

Dynamic Interactions Between Interest-Rate and Credit Risk: Theory and Evidence on the Credit Default Swap Term Structure*

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Abstract. This paper examines the interaction between default risk and interest-rate risk in determining the term structure of credit default swap spreads at different industry sectors and credit rating classes. The paper starts with a parsimonious three-factor interest-rate dynamic term structure and projects the credit spread at each industry sector and rating class to these interest-rate factors while also allowing the projection residual dynamics to depend on the level of the interest-rate factors. Estimation shows that credit risk exhibits intricate dynamic interactions with the interest-rate factors.

JEL Classification: E43, G12, G13, C51.

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1. Introduction

It is important to understand how credit risk interacts with interest-rate risk in determining the term structure of credit spreads on different reference entities. Nevertheless, limited data availability has severely hindered the understanding. Since defaults are rare events that often lead to termination or restructuring of the underlying reference entity, researchers need to rely heavily on cross-sectional averaging across different entities over a long history to obtain any reasonable estimates of statistical default probabilities. Although corporate bond prices contain useful information on the default probability and the pricing of credit risk, the information is often mingled with the pricing of interest-rate risk and other factors such as liquidity and tax.¹

The development in the credit derivatives market provides an excellent opportunity for a better understanding of the dynamics and pricing of credit risk, its interactions with interest-rate risks, and their impacts on the term structure of credit spreads. The most widely traded credit derivative is in the form of credit default swap (CDS), an over-the-counter contract that provides protection against credit risk. The protection buyer pays a fixed premium, often termed as the “CDS spread,” to the seller for a period of time. If a pre-specified credit event occurs, the protection buyer stops the premium payment and receives compensation from the protection seller. A credit event can be the bankruptcy of the reference entity, or default of a bond or other debt issued by the reference entity. If no credit event occurs during the term of the swap, the protection buyer continues to pay the premium until maturity. The CDS spreads are commonly set at inception to match the present values of the credit protection and premium payment so that no upfront payments are necessary.²

With a large data set on CDS spread quotes, this paper performs a joint analysis on the term structure of interest rates and credit spreads, with a focus on the dynamic interactions between the two sources of risk. The data set includes daily CDS spread quotes on hundreds of corporate companies and across six fixed maturities from one to ten years for each company. We classify the reference companies along two dimensions: (i) industry sectors and (ii) credit ratings. We also download from Bloomberg the eurodollar libor and swap rates of matching maturities and sample periods. Through model development and estimation, we address the following fundamental questions regarding credit risk and its dynamic interactions with

¹Many researchers strive to identify and distinguish the different components of corporate bond yields. Prominent examples include Fisher (1959), Jones, Mason, and Rosenfeld (1984), Longstaff and Schwartz (1995), Duffie and Singleton (1997), Duffie (1999), Elton, Gruber, Agrawal, and Mann (2001), Pierre, Goldstein, and Martin (2001), Delianedis and Geske (2001), Liu, Longstaff, and Mandell (2006), Eom, Helwege, and Huang (2004), Huang and Huang (2003), Collin-Dufresne, Goldstein, and Helwege (2003), Ericsson and Renault (2006), and Longstaff, Mithal, and Neis (2005).

²The CDS market is undergoing structural and contractual reforms. To reduce counterparty risk, the market is moving toward central clearing. To facilitate netting, the market is also moving toward contract specifications with upfront payments and fixed premium coupons of either 100 or 500 basis points. The data we have are CDS spreads that result in zero upfront payments.

interest-rate risk:

- How many factors govern the term structure of credit spreads?
- How do the credit-risk factors interact with interest-rate factors?
- How do the credit-risk dynamics and pricing differ across industry sectors and credit rating classes?

To address these questions, we develop a class of dynamic term structure models of interest-rate risk and credit risk. First, we model the term structure of the benchmark libor and swap rates using three interest-rate factors. Second, we assume that the default arrival intensities at each industry sector and credit rating class are governed by either one or two dynamic factors. To capture the dynamic interactions between the market-wide interest-rate risk and the specific credit risk at each industry sector and credit rating class, we project the instantaneous credit spread of each classified group onto the interest-rate factors and further allow the dynamics of the projection residuals to depend on the interest-rate factors. Through this specification, changes in the interest-rate factors affect both contemporaneous and subsequent changes in the credit-risk factors.

We estimate the models in a two-step sequential procedure. In the first step, we estimate the interest-rate factor dynamics using the benchmark libor and swap rates. In the second step, we take the interest-rate factors estimated from the first step as given, and estimate the credit-risk dynamics for each industry sector and credit rating class using the average CDS spreads for that sector and rating class across different maturities. At each step, we cast the models into a state-space form, obtain forecasts on the conditional mean and variance of observed series using a nonlinear filtering technique, and build the likelihood function on the forecasting errors of the observed series, assuming that the forecasting errors are normally distributed. We estimate the model parameters by maximizing the likelihood functions.

Estimation shows that one credit-risk factor can price the moderate-maturity CDS spreads well, but the performance deteriorates toward both ends of the credit spread curve. By contrast, two credit-risk factors can price the whole term structure of credit spreads well. Our estimation also shows that firms in different industry sectors and credit rating classes exhibit different credit-risk dynamics. Nevertheless, in all cases, credit risk shows intricate dynamic interactions with the interest-rate factors. Interest-rate factors both impact the credit spread contemporaneously, and affect subsequent changes in the credit-risk factors.

Our projection-based one-way interaction specification and our two-step sequential estimation procedure provide a viable channel for analyzing the dynamic interactions between market-wide risk factors on the one hand and industry/rating-specific risks on the other. To understand potential two-way interactions between

interest-rate risk and market-wide aggregate credit risk, we also consider an alternative two-way interactive dynamics and identify the two-way interaction through joint estimation of the interest-rate and the market-average CDS term structures. The joint estimation shows that the aggregate credit condition of the market can also influence the monetary policy and hence the benchmark interest-rate curve. In particular, worsening of the credit condition, represented by widening of average credit spreads, especially at long maturities, tends to lead to future easing in monetary policy and accordingly lowering of the current forward interest-rate curve. On the other hand, positive shocks to the short-term interest rate narrow the credit spread at long maturities, whereas positive shocks to long-term interest rates widen the credit spreads. These results highlight the intricate interactions between the two markets.

Understanding the dynamic interactions between interest rates and credit spreads has been a perennial topic in the literature as it has important implications for credit risk modeling. Early studies have often identified a negative relation between credit spreads and short-term interest rates (Duffee (1998)). To be consistent with this finding, many studies, e.g., Feldhütter and Lando (2008), Frühwirth, Schneider, and Sögner (2010), and Driessen (2005), directly incorporate a negative loading of the instantaneous interest rate into the credit spread specification.³ Compared to such practices, we explicitly recognize the fact that the instantaneous interest rate itself is driven by multiple risk factors, and that these different factors may have different impacts on the credit spreads (Wu and Zhang (2008)). Accordingly, we allow the instantaneous interest rate and credit spreads to depend on a common set of interest-rate factors with their different loadings directly estimated from the term structure of *libor/swap* rates and CDS spreads. We further allow the credit-risk factors to interact dynamically with the benchmark interest-rate factors. Thus, our modeling of the interactions between interest rates and credit spreads goes far beyond what has been done in the literature.

Our work constitutes the first comprehensive analysis of the joint term structure of interest rates and credit spreads using the CDS data. In other related studies, Skinner and Diaz (2003) analyze CDS prices from September 1997 to February 1999 for 31 CDS contracts. They compare the pricing results of the Duffie and Singleton (1999) and Jarrow and Turnbull (1995) models. Blanco, Brennan, and Marsh (2005) compare the CDS spreads with credit spreads derived from corporate bond yields and find that overall the two sources of spreads match each other well. When the two sources of spreads deviate from each other, they find that CDS spreads have a clear lead in price discovery. Longstaff, Mithal, and Neis (2005) regard the CDS spreads as purely due to credit risk and use the CDS spreads as a benchmark to identify the liquidity component of

³In proposing models for pricing interest-rate derivatives, Brigo and Alfonsi (2005) allow instantaneous correlation between the two Brownian motions driving the interest rate and the default arrival rate, and analyze numerically the effect of this correlation on calibration and pricing. Schneider, Sögner, and Věza (2010) analyze the loss given default and jumps in default risk using corporate CDS data, but they assume independence between interest rate and default risk.

corporate yield spreads. They find that a major portion of the corporate spread is due to credit risk. Hull, Predescu, and White (2004) examine the relation between the CDS spreads and announcements by rating agencies. Zhang (2008) uses sovereign CDS to study the case of Argentine default. Carr and Wu (2007) model and estimate the dynamic interaction between sovereign CDS spreads and currency options. Cremers, Driessen, Maenhout, and Weinbaum (2008) and Carr and Wu (2010, 2011) analyze the link between CDS spreads and stock option prices.

