1. Time step is 1 year, \( S = 20\% \), \( e^{-r \Delta t} = 0.95 \), \( q = 0 \)

\[ U = e^{\sigma \sqrt{\Delta t}} = e^{0.2 	imes \sqrt{1}} = 1.2214 \]

\[ d = \frac{1}{u} = 0.81873 \]

\[ S_0 = 100u. \]

(a) The stock price tree is

\[ \begin{array}{c}
100 \\
100x0.81873 \\
100x1.2214^2 \\
100 \\
\end{array} \]

\[ \Rightarrow \begin{array}{c}
100 \\
122.1403 \\
149.1825 \\
100 \\
81.87306 \\
67.0320 \\
\end{array} \]

(b) \[ p = \frac{e^{(r-q)\Delta t} - d}{U - d} = \frac{1}{0.95 - 0.81873} = 0.58087 \] (up probability)

Down probability = \( 1 - p = 1 - 0.58087 = 0.41913 \)

(c) European Put option at 2-yr maturity at \( K = 110 \). Option price tree

\[ \begin{array}{c}
0.95(0.58083 \times 110) \\
0.95(0.58083 \times 42.968) \\
= 29.817 \\
\end{array} \]

\[ \begin{array}{c}
\max(0, 110 - 149.1825) = 0 \\
\max(0, 110 - 42.968) = 42.968 \\
= 22.6269 \\
\end{array} \]
1. (c) Delta tree

\[
\frac{3.9817 - 22.6269}{22.1403 - 81.87368} = -0.4630
\]

\[
\frac{0-10}{149.1825-100} = -0.2033
\]

\[
\frac{0-42.968}{100-67.032} = -1
\]

1 year later

(c) Exercise value

\[
\max(0, 110-100) = 10
\]

\[
\max(0, 110-81.87368) = 28.1269
\]

\[
\max(0, 110-81.87368) = 42.968
\]

American value 1 year later = \( \max(\text{Exercise value, continuation value}) \)

2. (American value)

\[
\max(10, 0.95 \times 0.58887 \times 39.817 + 0.4113 \times 28.1269)
\]

\[
= \max(10, 3.3966)
\]

\[
= 3.3966
\]

\[
\max(3.9817) = 3.9817
\]

\[
\max(22.6269, 78.1269) = 28.1269
\]

\[
\max(22.6269, 42.968) = 42.968
\]

\[
\max(0, 3.9817) = 0
\]

\[
\max(22.6269, 78.1269) = 42.968
\]

\[
\frac{0-10}{149.1825-100} = -0.2033
\]

\[
\frac{0-42.968}{100-67.032} = -1
\]

\[
\max(0, 3.9817) = 0
\]

\[
\max(22.6269, 78.1269) = 42.968
\]
2. Zero rates, zero dividends: $F_t = S_t = 100, \ k = 90$

\[(a) \quad P = \left( F_t N(-d_1) + K N(-d_2) \right) e^{-rT}
\]

\[= -100 \times (1 - 0.75) + 90 \times (1 - 0.7) \]

\[= 90 \times 0.3 - 100 \times 0.25 = 2.00\]

**Delta**: $-N(-d_1) = -0.25$

(b) (i) 25-delta 10-year put. The same because it has the same delta.

(ii) 50-delta put is more sensitive because delta is larger.

(iii) 75-delta put is more sensitive.

3. (a) Stock price falls by $3$, portfolio falls by Delta = -200 million.

Hence, portfolio increases by 200 million.

(b) Volatility goes up by 0.01, portfolio changes by $0.01 \times 400 = 4$ million.

It is a value increase.

(c) Since we long 400 million Vega, need to short \(\frac{400}{20} = 20\) million contracts on the 20-vega put.

This generates a delta of \(-20 \times (-0.5) = 10\) million.

Total Delta is \(-200 + 10 = -190\) million.

Hence, long 190 million stock.
4. (a) Butterfly spread is positive \(\Rightarrow\) Fatter tails
(b) Risk reversal is negative \(\Rightarrow\) Negatively skewed