Assume a continuously compounding dollar interest rate of 5% for all maturities, whenever applicable.

1. (15) Under the Black-Scholes model, the stock price $S_t$ follows a geometric Brownian motion under
the risk-neutral measure,

$$dS_t/S_t = (r - q)dt + \sigma dW_t,$$

where $r$ is the continuously compounded interest rate and $q$ is the dividend yield. Assume that the stock
has an annualized dividend yield of 3% per year and an annualized volatility $\sigma = 20\%$. Consider the
log return $\ln(S_T/S_t)$ with a horizon of three months $(T - t = 3/12)$. What is the mean and variance of
the log return? What’s its distribution?

2. (10) Let $W_t$ denote a Brownian motion starting at zero. Compute the mean and volatility of the following
processes:

(a) $5\% + 20\%W_t$ with $t = 2$.

(b) $2\% + 10\%(W_T - W_t)$ with $t = 1$ and $T = 3$.

3. (30) Consider a two-year 25-delta call option on dollar price of pound (pound is the foreign currency).
The current pound interest rate is also 5% (the same as the dollar rate). The current exchange rate is
$2.00$ dollar. The strike of the option is $K = 2.52$, with $N(d_2) = 0.17$.

(a) Compute the value of the call option based on the Black-Scholes formula.

(b) Compute the value of a put option at the same strike and maturity.

(c) Compute the delta of the call option and the put option.

4. (20) Assume that you have an option portfolio with delta 10 and vega 200, and you want to alter your
delta and vega exposure using two liquid contracts. The first contract has a delta of 0.5 and vega of 2.
The second contract has delta of zero and vega of 6.

(a) How many of these two contracts do you need in order to balance your portfolio to delta and vega
neutral?

(b) Suppose you want to achieve delta neutral but are willing to have a vega exposure within $\pm 10$.
What are the minimum number of contracts you need to get your portfolio within your target
exposure range?

5. (15) The current spot price is $100$, consider three call options at 80, 100, and 120, which one has the
highest delta exposure? Which one has the highest vega exposure? How do your answers change if the
three options are put options?

6. (10) Consider a 25-delta put option on a stock. When the stock price goes up by one dollar, approxi-
mately how much will the put option price move? In what direction (up or down)?