The remainder of the paper is organized as follows. Section 2 describes the data set and documents the stylized evidence on the CDS spreads that motivates our theoretical efforts in Section 3, where we develop the dynamic term structure models that allow intricate dynamic interactions between interest-rate risk and credit risk. Section 4 describes our model estimation strategy. Section 5 discusses the estimation results. Section 6 explores potential two-way interactions between interest rates and market-average CDS spreads. Section 8 concludes.

2. Data and Evidence

We obtain daily CDS spread quotes from an investment bank at six fixed time-to-maturities at one, two, three, five, seven, and ten years from May 21, 2003 to October 8, 2007, spanning 1,142 business days. For each firm, the data set also contains an expected recovery rate estimate at each date. We obtain the credit rating information on each reference company from Standard & Poors, and the sector information from Reuters.

At each date, we divide the CDS data into four groups based on (i) two broad industry classifications: financial and corporate, and (ii) two broad credit rating groups: A (including A+ and A-) and BBB (including BBB+ and BBB-). Companies without credit rating information or with ratings above A or below BBB are excluded. For each group and at each maturity, we compute a weighted average CDS value, where the weight is computed based on the deviation of each quote from the median value. To reduce the impact of potential outliers, we set the weight to zero when a quote is 1.28 standard deviations away from the median. This criterion excludes about 10% of the quotes on each side of the spectrum. The particular choice of the weighting function and the truncation criterion are chosen based on the analysis of the data behavior and the stability of the resulting series. The number of quotes included in each average varies from a minimum of four quotes to a maximum of 299. The number of firms in our data set increases over time. Accordingly, the number of quotes included in the average also increases over time. The number of firms averages at 47 for financial firms with A rating, 43 for financial firms with BBB rating, 106 for corporate with A rating, and

204 for corporate with BBB rating.

Figure 1 plots the time series of the average CDS spreads at each industry sector and credit rating class. The six lines in each panel correspond to the six time series at different maturities, with the solid line denoting the one-year CDS series, the dash-dotted line denoting the 10-year CDS series, and the dashed lines for the intermediate maturities. In all four panels, the solid line for the one-year CDS series always stays at the bottom of the six lines whereas the dash-dotted line for the 10-year CDS series always stays at the top of the six lines, showing that the CDS term structure is always upward sloping during our sample period. The time series in Figure 1 show stronger co-movements across different maturities within the same panel (i.e., within the same industry sector and rating class) than co-movements across different panels.

[Figure 1 about here.]

Table I reports the summary statistics of the average CDS spreads under each industry sector and credit rating classification. The mean spreads are higher at longer maturities for all groups, generating an upward-sloping mean term structure. Within each industry sector and at each fixed maturity, the mean spreads are higher for BBB firms than for A firms. Within each credit rating class, the mean spreads are slightly higher for financial firms than for non-financial firms.

[Table I about here.]

The standard deviations of the spreads show an upward sloping term structure for the financial sector and the A rating class, but the term structure is either downward sloping or hump-shaped for other groups. The skewness and excess kurtosis estimates are mostly small. The daily autocorrelation estimates are between 0.98 and 0.99, indicating that the CDS spreads are highly persistent. The last row in each panel reports the summary statistics on the expected recovery rates, which average close to 40%, with small time variation.

To estimate the benchmark interest-rate dynamics, we obtain eurodollar libor and swap rates from Bloomberg that match the maturity and sample period of the CDS data. Figure 2 plot the time series of the interest-rate series, with the solid line denoting the 12-month libor, the dash-dotted line denoting the 10-year swap rate, and the dashed lines denoting swap rates with maturities from two to seven years. The interest rates show a steeply upward sloping term structure at the beginning of the sample but the term structure becomes flat since 2006, with the 12-month libor being close to or even higher than the 10-year swap rate.

[Figure 2 about here.]

Table II reports the summary statistics of the six interest-rate series. The interest rates average between 3.73% to 4.93%, with an average upward sloping term structure. The standard deviation estimates are downward sloping from 1.6 for the 12-month libor to 0.45 for the 10-year swap rate. The skewness and excess kurtosis estimates are small. The daily autocorrelation estimates range from 0.988 to 0.998, showing extremely high persistence for these time series.

[Table II about here.]

3. Modeling the Dynamic Interactions between Interest Rate and Credit Risk

We value the CDS contracts under the framework of Duffie and Singleton (1999) and Duffie, Pedersen, and Singleton (2003). Following the current industry standard, we define the benchmark instantaneous interest rate r_t based on the eurodollar libor and swap rates.⁴ Libor and swap rates contain a credit risk component. Using them as benchmarks, the estimated credit risk from CDS quotes can be regarded as relative credit risk.

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{Q})$ be a complete stochastic basis and \mathbb{Q} be a risk-neutral probability measure, under which the time- t fair value of a benchmark zero-coupon bond with maturity τ relates to the instantaneous benchmark interest rate by,

$$P(t, \tau) = \mathbb{E}_t \left[\exp \left(- \int_t^{t+\tau} r_u du \right) \right], \quad (1)$$

where $\mathbb{E}_t[\cdot]$ denotes the expectation operator under the risk-neutral measure \mathbb{Q} conditional on the time- t filtration \mathcal{F}_t .

To value a CDS contract, let λ_t^i to denote the intensity of a Poisson process that governs the default of a reference entity i . By modeling the dynamics of the Poisson intensities λ_i and their dynamic interactions with the benchmark interest rates, we determine the term structure of the CDS spreads for entity i . Let $S^i(t, \tau)$ denote the premium rate that a protection buyer should pay on reference entity i for a maturity of τ years to make the CDS contract worth zero at time t . Under the simplifying assumption of continuous payment, the time- t present value of the premium leg of the contract is given by,

$$\text{Premium}(t, \tau) = \mathbb{E}_t \left[S^i(t, \tau) \int_0^\tau \exp \left(- \int_t^{t+s} (r_u + \lambda_u^i) du \right) ds \right], \quad (2)$$

⁴Historically, researchers often use Treasury yields to define the instantaneous interest rate and the benchmark yield curve. Houweling and Vorst (2005) perform daily calibration of reduced-form models using credit default swap spreads and find that eurodollar swap rates are better suited than the Treasury yields in defining the benchmark yield curve.

and the present value of the protection leg is,

$$\text{Protection}(t, \tau) = \mathbb{E}_t \left[w^i(t, \tau) \int_0^\tau \lambda_{t+s}^i \exp \left(- \int_t^{t+s} (r_u + \lambda_u^i) du \right) ds \right], \quad (3)$$

where $w^i(t, \tau)$ denotes the time- t expected loss rate upon default on reference entity i over the horizon τ . By setting the present values of the two legs equal, one can solve for the CDS spread as,

$$S^i(t, \tau) = \frac{\mathbb{E}_t \left[w^i(t, \tau) \int_0^\tau \lambda_{t+s}^i \exp \left(- \int_t^{t+s} (r_u + \lambda_u^i) du \right) ds \right]}{\mathbb{E}_t \left[\int_0^\tau \exp \left(- \int_t^{t+s} (r_u + \lambda_u^i) du \right) ds \right]}, \quad (4)$$

which can be regarded as the weighted average of the expected default loss. In model estimation, we discretize the above equation according to quarterly premium payment intervals.

Under this framework, the benchmark libor and swap rate curve is determined by the dynamics of the instantaneous benchmark interest rate r . The CDS spreads of a certain reference entity are determined by the joint dynamics of the instantaneous interest rate r and the default arrival rate λ^i . We specify the two sets of dynamics in the following subsections.

3.1. BENCHMARK INTEREST-RATE DYNAMICS AND THE TERM STRUCTURE

To enhance model identification, we model the benchmark instantaneous interest-rate dynamics with a parsimonious dimension-invariant cascade structure developed by Calvet, Fisher, and Wu (2010). We apply a three-factor structure and specify the factor dynamics under the statistical measure \mathbb{P} as,

$$\begin{aligned} dx_{1,t} &= \kappa_r s_\kappa^2 (x_{2,t} - x_{1,t}) dt + \sigma_r dW_t^1, \\ dx_{2,t} &= \kappa_r s_\kappa (x_{3,t} - x_{2,t}) dt + \sigma_r dW_t^2, \\ dx_{3,t} &= \kappa_r (\theta_r - x_{3,t}) dt + \sigma_r dW_t^3. \end{aligned} \quad (5)$$

We set the instantaneous interest rate to the highest frequency component $r_t = x_{1,t}$, which mean reverts to a lower frequency stochastic factor $x_{2,t}$, which mean revert to an even lower frequency factor $x_{3,t}$, which reverts to a constant mean θ_r . The cascade structure naturally ranks the three interest-rate factors in terms of their relative frequency, which is distributed according to a power law scaling on the mean-reversion speeds, with $s_\kappa > 1$ denoting the scaling coefficient. For parsimony, we follow Calvet, Fisher, and Wu (2010) in assuming independent and identically distributed factor innovations and using one parameter σ_r to capture the instantaneous risk level, with $\mathbb{E}[dW_t^i dW_t^j] = 0$ for all $i \neq j$. We further assume that the market prices on all three Brownian risks are identical and constant at γ_r . With these assumptions, the interest rate statistical dynamics and term structure behavior are captured by merely five free parameters $(\kappa_r, s_\kappa, \theta_r, \sigma_r, \gamma_r)$.

