull spread

- Two calls: Long call at $K_1 = \$90$, short call at $K_2 = \$110$, short a bond with \$10 par.
- Two puts: Long a put at $K_1 = \$90$, short put at $K_2 = \$110$, long a bond with \$10 par.
- A call, a put, and a stock/forward: Long a put at $K_1 = \$90$, short a call at $K_2 = \$110$, long a forward at $K = 100$ (or long a stock, short a bond at \$100 par).



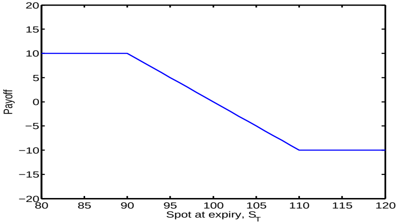
A general procedure to replicate payoffs

- Each kinky point corresponds to a strike price of an option contract.
- Given put-call party, you can use either a call or a put at each strike point.
- Use bonds for parallel shifts, use forwards for overall slope change.
- A general procedure using *calls*:
  - Starting from the left side of the payoff graph at $S_T = 0$ and progress to each kinky point sequentially to the right.
  - If the payoff at $S_T = 0$ is x dollars, long a zero-coupon bond with an x-dollar par value. [Short if x is negative].
  - If the slope of the payoff at $S_T = 0$ is s_0, long s_0 shares of a call/forward with a zero strike — A call at zero strike is the same as a forward at zero strike. [Short if s_0 is negative.]
  - Go to the next kinky point K_1. If the next slope (to the right of K_1 is s_1, long $(s_1 - S_0)$ shares of call at strike K_1. Short when the slope change is negative.
  - Go to the next kinky point K_2 with a new slope s_2, and long $(s_2 - s_1)$ shares of calls at strike K_2. Short when the slope change is negative.
  - Keep going until there are no more slope changes.

A general procedure to replicate payoffs

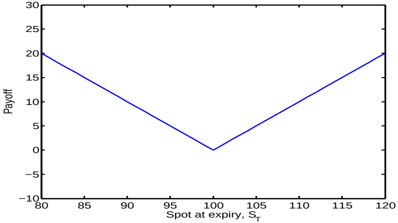
- A general procedure using *puts*:
  - Starting from the right side of the payoff graph at the highest strike under which there is a slope change. Let this strike be K_1.
  - If the payoff at K_1 is x dollars, long a zero-coupon bond with an x-dollar par value. [Short if x is negative].
  - If the slope to the right of K_1 is positive at s_0, long s_0 of a forward at K_1. Short the forward if s_0 is negative.
  - If the slope to the left of K_1 is s_1, short $(s_1 - s_0)$ shares of a put at K_1. Long if $(s_1 - s_0)$ is negative.
  - Go to the next kinky point K_2. If the slope to the left of K_2 is s_2, short $(s_2 - s_1)$ put with strike K_2.
  - Keep going until there are no more slope changes.
- A general procedure using *out-of-the-money options*
  - I'll leave this for the ambitious

Example: Bear spread



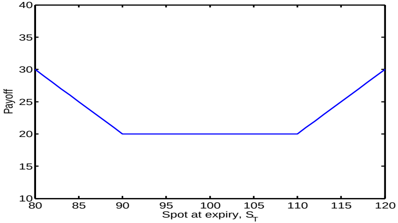
- How many (at minimum) options do you need to replicate the bear spread?
- Do the exercise, get familiar with the replication.
- Who wants a bear spread?

Example: Straddle



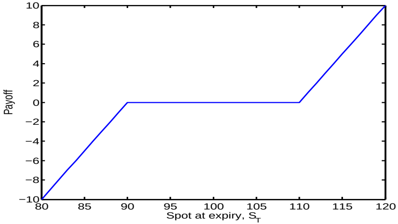
- How many (at minimum) options do you need to replicate the straddle?
- Do the exercise, get familiar with the replication.
- Who wants long/short a straddle?

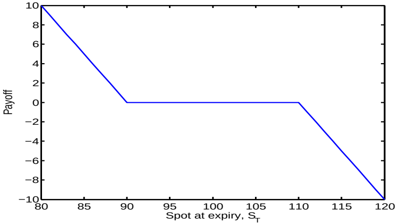
Example: Strangle



- How many (at minimum) options do you need to replicate the strangle?
- Do the exercise, get familiar with the replication.
- Who wants long/short a strangle?

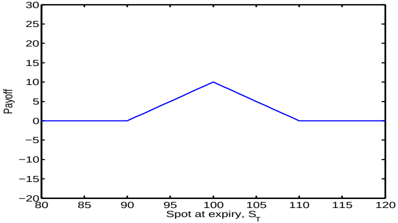
Example: Risk Reversal





- How many (at minimum) options do you need to replicate the risk reversal?
- Do the exercise, get familiar with the replication.
- Who wants long/short a risk reversal?

Example: Butterfly spread



- How many (at minimum) options do you need to replicate the butterfly spread?
- Do the exercise, get familiar with the replication.
- Who wants long/short a butterfly spread?

