1. Payoffs
   (a) Short 100 forward with $K = 120$
   Payoff = $(120 - S_T) \times 100$
   When $S_T = 100$,
   Payoff = $(120 - 100) \times 100 = 2000$

   (b) Long 50 call at $K = 120$, short 50 put at $K = 120$
   Payoff = $(Call - Put)^{50}$ = forward $\times 50$
   = $50 \left[ (S_T - K)^+ - (K - S_T)^+ \right]$
   = $(S_T - K)^+ \times 50$
   When $S_T = 100$,
   Payoff = $(100 - 120) \times 50 = -20 \times 50$
   = $-1000$

   (c) Long one put at $K = 90$, long one call at $K = 110$
   When $S_T = 100$, Payoff = 0.

   (d) Short one call at $K = 100$, long 2 calls at $K = 110$
   Payoff = $- (S_T - 100)^+ + 2 (S_T - 110)^+$
   When $S_T < 100$: Payoff = 0 + 0 = 0
   When $100 < S_T < 110$: Payoff = $- (S_T - 100) + 2 (110 - S_T)$
   When $S_T > 110$,
   Payoff = $- (S_T - 100) + 2 (S_T - 110) = S_T - 120$
   When $S_T = 100$, Payoff = 0.
2. Forward Pricing

(a) \( S_t = 100, \ r = 4\% \), dividend = 53 each half year, \( T-t = 1 \) year.

\[
F_{T-t} = S_t e^{r(T-t)} - D_1 e^{\frac{1}{2} r T} - D_2
\]

\[
= 100 e^{-0.04 \times 1} - 3 e^{-0.04 \times 0.5} - 3 = $98.02.
\]

(b) \( K = 110, \)

Value \( = e^{-r(T-t)} (F_{T-t} - K) = e^{-0.04 \times 1} (98.02 - 110) \)

\[
= -$11.51 \text{ per contract.}
\]

Value for 300 contracts = -$11.51 \times 300 = -$3,452.94

(c) Forward fair price is $98.02, The market is $100

Hence, short the forward at the market price of $100, (zero initial value)

Buy the stock and hold to maturity.

Cost is 100. Borrow this amount at 4% interest rate.

At expiry, receive 2 dividends worth of \( 3 e^{-0.04 \times 0.5} = 6.06 \)

Use the stock to cover the forward contract's need for a stock.

Receive the forward price of 100.

Repay borrowing cost of \( 100 e^{-0.04 \times 1} = 104.08 \)

Net benefit = 6.06 + 100 - 104.08 = 1.98.
3. Put-call parity

\[ K = 80, \quad F = 100, \quad C = 21.46. \]

(a) \( F > K \), in the money for the call option.

(b) Intrinsic value

\[ e^{-r(T-t)} (F-K) = e^{-0.04 \times 2} (100-80) = 18.46 \]

Hence, time value = \( C - \text{intrinsic value} = 21.46 - 18.46 = 3.00 \).

(c) The put is out of money, hence its intrinsic value is zero.

Its time value is the same as the call at $3.00.

So the spot put is worth $3 total.

(d) \( F > t \), call price $4, put price $1

(e) Vol 4, call price $4, put price $4

4. Monotonicity arbitrage

Call option price should decline monotonically with strike.

Here, \( C(90) = 5, \quad C(95) = 5.1 \). The monotonicity condition is violated.

Sell the $95 call, buy the $90 call. The net is $0.1.

At expiry, the payoff is:

\[ (S_T - 90)^+ + (S_T - 95)^+ \]

When \( S_T < 90 \), payoff = 0

When \( S_T \in (90, 95) \), payoff = \( S_T - 90 > 0 \) between (0, 5)

When \( S_T > 95 \), payoff = \( S_T - 90 - (S_T - 95) = 5 \).

The payoff is always zero or larger.
5. Creating strategies to match payoffs.

a) Use call strategy:
   
   At $S_T = 0$, Payoff = 80. Hence, long $80$ per bond.
   
   slope at $S_T = 0$ is $-1$. Hence, short 1 call at $K = 0$.
   
   at $S_T = 90$, slope changes from $-1$ to $1$. Hence, long 2 calls at $K = 90$
   
   at $S_T = 100$, slope changes from $1$ to $-1$. Hence, short 2 calls at $K = 100$

   at $S_T = 110$, slope changes from $-1$ to $1$. Hence, long 2 calls at $110$.

b) This is called a wrangle. It can also be replicated by:

   - Short 1 call at 100
   - Long 2 calls at 110
   - Short 1 put at 100
   - Long 2 puts at 90