In matrix notation, we can write the statistical dynamics for the interest-rate factors $X_t \equiv [x_{1,t}, x_{2,t}, x_{3,t}]^\top$ as,

$$dX_t = \kappa_X (\theta_r - X_t) dt + \sigma_r dW_t, \quad (6)$$

where the mean-reversion matrix is block diagonal,

$$\kappa_X = \begin{bmatrix} \kappa_r s_\kappa^2 & -\kappa_r s_\kappa^2 & 0 \\ 0 & \kappa_r s_\kappa & -\kappa_r s_\kappa \\ 0 & 0 & \kappa_r \end{bmatrix}, \quad (7)$$

and all three factors have the same long-run mean θ_r . Given the constant market price assumption, the factor dynamics under the risk-neutral measure \mathbb{Q} are given by,

$$dX_t = (C_X - \kappa_X X_t) dt + \sigma_r dW_t^\mathbb{Q}, \quad (8)$$

where the mean-reversion matrix remains the same and the constant vector C_X is given by,

$$C_X = [-\gamma_r \sigma_r, -\gamma_r \sigma_r, \kappa_r \theta_r - \gamma_r \sigma_r^2]^\top. \quad (9)$$

Under the above specifications, the time- t model value of the zero-coupon bond with time-to-maturity τ is exponential affine in the current level of the state vector, X_t ,

$$P(X_t, \tau) = \exp\left(-a(\tau) - b(\tau)^\top X_t\right), \quad (10)$$

where the coefficients solve the ordinary differential equations,

$$b'(\tau) = b_r - \kappa_X^\top b(\tau), \quad (11)$$

$$a'(\tau) = b(\tau)^\top C_X - \frac{1}{2} b(\tau)^\top b(\tau) \sigma_r^2, \quad (12)$$

starting at $b(\tau) = 0$ and $a(\tau) = 0$, with $b_r = [1, 0, 0]^\top$. The ordinary differential equations can be solved analytically (Calvet, Fisher, and Wu (2010)).

Given the solutions to the zero-coupon bonds, the model values for the libor and swap rates can be

computed as,

$$LIBOR(X_t, \tau) = \frac{100}{\tau} \left(\frac{1}{P(X_t, \tau)} - 1 \right), \quad SWAP(X_t, \tau) = 100h \times \frac{1 - P(X_t, \tau)}{\sum_{i=1}^{h\tau} P(X_t, i/h)}, \quad (13)$$

where τ denotes the time-to-maturity and h denotes the number of payments in each year for the swap contract. The day-count convention for libor is actual/360, starting two business days forward. For the U.S. dollar swap rates that we use, the number of payments is twice per year, $h = 2$, and the day-count convention is 30/360.

Historically, the spreads between libor and the corresponding overnight interest rate (OIS) swap rate are negligible as they average just about 10 basis points. The industry standard is to build one integrated interest-rate curve from quotes on all libor and swap rates. Quotes on eurodollar futures are sometimes also used to smooth the curve at intermediate maturities. During the 2007 financial crisis, the libor-OIS spread widened dramatically to as high as 3.5 percentage points. Swaps with different floating leg libor references often trade at significantly different levels, prompting the practice of building multiple interest-rate curves, with each curve corresponding to one libor tenor (Mercurio (2010a,b)). In this paper, we retain the simplifying assumption that there is one interest-rate curve underlying all libor and swap rates.

Our interest-rate dynamics specification belongs to the general three-factor Gaussian-affine class of dynamic term structure models, discussed more generally in Duffie and Kan (1996), Duffee (2002), and Dai and Singleton (2000, 2002). Gaussian-affine models are very tractable and can readily be linked to vector autoregressive (VAR) specifications in discrete time (e.g., Joslin, Singleton, and Zhu (2011)). We choose to apply strong structures to the dynamics for several reasons. First, the cascade interest-rate dynamics naturally rank the interest-rate factors according to their mean-reversion speeds, with the first factor capturing the highest frequency shocks and the last factor capturing shocks of the lowest frequency. Factor rotation is a common issue in general linear-Gaussian specifications, making the factor identification and interpretation difficult. The cascade structure that we adopt completely removes factor rotation and greatly enhances the economic interpretation of these factors. Second, the dimension-invariant assumptions allow us to use merely five parameters to model the interest-rate dynamics regardless of how many factors we incorporate into the system. The extreme parsimony of the specification allows us to identify the model parameters with strong statistical significance. Identification is a strong concern when estimating dynamics on highly persistent time series, such as interest rates and credit spreads (Duffee and Stanton (2008)). The parsimonious dimension-invariant structure provides a structural approach in mitigating the identification issues, especially when one needs to estimate high-dimensional models. Third, empirical analysis in Calvet,

Fisher, and Wu (2010) shows that the interest-rate data support the power-law scaling assumption across different frequencies. Thus, the specification achieves parsimony and dimension invariance while matching the observed behaviors of the data.

In reality, the pricing performance of a term structure model is mainly dictated by the number of factors than by the number of parameters. An ideal model should have as many factors as needed to match the data behavior while using as few free parameters as possible to enhance identification. The cascade dimension-invariant specification represents a new class of models that move toward this direction.

Empirically, Bikbov and Chernov (2004) show that all three-factor affine models generate similar performance in fitting the interest-rate term structure. In the Appendix, we also estimate a general three-factor Gaussian affine specification with 19 free parameters. The estimation results show that the pricing performance of this general specification is similar to the pricing performance of our parsimonious specification, as both models generate near perfect fitting to the observed interest-rate term structure.

3.2. CREDIT RISK DYNAMICS AND THE TERM STRUCTURE OF CDS SPREADS

To model the default arrival rate underlying each industry sector and credit rating class i , we first project the arrival rate λ_t onto the interest-rate factors X_t ,⁵

$$\lambda_t = \beta^\top X_t + y_t, \quad (14)$$

where β denotes the instantaneous response of the default arrival rate to the three interest-rate factors, and y_t denotes the projection residual, capturing the credit risk component that is contemporaneously orthogonal to the interest-rate factors.

Although y_t is contemporaneously orthogonal to the interest-rate factors by projection principles, the two can still interact dynamically. We consider a two-factor structure for the credit risk while allowing such dynamic interactions with the interest-rate factors. Under the statistical measure \mathbb{P} , we specify the credit-risk factor dynamics as,

$$\begin{aligned} dy_{1,t} &= \kappa_{y,1} (y_{2,t} - y_{1,t}) dt + \sum_{j=1}^3 \kappa_{yx,1}^j (x_{j+1,t} - x_{j,t}) dt + \sigma_y dW_t^4, \\ dy_{2,t} &= \kappa_{y,2} (\theta_y - y_{2,t}) dt + \sum_{j=1}^3 \kappa_{yx,2}^j (x_{j+1,t} - x_{j,t}) dt + \sigma_y dW_t^5, \quad x_4 = \theta_r, \end{aligned} \quad (15)$$

where the credit risk follows a two-factor cascade structure as $y_t = y_{1,t}$ mean reverts to a slower component $y_{2,t}$, which reverts to a constant mean θ_y , with $\kappa_{y,1} > \kappa_{y,2}$. Furthermore, the three interest-rate factors enter

⁵When no confusion shall occur, we suppress the industry sector/rating class reference i to reduce notation clustering.

the drifts of the two credit-risk factors to capture the dynamic interactions, with $\kappa_{yx,1}$ capturing how the deviation of each interest-rate factor from its lower-frequency trend predicts future movements in the first, higher-frequency credit risk factor y_t , and $\kappa_{yx,2}$ capturing the prediction of the three interest-rate factors on the lower-frequency credit-risk factor $y_{2,t}$. For parsimony, we assume independent and identically distributed credit risk Brownian innovations and use σ_y to capture the risk magnitude while extending the independence assumption $\mathbb{E}[dW_t^i dW_t^j] = 0$ for all $i \neq j$. We further assume constant and identical market price for the two credit Brownian risks γ_y . Thus, under the risk-neutral measure \mathbb{Q} , the drift of each of the two factors will be adjusted by $-\gamma_y \sigma_y$.