From butterflies to histograms/probabilities

- Suppose you construct a butterfly with the center strike at \$100, and with the two side strikes at \$99 and \$101. Then, you will get paid \$1 when the stock price reaches \$100 at expiry, and paid nothing if the stock price is either below \$99 or above \$101.
  - The price of the butterfly reflects the "risk-adjusted" probability that the stock price will fall between (99,101), times the present value of one dollar (discount).
- You can construct such butterflies with center strikes at \$80,\$81,$\cdots$, \$119, \$120, ...
- The cost/price of each fly reflects the probability of the stock price falling around the center strike of that fly.
- Thus, if you have options at all strikes, you can construct these butterflies and infer the probabilities of the future stock price reaching each price level.
- Breeden and Litzenberger (1978) for the underlying theory, and many following papers on practical implementation and applications ...

From call/put spreads to up/under odds

- Suppose you construct a call spread by long a call at $K = \$100$ and short a call at $K + \Delta K = \$101$. What is your payoff?
  - Let the strike distance ΔK shrink, and take $1/\Delta K$ shares of the spread, you obtain a synthetic *binary option* that pays off \$ iff the security price is greater than K.
  - The price of this spread represents market risk-adjusted expectation of the security price going above K.
- Example: What's the probability that TSLA price will be above \$400 one month from now? What odds are you willing to offer on TSLA?
- You can also construct probability of going below K via a put spread: Long a put at $K = \$100$ and short a put at $K - \Delta K = \$99$

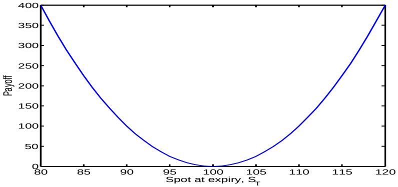
Replicate smooth terminal payoff function

If the payoff function $f(S_T)$ does not have "kinks" but is a smooth (differentiable) function:

$$
\begin{array}{rcll}
f(S_T) & = & f(F_t) & \text{bonds} \\
 & & +f'(F_t)(S_T - F_t) & \text{forwards} \\
 & & +\left\{ \begin{array}{l} \int_0^{F_t} f''(K)(K - S_T)^+ dK \\ \int_{F_t}^{\infty} f''(K)(S_T - K)^+ dK \end{array} \right\} & \text{OTM options}
\end{array}
$$

- With bonds, forwards, and European options, we can replicate any terminal payoff structures, smooth or kinky.
- The above formula is analogous to our "general approach" earlier on kinky payoffs, using out-of-money forward options.
- Reference: Carr and Madan, Optimal positioning in derivative securities, *Quantitative Finance*, 2001, 1, 19–37.

Smooth out the kinks: Can you replicate this?



- How many options do you need to replicate this quadratic payoff?
  - You need a continuum of options to replicate this payoff.
  - The weight on each strike K is $2dK$.
- Who wants long/short this payoff?
  - The variance of the stock price is $\mathbb{E}[(S_T - F_{t,T})^2]$.
- We usually talk about variance not on price changes but on returns.
  - Replicating return variance is harder, but doable ...
  - Variance swap contracts on major stock indexes are actively traded.
- Reference: Carr and Wu, Variance risk premiums, *Review of Financial Studies*, 2009, 22(3), 1131–1141.

VIX— CBOE's Volatility Index

- It is meant to capture the expected annualized volatility of the S&P 500 Index return over the next 30 days.
- It is created as the weighted average price of 30-day S&P 500 Index options across all strikes, with the weighting proportional to $1/K^2$.
- For history and technical details: Carr and Wu, A tale of two indices, *Journal of Derivatives*, 2006, 13(3), 13-29.

DOOM puts as credit insurance

- Assume: A company's stock price will stay about \$5 as long as the company is solvent, but will drop to zero upon bankruptcy.
  - Is this assumption reasonable? Make adjustments to make yourself comfortable.
- Under this assumption, we can create a pure *credit insurance* contract from deep-out-of-money (*DOOM*) American puts on the company's stock:
  - Select any DOOM put option with strike (K) below \$5, and take a position of $1/K$ on this put (P).
  - Or select two DOOM put options with strikes below \$5: $5 > K_2 > K_1$, and take a spread position $1/(K_2 - K_1)$ of $P_2 - P_1$.
  - *Payoff*: receive \$1 if and only if the company defaults before the option expires.
  - *Default probability*: $1/P$ or $(P_2 - P_2)/(K_2 - K_1)$ represents a market estimate of the default probability of this company before expiry.
- Reference: Carr and Wu, A simple robust link between American puts and credit protection, *Review of Financial Studies*, 2011, 24(2), 473–505.
- Application: CBOE's DOOM index...

Summary

- Identify what risk you want to bet on and what risk you want to hedge.
- Replicate your desired payoff structure using vanilla options and forwards.
- Requirements:
  - Given a payoff structure, you can design one replication strategy with options, forwards, and bonds.
  - Given a portfolio of options, forwards, and bonds, you can plot the payoff structure of the portfolio.
  - Given a specific scenario (e.g., $S_T = 110$), you can compute the terminal payoff value of a pre-specified portfolio.