1. Consider stock with a current price \( (S_t) \) of $100 and a constant annualized return volatility (\( \sigma \)) of 20%. The stock does not pay dividends. A risk-free zero-coupon bond with $1 par and one year maturity is worth $0.95 today.

(a) (10) Using the approach discussed in class, construct a two-step binomial tree to approximate the stock price dynamics, with each step being 1 year. List the stock price at each node at one and two years.

(b) (5) Compute the risk-neutral probability of going up and going down at each step.

(c) (20) Based on the binomial tree, compute (i) the current value and (ii) the delta of an European put option on the stock with a maturity of two years and a strike price of $110.

(d) (10) Based on the binomial tree, compute (i) the current value and (ii) the delta of an American put option on the stock with a maturity of two years and a strike price of $110.

2. Consider a European put option with the underlying security spot price being $100, strike pricing being $90, and time to maturity being one year. We also know that \( N(d_1) = 0.75 \) and \( N(d_2) = 0.7 \), and we further assume zero interest rates and zero dividends for this question.

(a) (10) Compute the Black-Scholes (i) value and (ii) delta of the European put option.

(b) (15) Compared to this put option, are the following put options more, less, or the same in terms of their sensitivity to the underlying price movement? (i) a 25-delta 10-year put option, (ii) a 50-delta 1-year put option, (iii) a 75-delta 1-year put option. [3 answers, one for each contract.]

3. You have a portfolio of options on the same stock, with a delta \(-200\) million and vega 400 million.

(a) (5) If the stock price suddenly falls by one dollar while the volatility does not change, how much do you expect your portfolio value to change?

(b) (5) If the stock price does not change but the volatility suddenly goes up by one percentage point (0.01, or 1%), how much do you expect your portfolio value to change?

(c) (10) If you want to alter your risk exposure using (i) the underlying stock and (ii) a put option with a delta of \(-0.5\) and a vega of 20. How many of these two contracts do you need to long or short in order to make your portfolio to delta and vega neutral?

4. You are trying to infer the return distribution of a currency at one month horizon. The one-month at-the-money straddle is quoted at 20%. The one-month 25-delta risk-reversal is quoted at \(-10\%\). The one-month 25-delta butterfly spread is quoted at 10%.

(a) (5) Compared to a normal distribution, does the currency return has fatter tails or thinner tails?

(b) (5) Compared to a normal distribution, is the currency return distribution negatively or positively skewed?