Existing studies often capture the interaction between interest rates and credit spreads through a direct loading of the instantaneous default rate on the instantaneous interest rate (e.g., Feldhütter and Lando (2008) and Frühwirth, Schneider, and Sögner (2010)),

$$\lambda_t = br_t + y_t, \quad (16)$$

where b is often set negative given the empirical evidence (Duffee (1998)). By contrast, our specification in (14) recognizes the fact that different frequency components of the interest-rate movements can have different impacts on the term structure of credit spreads. Furthermore, Equation (15) incorporates another layer of dynamic interaction absent from existing specifications in the literature. Together, our specifications allow the interest-rate factors to both impact the default arrival rate contemporaneously through the loading coefficients β and affect subsequent changes in the credit-risk factors through the interaction coefficients κ_{yx} . For model estimation, we consider both the two-factor specification in (15) and a one-factor special case by setting $y_{2,t} = \theta_y$.

We can write the joint risk-neutral dynamics of the interest-rate and credit risk factors in matrix form as,

$$Z_t \equiv \{ \{x_{j,t}\}_{j=1}^3, \{y_{k,t}\}_{k=1}^2 \}, \quad dZ_t = (C - \kappa Z_t) dt + \sqrt{\Sigma} dW_t^{\mathbb{Q}}, \quad (17)$$

where the constant vector is given by $C = [C_X^\top, -\gamma_y \sigma_y, \kappa_{y,2} \theta_y - \gamma_y \sigma_y]^\top$, the covariance matrix Σ is diagonal with the first three diagonal elements being σ_r^2 and the remaining two diagonal elements being σ_y^2 , and the

mean-reverting matrix κ is given by

$$\kappa = \begin{bmatrix} \kappa_r s_\kappa^2 & -\kappa s_\kappa^2 & 0 & 0 & 0 \\ 0 & \kappa_r s_\kappa & -\kappa_r s_\kappa & 0 & 0 \\ 0 & 0 & \kappa_r & 0 & 0 \\ \kappa_{yx,1}^1 & \kappa_{yx,1}^2 - \kappa_{yx,1}^1 & \kappa_{yx,1}^3 - \kappa_{yx,1}^2 & \kappa_{y,1} & -\kappa_{y,1} \\ \kappa_{yx,2}^1 & \kappa_{yx,2}^2 - \kappa_{yx,2}^1 & \kappa_{yx,2}^3 - \kappa_{yx,2}^2 & 0 & \kappa_{y,2} \end{bmatrix}. \quad (18)$$

With this compact matrix notation, the present value of the premium leg of the CDS contract becomes,

$$\text{Premium}(Z_t, \tau) = \mathbb{E}_t \left[S(Z_t, \tau) \int_0^\tau \exp \left(- \int_t^{t+s} (b_Z^\top Z_u) du \right) ds \right], \quad (19)$$

with $b_Z = [(b_r + \beta)^\top, 1, 0]^\top$. The solution is exponential affine in the state vector Z_t ,

$$\text{Premium}(Z_t, \tau) = S(Z_t, \tau) \int_0^\tau \exp \left(-a(s) - b(s)^\top Z_t \right) ds, \quad (20)$$

where the coefficients $a(s)$ and $b(s)$ are determined by the following ordinary differential equations:

$$a'(s) = b(s)^\top C - \frac{1}{2} b(s)^\top \Sigma b(s), \quad b'(s) = b_Z - \kappa^\top b(s), \quad (21)$$

subject to the boundary conditions $a(0) = 0$ and $b(0) = 0$.

The present value of the protection leg becomes,

$$\text{Protection}(Z_t, \tau) = \mathbb{E}_t \left[w(t, \tau) \int_0^\tau \left(d_Z^\top Z_{t+s} \right) \exp \left(- \int_t^{t+s} (b_Z^\top Z_u) du \right) ds \right], \quad (22)$$

with $d_Z = [\beta^\top, 1, 0]^\top$. The solution is,

$$\text{Protection}(Z_t, \tau) = w(t, \tau) \int_0^\tau \left(c(s) + d(s)^\top Z_t \right) \exp \left(-a(s) - b(s)^\top Z_t \right) ds, \quad (23)$$

where the coefficients $[a(s), b(s)]$ are determined by the ordinary differential equations in (21) and the coefficients $[c(s), d(s)]$ are determined by the following ordinary differential equations:

$$c'(s) = d(s)^\top \theta - b(s)^\top \sigma d(s), \quad d'(s) = -\kappa^\top d(s), \quad (24)$$

with $c(0) = 0$ and $d(0) = d_Z$. The CDS spread can then be solved by equating the values of the two legs,

$$S(Z_t, \tau) = \frac{w(t, \tau) \int_0^\tau (c(s) + d(s)^\top Z_t) \exp(-a(s) - b(s)^\top Z_t) ds}{\int_0^\tau \exp(-a(s) - b(s)^\top Z_t) ds}. \quad (25)$$

4. Estimation Strategy

We estimate the dynamics of benchmark interest-rate risk and credit risk in two consecutive steps, all using a quasi-maximum likelihood method. At each step, we cast the models into a state-space form, obtain forecasts on the conditional mean and variance of observed interest rates and credit default swap spreads using a nonlinear filtering technique, and build the likelihood function on the forecasting errors of the observed series, assuming that the forecasting errors are normally distributed. The model parameters are estimated by maximizing the likelihood function.

In the first step, we estimate the interest-rate factor dynamics using libor and swap rates. In the state-space form, we regard the interest-rate factors (X) as the unobservable states and specify the state-propagation equation using an Euler approximation of the statistical dynamics of the interest-rate factors in equation (5):

$$X_t = A_X + \Phi_X X_{t-1} + \sqrt{Q_X} \varepsilon_{xt}, \quad (26)$$

where ε_{xt} denotes a three-dimensional i.i.d. standard normal innovation vector and $\Phi_X = \exp(-\kappa_X \Delta t)$, $A_X = (I - \Phi_X) \mathbf{e} \theta_r$, and $Q_X = I \Delta t \sigma_r^2$, with I denoting an identity matrix, \mathbf{e} denoting a vector of ones, and $\Delta t = 1/252$ denoting the daily frequency. The measurement equations are constructed based on the observed libor and swap rates, assuming additive, normally-distributed measurement errors,

$$m_t = \begin{bmatrix} LIBOR(X_t, i) \\ SWAP(X_t, j) \end{bmatrix} + e_t, \quad cov(e_t) = \mathcal{R}, \quad \begin{array}{l} i = 12 \text{ months,} \\ j = 2, 3, 5, 7, 10 \text{ years.} \end{array} \quad (27)$$

In the second step, we take the estimated interest-rate factor dynamics in the first step as given, and estimate the credit-risk factor dynamics (Y) at each industry sector and credit rating class using the six average credit default swap spread series for each group. The state-propagation equation is an Euler approximation of statistical dynamics of the credit-risk factors,

$$Y_t = A_Y + \Phi_Y Y_{t-1} + \kappa_{yx} X_{t-1} \Delta t + \sqrt{Q_Y} \varepsilon_{yt}, \quad (28)$$

with $\Phi_Y = \exp(-\kappa_Y \Delta t)$, $\kappa_Y = [\kappa_{y,1}, -\kappa_{y,1}; 0, \kappa_{y,2}]$, $A_Y = (I - \Phi_Y) \mathbf{e} \theta_y$, and $Q_Y = I \Delta t \sigma_y^2$. The measurement

equations are defined on the CDS spreads at the six maturities,

$$m_t = S^i(Z_t, \tau) + e_t, \quad \text{cov}(e_t) = \mathcal{R}, \tau = 1, 2, 3, 5, 7, 10 \text{ years}, \quad (29)$$

where $i = 1, 2, 3, 4$ denotes the i th industry sector and credit rating class. We repeat this step eight times, for both one and two credit risk factors and for each of two industry sectors and two credit rating classes.

Given the definition of the state-propagation equation and measurement equations at each step, we use an extended version of the Kalman filter to filter out the mean and covariance matrix of the state variables conditional on the observed series, and construct the predictive mean and covariance matrix of the observed series based on the filtered state variables. Then, we define the daily log likelihood function assuming normal forecasting errors on the observed series:

$$l_t(\Theta) = -\frac{1}{2} \log |\bar{V}_t| - \frac{1}{2} \left((m_t - \bar{m}_t)^\top (\bar{V}_t)^{-1} (m_t - \bar{m}_t) \right), \quad (30)$$

where \bar{m} and \bar{V} denote the conditional mean and variance forecasts on the measurement series, respectively. The model parameters, Θ , are estimated by maximizing the sum of the daily log likelihood values,

$$\Theta \equiv \arg \max_{\Theta} \mathcal{L}(\Theta, \{m_t\}_{t=1}^N), \quad \text{with} \quad \mathcal{L}(\Theta, \{m_t\}_{t=1}^N) = \sum_{t=1}^N l_t(\Theta), \quad (31)$$

where $N = 1, 142$ denotes the number of observations for each series. For each step, we assume that the measurement errors on each series are independent and identically distributed.

5. Term Structure of Interest Rates and Credit Spreads

First, we discuss the estimated dynamics and the term structure behavior of the benchmark interest rates. Then, we analyze how the interest-rate factors interact with the default risk to determine the CDS term structure behavior in each industry sector and rating class.

