Assume a continuously compounding dollar interest rate of 5% for all maturities, whenever applicable.

1. Consider a non-dividend paying stock, with a current price ($S_t$) of $100 and a constant annualized return volatility ($\sigma$) of 20%. The stock price evolves according to the geometric Brownian motion as assumed by Black, Scholes, and Merton. We want to price a three-month ($T - t = 3/12$) European call option on the stock with a strike price ($K$) of $105.

(a) (5) Is this option in the money, out of the money, or at-the-money?

(b) (10) Using the approach discussed in class, construct a two-step binomial tree to approximate the stock price dynamics, with each step being 1.5 months.

(c) (10) Compute (i) the current value and (ii) the delta of the call option based on the two-step binomial tree (using any approach you like)?

(d) (15) Compute (i) the current value, (ii) the delta, and (iii) the vega of the call option based on the Black-Scholes formula. I have: $N(d_1) = 0.3772$ and $N(d_2) = 0.3398$.

(e) (10) Based on your calculated delta and vega numbers from the Black-Scholes formula, how much will the call option value change approximately if the stock price goes up by $1? How much will the call value change if the stock return volatility increase from 20% to 21%?

(f) (15) Given the assumptions, calculate the stock return's volatility (standard deviation) over horizons of three months, one year, and two years, respectively.

2. (20) Assume that you have a portfolio with delta 30 and vega 400, and you want to alter your delta and vega exposure using (i) the underlying stock and (ii) a delta-neutral straddle with a vega of 20.

(a) How many of these two contracts do you need to balance your portfolio to delta and vega neutral?

(b) Suppose you want to constrain the delta exposure of your portfolio to be within ±10 and the vega exposure to be within ±100. What are the minimum number of contracts you need to get your portfolio within your target exposure range?

3. (15) We have three over-the-counter quotes on one-year dollar-yen options (dollar is home currency): (i) delta-neutral straddle at 15%, (ii) 25-delta risk-reversal at −2%, and (iii) 25-delta butterfly spread at 1%.

(a) Which of the three contracts has the highest delta exposure? Which of the three contracts has the highest vega exposure?

(b) Based on the quotes, do you think the one-year risk-neutral dollar-yen return distribution is positively skewed, negatively skewed, or symmetric? Why (which quote gives you the information)?

(c) Based on the quotes, do you think the one-year risk-neutral dollar-yen return distribution has fatter, thinner, or the same tails as a normal distribution does?