1. If you long a forward at $K = 100$, the payoff is $(S_T - 100)$.

   (a) If $S_T = 80$, the payoff is $-20$.
   If $S_T = 120$, the payoff is $120 - 100 = 20$.

   ![Diagram](image1)

   (b) With a short position, the payoff is $(100 - S_T)$.
   Hence, for (a), the payoff is $20$ when $S_T = 80$, $-20$ when $S_T = 120$.

   ![Diagram](image2)

2. If you long a zero-coupon bond with par $550$, the payoff is $550$, regardless of the stock index (or anything else).

   (a) $550$, $-550$
   (b) ![Diagram](image3)
   (c) $-550$, $-550$. 

   ![Diagram](image4)
3. If you long a stock, the payoff is $ST$ when there is no dividend.

(a) $56, \quad 65$
(b) $ST$
(c) $56, \quad -65$

4. Long a call with $K = 80$, payoff $= (S_T - 80)^+$

(a) $(100 - 80)^+ = 20, \quad (90 - 80)^+ = 10$
(b) Payoff
(c) Short is just negative of everything, payoff $= -(S_T - 80)^+$

(c) $100 - 80)^+ = -20, \quad (90 - 80)^+ = -10$

(d) Strike is $K = 80$, if H/S in the money (spot or forward),

Intrinsic value $= e^{-\gamma} (F - K)^+$

$= e^{-0.05} (101 - 80)^+ = 19.98$

(e) It's out of the money, intrinsic value $= 0$. 
5. Long a put at $K = 80$, payoff $= (80 - S_T)^+$

(a) $(80 - 100)^+ = 0$  
(b) $(80 - 90)^+ = 0$

(c) $0, 0$

(d) out of the money  
(e) in the money, intrinsic value $= e^{-0.051} (K - F)^+$

$= e^{-0.051} (80 - 60.5)^+ = 18.55$

6. My way of doing this is to just write out the payoff for each derivative, and then consolidate:

(a) long 1 forward at 100 : $+ (S_T - 100)$
(b) short 1 call at 100 : $- (S_T - 100)^+$
(c) long 1 put at 100 : $+ (100 - S_T)^+$

It is not easy to consolidate $(S_T - K)^+$ and $(K - S)^+$ because the positive part of argument makes it no longer linear operation. What you can do is to divide the payoff to intervals at the strikes. In this case, there is only one strike at 100. Hence, you can analyze the payoff:

If $S_T > 100$ : $+ (S_T - 100) - (S_T - 100)^+ + 0 = 0$

Note that given that $S_T > 100$, I can drop the "positive part" argument.

If $S_T < 100$, $+ (S_T - 100) - 0 + (100 - S_T) = 0$
6. So the payoff is zero at all possible \( S_t \).

7. This one is a little harder, but you can follow the same procedure.

(a) Long 2 forwards at 100
(b) Long 1 call at 80:
(c) Long 1 put at 100
(d) Short 1 put at 80

\[ + Z(S_t - 100) + (S_t - 80)^+ (100 - S_t)^+ + (80 - S_t)^+ \]

I have 2 Strangles 80 & 100. So I need to analyze the payoff at 3 regions: 0–80, 80–100, 100–infinity

\( S_t \in (0, 80) \):
\[ Z(S_t - 100) + 0 + (100 - S_t) - (80 - S_t) \]
\[ = 2S_t - S_t - 200 + 100 - 80 \]
\[ = 2S_t - 180 \]

\( S_t \in (80, 100) \):
\[ Z(S_t - 100) + (S_t - 80) + (100 - S_t) - 0 \]
\[ = 2S_t + S_t - S_t - 200 - 80 + 100 \]
\[ = 2S_t - 180 \]

\( S_t \in (100, 100) \):
\[ Z(S_t - 100) + (S_t - 80) + 0 - 0 \]
\[ = 2S_t + S_t - 200 - 80 \]
\[ = 3S_t - 280 \]

Answers:
\( S_t = 90 \): Payoff = 2(90-180) = -40
\( S_t = 110 \): Payoff = 3(110-280) = 50
8. (a) [Diagram showing a straddle]
   (b) [Diagram showing a forward]
   (c) [Diagram showing a risk reversal]
   (d) [Diagram showing a strangle]
   (e) [Diagram showing a hypothetical payoff]
   (f) [Diagram showing another hypothetical payoff]

Name (straddle, forward, ...) is not important.

9. (a) \( F = S e^{(r_d - r_f)(T-t)} = 2 e^{(0.05 - 0.06) \times 2} = 1.96 \)

(b) If the forward is $2, short forward, long replicating portfolio (buy currency & hold).

Today: Sign contract (short forward)

   Borrow $2, buy pound, same pound in bank.
   Total cashflow = 0.

End: Use the pound to cover the short forward contract.

Receive $2 from re-forward ($2 = 2).
9. (b) Today:
(1) Sign 1 short forward contract
(2) Buy \( e^{-0.06 \times 2} \) fraction of pound and save the pound in the bank.
This will become 1 pound at expiry.
(3) Borrow 2 \( e^{-0.06 \times 2} \) to cover the cost of buying the pound.

Total cashflow is zero.