5.1. DYNAMICS AND TERM STRUCTURE OF BENCHMARK INTEREST RATES

Table III reports the summary statistics of the pricing errors on the libor and swap rates from the three-factor cascade term structure model. Using three factors to explain six interest-rate time series, the model is able to achieve a near-perfect fitting on the interest-rate term structures. The mean pricing errors average close to zero, and the standard deviation of the pricing errors average just about one basis point. Over the whole

sample of 1,142 days, the largest pricing error is merely 7.55 basis points on the two-year swap rate. The last column of Table III reports the explained variation on each series, defined as one minus the ratio of pricing error variance to the variance of the original data series. The explained variation averages at 99.98%.

[Table III about here.]

The near-perfect fitting of the term structure model provides a solid starting point for pricing the credit default swaps in the next step. As shown in the pricing relation in (4), mispricing in interest rates can potentially generate distortions in the pricing of the credit default swaps. Therefore, it is important to start with a near-perfect pricing on the benchmark interest rate to avoid distortions on the estimated credit-risk dynamics.

Table IV reports the estimates and the standard errors (in parentheses) on the five parameters that govern the dynamics and term structure of the benchmark libor and swap rates. Due to the extreme parsimony of the specification, all five parameters are estimated with high statistical significance. The mean-reversion speed of the lowest-frequency interest-rate factor is $\kappa_r = 0.0788$. The reciprocal of the mean-reversion speed has the unit of time (in years). The longer the time, the slower the mean reversion is. This lowest frequency component corresponds to a frequency cycle of $1/0.0788 = 12.7$ years. The power scaling coefficient between the different mean-reversion speeds is estimated at $s_\kappa = 5.0316$, implying mean-reversion speeds of 0.3963 and 1.9941 for the two higher frequencies, respectively. Thus, the highest frequency corresponds to a cycle of half a year whereas the middle frequency corresponds to a cycle of 2.5 years. Intuitively, these different frequency components control the interest-rate behavior at different segments of the yield curve.

[Table IV about here.]

To better understand the effects of the different frequency components on the yield curve, it is useful to write the instantaneous forward rate curve as a function of the three frequency components,

$$f(X_t, \tau) \equiv -\frac{\partial \ln P(X_t, \tau)}{\partial \tau} = a'(\tau) + b'(\tau)^\top X_t, \quad (32)$$

which follows from the bond pricing solution in (10). The loading coefficients $b'(\tau) = e^{-\kappa_X^\top \tau} b_r$ capture the contemporaneous responses of the instantaneous forward rate curve to the three interest-rate frequency components. The response function of the forward rate curve is purely determined by the mean-reversion matrix of the interest-rate factors (κ_X) and the linkage between the short rate and the factors (b_r). Figure 3 plots the contemporaneous response function computed based on the coefficient estimates. The solid line

denotes the loading of the highest frequency component $x_{1,t}$, which starts at one at zero maturity as we set the instantaneous interest rate to this component, $r_t = x_{1,t}$, but the loading declines quickly as the forward rate maturity increases. The dashed line denotes the loading of the intermediate frequency component $x_{2,t}$, which shows a hump shape that peaks around one-year maturity. The dash-dotted line depicts the loading of the lowest frequency component $x_{3,t}$ on the forward rate curve, which also shows a humped shape and peaks around 5.6 years. Under the cascade structure, Calvet, Fisher, and Wu (2010) show that the loading patterns are hump-shaped for all but the highest frequency component, with the humps peaking at longer maturities for slower frequency components.

[Figure 3 about here.]

Table IV reports a long-run mean estimate for the short rate at 1.57%, and an instantaneous volatility for the three risk factors at 1.61%. The market price of risk is estimated to be negative at $\gamma_r = -0.2372$, which contributes to the observed upward sloping mean term structure. In the absence of a risk premium, the convexity effect drags the long-term rate lower than the short-term rate on average. A negative market price helps to overcome the convexity effect to generate the upward sloping mean yield curve observed in the data.

5.2. DEFAULT ARRIVAL DYNAMICS AND THE TERM STRUCTURE OF CREDIT SPREADS

We estimate two models for the CDS term structure at each industry sector and rating class, one with one credit-risk factor, and the other allowing two credit-risk factors, where the first factor $y_{1,t}$ mean reverts to a lower-frequency stochastic tendency $y_{2,t}$. Tables V reports the summary statistics of the pricing errors on the CDS spreads at each sector and rating class and for each model. The model with one credit-risk factor can price the intermediate maturity CDS spread reasonably well, as the explained variation for the three-year CDS ranges from 96% to 97%, but the model performance deteriorates at the two ends of the CDS curve. On average, the explained variation ranges from 85.91% for the Financial/BBB classification to 92.59% for the Financial/A classification.

[Table V about here.]

The specification with two credit-risk factors allows separate variations for the instantaneous default arrival rate and its long-term tendency, and accordingly, separate variations for the short- and long-term CDS spreads. As a result, the two-factor model can price the CDS term structure well at both short and long maturities. The lowest explained variation is over 95% and the highest explained variation is over 99%. The

performance comparison suggests that it is important to allow separate variations for short- and long-term CDS spreads to accommodate both the level and the slope changes in the CDS term structure.

Tables VI and VII report the parameter estimates and standard errors on the dynamics of the default arrival rate and its interactions with the interest-rate factors for each industry sector and credit rating class. Tables VI reports the estimates for the one-factor credit-risk specification. Tables VII reports the estimates for the two-factor credit-risk specification. The estimates in the two tables reveal intricate dynamic interactions between the credit-risk factors and the interest-rate factors. The three β coefficients measure the contemporaneous response of the default arrival rate to the three frequency components in the interest-rate movements, whereas the κ_{xy} matrix captures the predictive power of interest-rate factors on the default-risk factors. Under the one-factor credit-risk specification, the most significant contemporaneous response comes from the highest-frequency interest-rate factor (β_1), whereas the most significant predictive impact comes from the lowest-frequency long-term trend in the interest-rate movement (κ_{yx}^3).

[Table VI about here.]

Under the two-factor credit-risk specification, where the short- and long-term CDS spreads are allowed to vary separately, the interactions become more complicated, and more contemporaneous loading coefficients become statistically significant. For the predictive coefficients κ_{yx} , the high-frequency interest-rate factor predicts the high-frequency credit risk factor better as $\kappa_{yx,1}^1$ has more significant estimates than does $\kappa_{yx,2}^1$. By contrast, the low-frequency interest-rate factor predicts the low-frequency credit-risk factor better as $\kappa_{yx,2}^3$ has more significant elements than does $\kappa_{yx,1}^3$.

[Table VII about here.]

Early studies have identified negative contemporaneous relations between credit spreads and short-term interest rates. Tables VI and VII show that the loading coefficient estimates β often show different signs for the three interest-rate factors and that the signs can switch depending on the number of credit-risk factors. The estimates on the predictive matrix κ_{xy} also vary depending on the particular frequency components and the industry sector/credit rating classification. Thus, a simple instantaneous loading on the short rate represents an over simplification.

When we allow two credit-risk factors, the higher-frequency component has a mean-reversion speed estimate $\kappa_{y,1}$ between 0.13 to 0.27, corresponding to a cycle of 4-8 years. By contrast, the mean-reversion speed estimates for the lower-frequency component are close to zero, suggesting near random walk behavior for the stochastic trend of the default arrival rate. The low-mean reversion speed, coupled with the negative

market price of credit risk estimates for all industry sectors and rating classes, contributes to the steeply upward sloping CDS term structure observed in the data.

The instantaneous volatility estimates are fairly stable across different industry sector and rating classes, and the long-run mean estimates for the default arrival rate θ_y are lower for the A rating class than for the BBB rating class, reflecting the average CDS level difference between the two rating classes.

To understand how the different interest-rate and credit-risk factors interact with one another to influence the credit spread term structure, we can look into the behavior of the loading coefficients $d(s)$ in the CDS spread solution in (25). The coefficients can be interpreted as the contemporaneous response of the forward default arrival rate curve to the interest-rate and credit-risk factors Z_t . Figure 4 plots the loading coefficients for the four industry sector and credit rating classifications. The panels on the left hand side capture the contributions from the interest-rate factors, whereas the panels on the right hand side capture the contributions from the two credit-risk factors. Each row corresponds to one industry sector and credit rating classification. To make the responses to the interest-rate and credit-risk factors comparable, we multiply the first three elements of the coefficients $d(s)$ by $\sigma_r \times 10000$ and the last two elements by $\sigma_y \times 10000$, so that each line represents the response (in basis points) of the forward default arrival rate to one standard deviation shock from each factor. In each panel, the frequencies decline from the solid line to the dashed line and then to the dash-dotted line.

[Figure 4 about here.]

