1. \( r = 5\% \), \( q = 3\% \), \( \sigma = 20\% \), \( T-t = \frac{T}{12} \)

\[ \log \frac{S_T}{S_t} \sim \phi \left( (r-q) \left( \frac{T}{12} \right) - \frac{1}{2} \sigma^2 \left( \frac{T}{12} \right), \ 0^2 \left( \frac{T}{12} \right) \right) \]

mean: \( (5\%-3\%) \times \frac{3}{12} - \frac{1}{2} \times 0.2^2 \times \frac{3}{12} = 0 \)

variance: \( 0.2^2 \times \frac{3}{12} = 0.01 \)

normal distribution

2. (a) \( 5\% + 20\% \) \( W_t \), \( t=2 \):

mean: \( 5\% \)

volatility: \( \sqrt{(20\%)^2 \times 2} = \sqrt{2} \times 20\% = 28.28\% \)

(b) \( 2\% + 13\% \) \( (W_T - W_t) \), \( t=1 \), \( T=3 \)

mean: \( 2\% \)

volatility: \( 10\% \sqrt{3-1} = 14.14\% \)

3. (a) \( C_t = S_t \sum_{0}^{r} e^{-\frac{q(t)}{2}} N(d_1) = K e^{-\frac{r(t)}{2}} N(d_2) \), \( r_t = r = 5\% \)

\[ \Delta_t = \frac{N(d_1)}{N(d_2)} = 0.25 \]

\[ N(d_1) = 0.17 \], \( S_t = 2 \), \( K = 2.52 \), \( t = 2 \)

\[ \text{Hence, } C_t = 2 \times 0.25 - 2.52 \times e^{-0.05 \times 2} \times 0.17 = 0.11 \]
3. (b) \[ C_e - P_e = e^{-\Delta t} (F_e - k) \]

\[ P_e = C_e - e^{\Delta t} (F_e - k) \]

\[ = 0.11 - e^{-0.05 \times 2} (2 - 2.5^2) = 0.58 \]

Note: \( F_e = s_e C_e = 2e^{(1.65 - 1.5) \times 2} = 2 \).

(c) Delta of call = \( e^{-\Delta t} N(d_1) = 0.25 \)

Delta of put = \(-e^{\Delta t} N(-d_1) = -e^{\Delta t} (1 - N(d_1)) \)

\[ = -e^{\Delta t} + e^{\Delta t} N(d_1) = -e^{\Delta t} + 0.25 \]

\[ = -0.655 \]

4. (a) \begin{align*}
\begin{cases}
   n_1 \times 0.5 + n_2 \times 0 + 10 = 0 \quad (1) \\
   n_1 \times 2 + n_2 \times 6 + 200 = 0 \quad (2)
\end{cases}
\end{align*}

From (1), \( n_1 = -10/0.5 = -20 \)

From (2) \( n_2 = \frac{-200 - n_1 \times 2}{6} = \frac{-200 + 20 \times 2}{6} = -26.67 \).

(b) To achieve delta neutral, you need \( n_1 = -20 \).

Then, you want \[ |(2) - 20x2 + n_2 \times 6 + 200| \leq 10 \]

\[ |n_2 \times 6 + 160| \leq 10 \]
4(b) \[ |N_2 \times 6 + 160| \leq 10 \]
\[-10 \leq 6N_2 + 160 \leq 10 \]
\[ N_2 = \frac{10 - 160}{6} = -25 \overset{\text{V}}{\text{ or }} \]
\[ N_2 = \frac{-10 - 160}{6} = -28.33 \]

We need a short 25 contracts at minimum to keep portfolio vega within ±10.

5. \[ S_t = 100. \text{ strike 80, 100, 120 call.} \]

Call payoff \((S_t - K)^+\) 
- 80 strike = in-the-money
- Highest delta

100 strike has the highest vega (at the money).

For put options, 120 strike has the highest delta (at-the-money).
100 strike has the highest vega.

6. When stock price goes up by one dollar,
- The put option goes down by approximately delta, hence 25 calls.