1 List of Topics

1. Payoffs: forward, call, put, stock(spot), bond
   (a) Can plot/describe the payoff of any combinations/portfolios of the above instruments.
      • When the portfolio contains options, it is important to break down the analysis at each strike
        (when the underlying security price is above or below each strike).
   (b) Can deal with concrete numbers
   (c) Given a targeted payoff structure (a plot), can replicate the payoff using the above instruments.
      • The replication strategy is in general not unique.
      • When things are not as clear, follow the general strategies of using call options only or using
        put options only.
      • Once you have formed the portfolio, analyze the payoff from the portfolio and double-check
        to make sure that it matches the targeted profile.

2. Pricing: forward pricing (buy and carry), bond pricing (discounting with continuously compounded
   interest rate)

3. No-arbitrage trading:
   (a) Master the idea of “Buy low, sell high.”
   (b) Can figure out the payoffs of the strategy today and at expiry to show why it is an arbitrage.

4. Option behaviors
   (a) The basic definitions of calls, puts; American v. European; out-of-the-money, in-the-money, 
       at-the-money; intrinsic value v. time value.
   (b) Put-call parity
   (c) Lower and upper bounds
   (d) Its relation with spot, strike, and volatility.
   (e) When the market quotes violate the parity conditions, the lower/upper bounds, and/or its de-
       pendence structure on strike and maturity, think of ways (arbitrage strategies) to exploit the
       violations.

2 Clarification on intrinsic value v. time value

For a time-$t$ European call option at strike $K$ and maturity $T$, its intrinsic value is

$$\text{Intrinsic value of call} = e^{-r(T-t)}(F_{t,T} - K)^+,$$
where \( F_{t,T} \) denote the time-\( t \) forward price of maturity \( T \). In the early sections of the semester, I sometimes use \((S_t - K)^+\) to approximate the intrinsic value. But to be precise, we cannot exercise now. Hence, the comparison should be with the relevant forward price, not the current spot.

Furthermore, once we use the more exact definition, the time value of the option, \( TV_t(K, T) \), is simply the difference between the option price \( c_t \) and the intrinsic value,

\[
c_t(K, T) = e^{-r(T-t)}(F_{i,T} - K)^+ + TV_t(K, T).
\]

For a put option, the intrinsic value is analogously defined as

\[
\text{Intrinsic value of put} = e^{-r(T-t)}(K - F_{i,T})^+,
\]

and the put option value can also be decomposed as the intrinsic value and the time value,

\[
p_t(K, T) = e^{-r(T-t)}(K - F_{i,T})^+ + TV_t(K, T).
\]

In particular, the call and put options with the same strike and maturity have the same time value \( TV_t(K, T) \). We thus have the put-call parity:

\[
c_t(K, T) - p_t(K, T) = e^{-r(T-t)}(F_{i,T} - K)^+ - e^{-r(T-t)}(K - F_{i,T})^+ = e^{-r(T-t)}(F_{i,T} - K).
\]