The plots on the left hand side show that interest-rate movements can have significant contributions to the credit spread term structure. The exact shape of the response function, however, varies across different industry sector and credit rating classifications. A positive interest-rate shock can induce either narrowing or widening of the credit spread, depending on where the interest-rate shock comes from (in terms of interest-rate maturity) and where the credit spread is measured (in terms of both the credit spread maturity and the industry sector and credit rating classification).

On the other hand, the responses of the credit spreads to the credit-risk factors are all positive and the term structure of the response is mainly determined by the relative persistence of the two credit-risk factors. The responses to the high-frequency credit-risk component start at a positive level by design and decline exponentially with increasing maturity. By contrast, the responses to the stochastic trend in the credit risk start at zero and increase gradually as the maturity increases.

6. Two-way Dynamic Interactions and Joint Estimation

To capture the interactions between market-wide interest-rate movements and credit-spread variations at each industry sector and credit rating class, we first project the credit spread of each classification onto the interest-rate factors and then allow the dynamics of the orthogonalized credit-risk factors to depend on the interest-rate factors. This one-way interaction specification allows separate estimation of the benchmark interest-rate term structure from the estimation of the credit spread term structure for different industry sectors and credit rating classes. The sequential procedure can be readily applied to CDS term structures on individual reference companies.

Nevertheless, it is reasonable to think that monetary policies and therefore the term structure of the benchmark interest rates can also respond to the aggregate credit condition of the market. To capture such consideration, one needs to incorporate two-way dynamic interactions between the interest-rate factors and systematic, market-wide credit-risk variations.

To analyze the two-way dynamic interactions, we first average CDS spreads across all four industry sector and credit rating classes to obtain a market-wide CDS term structure. Then, we allow two-way interactions between the benchmark interest-rate factors and the default arrival rates underlying this market-wide credit spread movements. We maintain the same contemporaneous projection,

$$r_t = x_{1,t}, \quad \lambda_t = \beta^\top x_t + y_{1,t},$$

but allow two-way dynamic interactions between the interest-rate factors and the credit risk factors,

$$dx_{j,t} = \kappa_r \delta_\kappa^{(3-j)} (x_{j+1,t} - x_{j,t}) dt + \sum_{i=1}^2 \kappa_{xy,i}^j (y_{i+1,t} - y_{i,t}) dt + \sigma_r dW_t^j, \quad j = 1, 2, 3, \quad (33)$$

$$dy_{i,t} = \kappa_{y,i} (y_{i+1,t} - y_{i,t}) dt + \sum_{j=1}^3 \kappa_{yx,i}^j (x_{j+1,t} - x_{j,t}) dt + \sigma_y dW_t^4, \quad i = 1, 2, \quad (34)$$

with $x_4 = \theta_r$ and $y_3 = \theta_y$. In the above specification, κ_{xy} captures the prediction of the two credit-risk factors on the three interest-rate factors whereas κ_{yx} capture the prediction of the three interest-rate factors on the two credit-risk factors. In matrix notation, we can write the factor dynamics, $Z_t \equiv \{ \{x_{j,t}\}_{j=1}^3, \{y_{k,t}\}_{k=1}^2 \}$, under the statistical measure \mathbb{P} as,

$$dZ_t = \kappa(\theta - Z_t) dt + \sqrt{\Sigma} dW_t, \quad (35)$$

where the long-run mean vector $\theta = [\theta_r, \theta_r, \theta_r, \theta_y, \theta_y]^\top$ and the diagonal covariance matrix $\Sigma = \langle \sigma_r^2, \sigma_r^2, \sigma_r^2, \sigma_y^2, \sigma_y^2 \rangle$

remain the same as before. The new mean-reverting matrix becomes,

$$\kappa = \begin{bmatrix} \kappa_r s_\kappa^2 & -\kappa s_\kappa^2 & 0 & \kappa_{xy,1}^1 & \kappa_{xy,2}^1 - \kappa_{xy,1}^1 \\ 0 & \kappa_r s_\kappa & -\kappa_r s_\kappa & \kappa_{xy,1}^2 & \kappa_{xy,2}^2 - \kappa_{xy,1}^2 \\ 0 & 0 & \kappa_r & \kappa_{xy,1}^3 & \kappa_{xy,2}^3 - \kappa_{xy,1}^3 \\ \kappa_{yx,1}^1 & \kappa_{yx,1}^2 - \kappa_{yx,1}^1 & \kappa_{yx,1}^3 - \kappa_{yx,1}^2 & \kappa_{y,1} & -\kappa_{y,1} \\ \kappa_{yx,2}^1 & \kappa_{yx,2}^2 - \kappa_{yx,2}^1 & \kappa_{yx,2}^3 - \kappa_{yx,2}^2 & 0 & \kappa_{y,2} \end{bmatrix}. \quad (36)$$

By maintaining the same market price of risk assumption, we can write the risk-neutral factor dynamics as,

$$dZ_t = (C - \kappa Z_t) dt + \sqrt{\Sigma} dW_t^\mathbb{Q}, \quad (37)$$

where the constant term in the risk-neutral drift is given by,

$$C = \begin{bmatrix} \kappa_{xy,2}^1 \theta_y - \gamma_r \sigma_r \\ \kappa_{xy,2}^2 \theta_y - \gamma_r \sigma_r \\ \kappa_{xy,2}^3 \theta_y + \kappa_r \theta_r - \gamma_r \sigma_r \\ \kappa_{yx,1}^3 \theta_r - \gamma_y \sigma_y \\ \kappa_{yx,2}^3 \theta_r + \kappa_{y,2} \theta_y - \gamma_y \sigma_y \end{bmatrix}. \quad (38)$$

The bond pricing and CDS valuation take similar forms and the coefficients solve analogous ordinary differential equations, only with variations in the elements of the matrix κ and the vector C .

With the two-way interactions, we estimate the interest-rate and credit-risk dynamics jointly using libor and swap rates as well as the average CDS spreads. In this case, the state propagation is constructed based on an Euler approximation of the statistical factor dynamics in (35) and the measurement equations include both the six libor and swap rate series and the six average CDS spread series. Table VIII reports the summary statistics of the pricing errors on the interest rates and CDS spreads from the joint estimation. The five-factor model explains all interest-rate series over 99.9%, with an average of 99.98%. The model explains all CDS spread series over 96%, with an average of 98.58%. By allowing two-way interactions, the performance on the benchmark interest rates becomes slightly better than the original three-factor specification. The performance on the CDS spreads is also better than the performance from the two-stage estimation on each of the four industry sector and credit rating classifications.

[Table VIII about here.]

Table IX reports the model parameter estimates and standard errors from this joint estimation. Incorporating two-way interactions leads to some rotations of the factors. As a result, the frequencies of the interest-rate factors spread further apart with the scaling coefficient becoming three times as large at $s_{\kappa} = 16.839$ and the lowest frequency component becoming even more persistent at $\kappa_r = 0.0067$, corresponding to a frequency cycle of 150 years. On the other hand, the frequencies of the two credit-risk factors become closer to each other, with the mean-reversion speeds at 0.18 and 0.16, respectively. Through further cross-sectional averaging, the instantaneous volatility estimate for the credit-risk factors becomes lower at $\sigma_y = 0.0006$, while the instantaneous volatility for the interest-rate factors becomes larger at $\sigma_r = 0.0206$. The estimates for the market prices of both interest-rate risks and credit risks remain negative.

[Table IX about here.]

The last panel in Table IX reports the parameters that govern the dynamic interactions between the interest-rate and credit-risk factors. Contemporaneously, the highest frequency component of the interest-rate movement loads positively on the default rate, but the two lower frequency interest-rate factors load negatively on the default rate. Predictively, the two credit-risk factors generate significant predictions on the interest rate movements at both the short and the long term, more so at the short term. The interest-rate factors, especially the intermediate frequency component, also predict strongly on the movements of the two credit-risk factors. Therefore, the estimates suggest intricate dynamic interactions between the interest-rate and the credit-risk markets.

To see how the interest-rate and credit-risk factors affect the benchmark interest-rate term structure, Figure 5 plots the contemporaneous responses of the instantaneous forward interest rate curve to the five interest-rate and credit risk factors. The three lines in the left panel plot the responses to the three interest-rate factors. The two lines in the right panel plot the responses to the two credit-risk factors. In each panel, the frequency declines from the solid line, to the dashed line, and then to the dash-dotted line. Again, to make the responses comparable for the two sets of factors, we multiply the first three elements of $b'(\tau)$ by $\sigma_r \times 10000$ and multiply the last two elements of $b'(\tau)$ by $\sigma_y \times 10000$ so that each line represents the forward interest-rate curve response in basis points to one standard deviation movements in each of the five factors. In the left panel, the highest frequency interest-rate factor starts at σ_r at zero maturity by design and declines rapidly as the forward rate maturity increases. However, through interactions with the credit-risk dynamics, the response reaches a bottom at 1.96-year maturity and starts to go up again after that. The responses to the other two frequency components both start at zero by design, and generate hump-shaped term structures for the response, which peak at 1.35 and 4.58 years, respectively. In the right panel, the two credit-risk factors

have zero contribution to the instantaneous interest rate, but non-zero contribution to forward rates at other maturities. Due to the dynamic interactions, the contributions start positively, but then become negative at longer maturities, more so for the lower-frequency credit-risk factor. The response functions suggest that a systematic widening of credit spreads and hence a worsening of the credit condition in the market, especially at longer terms, tend to be associated with subsequent easing in monetary policy and hence lowering of the forward rates.