Expire:
- Use the 1 pound from (2) to cover the 1 pound from (1).
- Receive $2 from the forward contract.
- Need to pay back the borrowing + interest (dollar interest)

\[
(2 \times e^{-0.06 \times 2}) \times e^{0.05 \times 2} = 2 e^{0.05 \times 2 - 0.06 \times 2} = 1.96
\]

Net cashflow is \$2 - 1.96 = \$0.04.

(c) If forward is \$1.50, Long forward, short replication.

Today:
(1) Sign 1 Long forward at \$1.50.
(2) Short sell \( e^{-0.06 \times 2} \) fraction of pound -> becomes 1 pound at expiry.
(3) Receive 2 \( e^{-0.06 \times 2} \) cash today from short selling, save it in the bank, it will become (2 \( e^{-0.06 \times 2} \) \( e^{0.05 \times 2} \) = \$1.96 at expiry.

Expire:
- Pay \$1.50, receive one pound, pay one pound to the pound lender.
- Receive 1.96 from the bank (short sell receipt).

Net cashflow = 1.96 - 1.50 = \$0.46.
10. 
(a) \( F = 640 e^{0.04 \times 0.5} = 652.93 \)
\[ F = 640 e^{0.04 \times 4} = 751.05 \]

(b) If half-year forward is \( 670 \). Short this forward, buy Google stock.

Today:
1) Sign forward short at \( 670 \)
2) Buy Google at \( 640 \)
3) Borrow \( 640 \) to cover the cost

Expiration (half-year): Use google stock to cover short forward
Receive \( 670 \)
Return borrowed money + interest: \( 640 e^{-0.04 \times 0.5} = 652.93 \)
Net cashflow \( 670 - 652.93 = 17.07 \)

11. 
\[ PV_1 = 100 e^{-0.05 \times 2} = 90.48 \]
\[ PV_2 = 50 e^{-0.05 \times \frac{3}{12}} = 49.38 \]

12. Since there is no model assumption, we can only know its lower and upper bound.

Lower bound: \( (F-K) e^{-r(T-t)} = (120-100) e^{-0.05 \times 1} = 9.02 \)
Upper bound: \( F e^{-r(T-t)} = 120 e^{-0.05 \times 1} = 114.15 \)

(a) Option quote = \$18. Lower than the lower bound.
Buy option, Short forward. Cash-flow = \(-18 + 19.02 = 1.02 \)

Expiration: If \( St > 100 \), \((S_t-100) + (100 - St) = 0 \)
If \( St < 100 \), \(0 + 100 - St > 0 \)
12. (b). If call is at 121, it's higher than the upper bound.

Sell the call, long the forward which is delivered price.

\[ +121 - 114.15 = 6.85 \]

At expiry: \((S_T - 100)^+ + S_T\)

- When \(S_T > 100\): \(-S_T + 100 + S_T = 100\)
- When \(S_T < 100\): \(0 + S_T > 0\)

make money at both today & at expiry.

13. \(106 < 120\)

\(r = 5\%\)

(a) Call with \(K = 110\).

\((S_T - K)^+\)

\[ \begin{align*}
120 & \quad \text{(if } S_T > 110) \text{) +}\nonumber \\
100 & \quad \text{call} \nonumber \\
80 & \quad \text{(if } S_T < 110) \text{) +}\nonumber 
\end{align*} \]

Replication:

\[ \begin{align*}
10 &= \Delta \times 120 + D \\
0 &= \Delta \times 80 + D \\
\end{align*} \]

\[ \Delta = \frac{10 - 0}{120 - 80} = \frac{1}{4} \]

\[ D = -\frac{1}{4} \times 80 = -20 \]

Call = Replication Value = \(\Delta \times 100 + D \times e^{-0.05 \times 0.5}\)

\[ = \frac{1}{4} \times 100 - 20 \times e^{-0.025} = 5.49 \]
13. (b) Put with $K = 100$, $(100 - S_t)^+ \\ \begin{align*} 
120 \\
100 \\
\text{Put} \\
80 \\
(100 - 80)^+ = 20 
\end{align*}
\\
\text{Replication:} \quad \begin{align*} 
0 &= \Delta 120 + D \\
20 &= \Delta 80 + D 
\end{align*}
\\
\Delta = \frac{-20}{120 - 80} = -\frac{1}{2} \\
D = \frac{1}{2} \times 120 = 60 \\
\text{Put} = \text{Replication portfolio value} = \Delta \times 100 + D e^{-0.05 \times 0.5} \\
&= -\frac{1}{2} \times 100 + 60 e^{-0.05 \times 0.5} = 81.52.
\\
(c) If the call quote is \$4, \quad \begin{align*} 
\text{Buy the call, } -4 \\
\text{Sell the replicating portfolio } + 5.49 \\
\left( \begin{array}{c} 
\frac{1}{D} \\
\text{or sell the security } + \frac{1}{2} \times 100 \\
\end{array} \right) \\
\text{and buy 20-per stock: } -20 e^{-0.05 \times 0.5} \\
\text{At expiry, the payoff cancels.}
\end{align*}
\\
(d) If put is at \$9, \quad \text{Sell the put } + 9 \\
\text{Buy the replicating portfolio : } -81.52 \\
\quad + 0.48 \\
\text{No cash flow at expiry.