1. Short 2y Forward with $K=100.$
   Payoff is $K - ST$

(a) $100 - 90 = 10$
   $100 - 100 = 0$
   $100 - 110.52 = -10.52$
   $100 - 120 = -20$

(b) $\text{Payoff}$

(c) $S_t = 50, D_e = 5, T-t = 2$
   
   $F_t = (S_t - D_e)e^{-0.05 \times 2} = (50 - 5)e^{-0.05 \times 2} = 49.73$

(d) $e^{-0.05 \times 2} (K - F_t) = e^{-0.05 \times 2} (100 - 49.73) = 45.48$

(e) Market: $50$, higher than the buy&carry cost of $49.73$
   
   Hence, short the forward (high value), and buy the stock today.

   Buying the stock costs $450 today, which we borrow from the bank at 5% interest.

   At expiry (2 yr later), I own the stock and have received dividends worth $5 \times e^{0.05 \times 2} = 5.52$ (Each time I receive dividend, I save it in the bank earning 5% interest).

   From the short forward position, I need to sell the stock (so the stock is gone), and I receive the strike of $450 (market quote).

   The borrowed money now costs me $50 e^{0.05 \times 2} = 55.26$.

   In total, I owe $55.26 to the bank, and I receive $50 from forward, and 5.52 from dividend. The net is $0.27. That's the sure money I make.
2. \( S_t = 2, \quad r_d = 5\%, \quad r_f = 6\%. \quad T-t = 5. \)

(a) \( F_t = S_t e^{(r_d - r_f)(T-t)} = 2 \times e^{(0.05 - 0.06)(5)} = 1.9025. \)

(b) Market: $2, higher than the cost of buy and carry.

So short forward, buy and carry pound. Exact procedure is:

Today: short forward at $2. No upfront cost.

Buy \( e^{-0.06 \times 5} = e^{-0.3} = 0.7408 \) share of a pound. This will grow in the bank to become 1 pound at 5 year maturity.

The cost of pound purchase is \( 0.7408 	imes 2 = \$1.4816 \), which I borrow from the bank at 5% dollar interest.

Expiration (5 yr later): the pound will grow into one pound in 5 years at 6% rate.

\[ \frac{1}{0.7408} \]

The dollar borrowing of $1,4816 will grow to \( 1,4816 \times e^{0.05 \times 5} = 1,9025 \).

That's what I need to pay back.

From the forward contract, I pay 1 pound - hence, pound is gone, and I receive the $2 strike.

My total is $2 sales revenue plus $1,9025 debt, thus, a net of $0.0975 profit with no pound position left.
3. Long put at $K = 100$. Payoff = $(K - S_T)^+$

<table>
<thead>
<tr>
<th>$S_T$</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>$(100-90)^+ = 10$</td>
</tr>
<tr>
<td>95</td>
<td>$(100-95)^+ = 5$</td>
</tr>
<tr>
<td>100</td>
<td>$(100-100)^+ = 0$</td>
</tr>
<tr>
<td>110</td>
<td>$(100-110)^+ = 0$</td>
</tr>
</tbody>
</table>

4. Long put at 20, short a 10.

$(20-S_T)^+ - (10-S_T)^+$

when $S_T < 10$, Payoff = $(20-S_T) - (10-S_T) = 10$

When $S_T$ between (10, 20), Payoff = $(20-S_T) - 0 = 20 - S_T$

When $S_T > 20$, Payoff = 0.

(a) Answer:

<table>
<thead>
<tr>
<th>$S_T$</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
</tr>
</tbody>
</table>

(c) If stock price > $30$ before default, the portfolio is worth $30$.

If the stock price < $2$ after default, the portfolio is worth $10$.

So the payoff of the portfolio is either $0$ (no default) or $10$ (default), regardless of how the stock price vary over time.

(ii) Long $K_1$, short $K_2$ such that both $K_1$, $K_2$ are within (2, 30) range and $K_1 - K_2 = 10$. Examples: Long put at 30, short put at 20, or long at 25, short at 15.
5. Put with $K=100$. $F=90$, $P_e=15$, $t=1$

(a) $K-F<0$, out of money, In the money.

(b) Intrinsic value is zero.

Timing value is $t/2 = 0.5$. $e^{-rT} = e^{-0.05*1} = 0.951$.

Time value is $15 - 9.51 = 5.49$.

6. $F=120$, $(T-t)=2$, $P_e=17$, $K=100$.

Lower bound: $(F-K)e^{-(T-t)} = (120-100)e^{-0.05*2} = 18.0967$.

Upper bound: $F e^{-(T-t)} = 120 e^{-0.05*2} = 120 e^{-0.1} = 108.58$.

Call option price is lower than lower bound. **Arbitrage!**

- Buy the call, cost $17.
- Sell a forward with the same strike of 100, the value of this short forward is $(K)(120-100)e^{-0.05*2} = 18.0967$.

Hence, you receive $18.0967 by selling.

The net profit today is $18.0967 - $17 = 1.0967$.

At Expiry, your payoff is $(K-S_T) + (S_T-K)^+ = 0$.

When $K > S_T$, payoff = $K - S_T + 0 = K - S_T > 0$

When $K < S_T$

This is an arbitrage because you receive money (1.0967) today and you get a put option!