[Figure 5 about here.]

To see how the interest-rate and credit-risk factors affect the average credit spread term structure, Figure 6 plots the contemporaneous responses of the forward default arrival rate curve to the five interest-rate and credit risk factors. Similar to the layout in Figure 5, the three lines in the left panel plot the responses to the three interest-rate factors, and the two lines in the right panel plot the responses to the two credit-risk factors. In each panel, the frequency declines from the solid line, to the dashed line, and then to the dash-dotted line. Again, to make the responses comparable for the two sets of factors, we multiply the first three elements of $d(s)$ by $\sigma_r \times 10000$ and multiply the last two elements of $d(s)$ by $\sigma_y \times 10000$ so that each line represents the forward default rate response in basis points to one standard deviation movements in each of the five factors.

[Figure 6 about here.]

In the left panel, the loadings of the three interest-rate factors at zero credit-spread maturity are determined by the estimates of the contemporaneous coefficient β . At longer maturities, the dynamic interactions also play a role. Overall, the credit spread curve responds negatively to the high-frequency and hence short-term interest-rate shocks, but positively to the low-frequency and hence long-term forward rate shocks.

In the right panel, the loadings of the two credit-risk factors on the credit spread term structure are largely governed by the mean-reversion speeds of each risk factor. The loading to the high-frequency factor (solid line) starts at σ_y by design and declines monotonically as maturity increases. The loading to the stochastic tendency (dashed line) starts at zero and shows a hump-shaped term structure that peaks at 5.3 years.

7. Sub-sample Analysis

To gauge the stability of the dynamic interactions between the interest rate and credit markets, we perform estimation on sub-samples of the data. We divide the sample into two equal-length periods, with the first

sample from May 21, 2003 to July 28, 2005, and the second sample from July 29, 2005 to October 8, 2007. Each sub-sample spans 571 business days. In this analysis, we focus on the joint estimation of the two-way interaction specified in Section 6. From the time-series plots in Figures 1 and 2, we observe distinct behaviors for both interest rates and credit spreads during the two sub-sample periods. During the first half of the sample, the interest-rate term structure is steep and the short-term interest rate is trending upward while the long-term rate stays within a narrower band. The CDS spreads have a declining trend. During the second half, the interest-rate term structure is much more flat and the rates move within a much narrower range. On the other hand, the CDS spreads have a steeper term structure but experience less intertemporal movements.

Table X reports the parameter estimates during the two sub-sample periods. The different interest-rate and CDS behaviors during the two sub-samples lead to different parameter estimates. The estimate for σ_y in the second half is about half as much as the estimate for the first half, reflecting the smaller intertemporal CDS movements in the second half. On the other hand, the second-half estimates for the mean-reversion speeds are smaller, and the estimate for the market price of credit risk is larger to accommodate the steeper CDS term structure. The opposite is true for the interest-rate term structure, as the second-half estimate for the market price estimate is smaller in line with the flatter interest-rate term structure in the second half. The estimates for the interactive coefficients also show variations over the two sample periods.

[Table X about here.]

To see how the parameter variations affect the response functions, Figure 7 plots the responses of the forward interest-rate curve to one standard deviation shocks in each of the five factors. The top two panels denote the response functions estimated from the first half of the sample, whereas the bottom two panels plot the response functions estimated from the second half. While there are quantitative differences, the patterns of the response functions also show strong similarities across the two sample periods. The responses to the three interest-rate factors are largely driven by their corresponding mean-reversion speeds, as we have observed from Figure 5 based on the full-sample estimation. The responses to the two credit-risk factors start at zero, but become negative at long maturities, more so for the stochastic trend factor, under both time periods.

[Figure 7 about here.]

Figure 8 plots the responses of the forward default arrival rate to one standard deviation shocks in each of the five factors based on parameter estimates from the two sub-sample periods. Again, despite variations

in parameter estimates, the general patterns of the interactions are similar across the two sub-samples. The responses to the two credit-risk factors are mainly dictated by their respective mean-reversion speeds. The responses to the highest-frequency interest-rate factor are largely negative at both sample periods, whereas the responses to the lowest-frequency interest-rate factor are mostly positive.

[Figure 8 about here.]

8. Conclusion

Exploiting information in the credit default swap term structure, we study how default risk interacts with interest-rate risk to determine the term structure of credit spreads. To analyze the interaction between market-wide interest-rate movements and industry/rating specific credit-spread changes, we project the credit spreads onto the interest-rate factors and further allow the projection residuals to interact dynamically with the interest-rate factors. As a result, interest-rate factors not only affect contemporaneous credit-spread movements, but also predict future credit-risk dynamics. In line with this one-way interaction, we propose a sequential estimation procedure, in which the first step identifies the dynamics of the benchmark interest-rate factors and the following steps identify the dynamics of the default arrival rate for each industry sector and credit rating class. This procedure can be readily applied to CDS term structure analysis on individual reference companies. On the other hand, to analyze potential two-way interactions between the interest-rate term structure and market-wide credit conditions, we also propose a joint identification procedure for the interest rates and market-average CDS spreads that can accommodate two-way dynamic interactions between interest-rate and credit-risk factors.

The estimation results show that the two markets present intricate dynamic interactions. Different frequency components in the interest-rate movements impact the CDS term structure differently at different industry sectors and credit-rating classes. When we analyze the two-way interactions between interest rates and market-average CDS spreads, we find that the aggregate credit condition of the market can also influence the monetary policy and hence the benchmark interest-rate curve. In particular, worsening of the credit condition, represented by a widening of credit spreads, especially at long maturities, tends to lead to future easing in monetary policy and accordingly lowering of the current forward interest rate curve. On the other hand, positive shocks to the instantaneous interest rate narrow the credit spread at long maturities, whereas positive shocks to long-term interest rates widen the credit spreads. These results highlight the intricate interactions between the two markets.

Appendix: Performance Comparison to a General Gaussian Affine Model

Our term structure model is an extremely parsimonious specification that belongs to the general Gaussian-affine class. Compared to a general, unrestricted specification, we apply structural constraints that completely remove factor rotation and make the factors economically meaningful as they are ranked by the frequencies of the shocks. The extreme parsimony also allows us to identify the parameters with strong statistical significance, thus mitigating the commonly experienced identification issues. As shown by Bikbov and Chernov (2004), the pricing performances of most three-factor affine models are similar. We expect our structural constraints to improve model identification and factor interpretability while without deteriorating the pricing performance on the interest rate term structure.

To verify this expectation, we estimate a general three-factor Gaussian affine specification in this appendix. Let $X_t \in \mathbb{R}^3$ denote the three-dimensional state vector. We specify the instantaneous interest rate $r(X_t)$ as a general affine function of the state vector,

$$r(X_t) = a_r + b_r^\top X_t. \quad (39)$$

We further specify the factor dynamics as following a general Ornstein-Uhlenbeck process under the statistical measure \mathbb{P} ,

$$dX_t = -\kappa X_t dt + dW_t. \quad (40)$$

We allow an affine market price of risk as in Duffee (2002),

$$\gamma(X_t) = \lambda_1 + \lambda_2 X_t. \quad (41)$$

For identification, we normalize the state vector to have a zero long-run statistical mean and an identity instantaneous covariance matrix, we restrict the affine loading coefficient to be positive $b_r \geq 0$, and we further restrict the mean-reverting matrix κ and the market price matrix λ_2 to be lower triangular. The model has 19 free parameters compared to five in our specification.

Under the general specification, the value for zero-coupon bonds retains the exponential-affine form of equation (10), where the affine coefficients satisfy the following ordinary differential equations,

$$a'(\tau) = a_r - b(\tau)^\top \lambda_1 - b(\tau)^\top b(\tau)/2, \quad b'(\tau) = b_r - (\kappa + \lambda_2)^\top b(\tau), \quad (42)$$

starting at $a(0) = 0$ and $b(0) = 0$.

We estimate the general specification on the same set of libor and swap rates over the same sample period. Table XI reports the parameter estimates on the general Gaussian-affine model. The maximized log likelihood is 20,475, higher than the maximized log likelihood from our parsimonious specification at 20,450. Given the large amount of observation (1142 days), the likelihood ratio difference is statistically significant. Nevertheless, when we compute the summary statistics on the pricing errors from this general specification, as shown in Table XII, the pricing performance is very much the same as that from our parsimonious specification in Table III. The average fitting error is actually slightly larger for the general specification. The indistinguishable pricing performance comes from two main reasons. First, the pricing performance is mainly determined by the number of factors, which are allowed to vary with the interest rates, rather than the number of parameters, which are fixed over the whole sample period. The two models have the same number of factors, but only differ in number of parameters. Second, the restricted model already generates near-perfect fitting (see Table III), the extra parameters cannot possibly add much as a result.

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Table I. Summary statistics of credit default swap spreads

Entries report the summary statistics of the average CDS spreads in basis points at four industry sector and credit rating classifications. The statistics include the sample average (Mean), standard deviation (Std), skewness (Skew), excess kurtosis (Kurtosis), and daily autocorrelation (Auto). The last row in each panel reports the summary statistics of the expected recovery rate. Data are daily from May 21, 2003 to October 8, 2007, 1,142 observations for each series.

Maturity	Mean	Std	Skew	Kurtosis	Auto
Sector: Financial; Rating: A					
1	11.96	6.35	1.28	1.25	0.984
2	15.96	6.81	0.97	0.16	0.990
3	20.02	7.18	0.87	-0.03	0.991
5	28.42	8.15	0.57	-0.54	0.993
7	33.84	8.40	0.51	-0.50	0.990
10	41.62	9.21	0.25	-0.59	0.989
R	39.48	0.85	0.42	1.78	0.967
Sector: Corporate; Rating: A					
1	9.19	5.20	0.97	-0.48	0.995
2	12.57	5.34	0.90	-0.42	0.996
3	15.85	5.10	0.81	-0.26	0.995
5	23.17	4.20	0.28	-0.38	0.994
7	29.15	3.67	0.15	-0.55	0.990
10	36.83	3.83	0.39	-0.25	0.984
R	39.54	0.77	0.80	1.86	0.978
Sector: Financial; Rating: BBB					
1	18.57	9.76	1.13	0.40	0.993
2	24.89	8.91	1.11	0.81	0.993
3	32.14	8.73	0.85	0.54	0.993
5	47.33	8.53	0.90	0.88	0.989
7	54.68	7.75	0.74	0.48	0.987
10	64.07	8.25	1.03	1.85	0.982
R	39.58	1.11	0.99	1.83	0.983
Sector: Corporate; Rating: BBB					
1	18.53	11.59	1.16	0.09	0.994
2	24.25	10.98	1.10	0.04	0.994
3	30.86	10.98	1.17	0.48	0.994
5	44.16	8.90	1.17	0.94	0.992
7	52.60	6.75	0.69	0.21	0.989
10	62.78	6.05	0.27	-0.04	0.987
R	39.58	0.82	0.56	1.98	0.978

Table II. Summary statistics of libor and swap rates

Entries report the summary statistics of the eurodollar libor at one-year maturity and swap rates at two, three, five, seven, and ten years. Mean, Std, Skew, Kurtosis, and Auto denote the sample estimates of the mean, standard deviation, skewness, excess kurtosis, and the first-order daily utocorrelation, respectively. Data are daily from May 21, 2003 to October 8, 2007, 1,142 observations for each series.

Maturity	Mean	Std	Skew	Kurtosis	Auto
1	3.73	1.60	-0.37	-1.46	0.998
2	3.95	1.29	-0.46	-1.23	0.997
3	4.15	1.06	-0.48	-1.02	0.996
5	4.47	0.75	-0.44	-0.58	0.994
7	4.70	0.58	-0.39	-0.18	0.992
10	4.93	0.45	-0.32	0.19	0.988

Table III. Summary statistics of pricing errors on the libor and swap rates

Entries report the summary statistics of the pricing errors on the eurodollar libor and swap rates under the three-factor cascade term structure model. We define the pricing error as the difference between the observed interest-rate quotes and the model-implied fair values, in basis points. The columns titled Mean, Std, Auto, Max, and VR denote, respectively, the sample mean, the standard deviation, the first-order daily autocorrelation, the maximum absolute error, and the explained percentage variance, defined as one minus the ratio of pricing error variance to interest-rate variance, in percentages.

Maturity	Mean	Std	Auto	Max	VR
1	-0.02	1.15	-0.06	4.43	99.99
2	0.20	1.57	0.53	7.55	99.99
3	-0.31	0.90	0.54	3.89	99.99
5	0.22	1.27	0.87	7.29	99.97
7	-0.05	0.77	0.87	3.02	99.98
10	-0.02	1.21	0.89	4.40	99.92
Average	0.00	1.14	0.60	5.10	99.98

Table IV. Parameter estimates on the cascade interest-rate term structure

Entries report the estimates and the standard errors (in parentheses) of the five parameters that determine the dynamics and term structure of the benchmark libor and swap interest rates.

Parameter	κ_r	s_κ	θ_r	σ_r	γ_r
Estimates	0.0788	5.0316	0.0157	0.0161	-0.2372
Std Err	(0.0013)	(0.0875)	(0.0011)	(0.0001)	(0.0060)

Table V. Summary statistics of pricing errors on CDS spreads

Entries report the summary statistics of the pricing errors on the CDS spreads under both the one-factor and the two-factor credit-risk specifications. The pricing error is defined as the difference between the spread quotes and the model-implied fair values, in basis points. The columns titled Mean, Std, Auto, Max, and VR denote, respectively, the sample mean, the standard deviation, the first-order daily autocorrelation, the maximum absolute error, and the explained percentage variance, defined as one minus the ratio of pricing error variance to the variance of the original series.

Sector/Rating	Financial/A					Corporate/A					Financial/BBB					Corporate/BBB				
	Mean	Std	Auto	Max	VR	Mean	Std	Auto	Max	VR	Mean	Std	Auto	Max	VR	Mean	Std	Auto	Max	VR
One credit risk factor																				
1	-0.46	2.45	0.92	10.18	84.86	-0.14	1.63	0.98	4.85	89.99	0.02	3.56	0.97	13.61	86.49	0.22	2.60	0.96	7.51	94.85
2	-0.39	1.93	0.92	7.70	91.83	-0.14	1.11	0.97	4.56	95.59	-1.14	2.45	0.95	10.71	92.31	-0.83	1.71	0.91	8.47	97.53
3	-0.08	1.18	0.83	5.60	97.25	-0.22	0.96	0.96	3.10	96.37	-0.70	1.43	0.88	5.66	97.26	-0.58	1.94	0.95	6.16	96.80
5	1.31	1.57	0.91	8.48	96.23	0.57	0.85	0.94	3.22	95.86	2.72	2.42	0.91	12.77	91.90	1.42	1.74	0.92	8.02	96.04
7	0.26	1.93	0.88	8.90	94.64	0.37	1.15	0.94	5.96	89.94	0.40	3.42	0.95	16.87	80.34	0.32	2.30	0.95	10.39	88.08
10	-0.80	2.79	0.93	10.15	90.72	-0.51	1.91	0.95	6.98	74.70	-1.56	4.72	0.95	20.46	67.19	-0.95	3.73	0.97	14.90	61.25
Average	-0.03	1.98	0.90	8.50	92.59	-0.01	1.27	0.96	4.78	90.41	-0.04	3.00	0.94	13.35	85.91	-0.07	2.34	0.95	9.24	89.09
Two credit risk factors																				
1	0.27	0.82	0.55	8.03	98.29	0.24	0.42	0.83	1.71	99.34	0.97	0.96	0.75	4.89	99.02	0.76	1.01	0.83	4.98	99.22
2	-0.30	0.80	0.63	5.17	98.59	-0.17	0.32	0.79	1.40	99.63	-1.20	0.93	0.74	6.52	98.89	-0.96	0.73	0.67	7.80	99.55
3	-0.44	0.71	0.61	4.86	98.99	-0.44	0.39	0.81	1.60	99.42	-1.37	0.99	0.81	5.19	98.70	-1.01	1.01	0.90	4.91	99.14
5	0.61	1.18	0.86	5.26	97.89	0.28	0.49	0.85	1.78	98.63	1.72	1.56	0.81	10.55	96.60	1.00	1.21	0.90	6.97	98.10
7	-0.16	1.00	0.65	5.62	98.57	0.24	0.48	0.74	2.55	98.28	-0.05	1.48	0.82	7.18	96.32	0.29	1.11	0.87	6.19	97.24
10	-0.05	1.10	0.66	6.74	98.55	-0.18	0.67	0.73	3.44	96.87	-0.23	1.57	0.69	8.11	96.36	-0.29	1.23	0.86	4.88	95.74
Average	-0.01	0.94	0.66	5.95	98.48	-0.00	0.46	0.79	2.08	98.70	-0.03	1.25	0.77	7.07	97.65	-0.03	1.05	0.84	5.96	98.17

Table VI. One-factor default arrival dynamics and the term structure of credit spreads

Entries report the parameter estimates and the standard errors (in parentheses) that determine the one-factor default arrival dynamics and the term structure of credit spreads. Estimation is based on CDS spreads at each of the four industry and credit rating classes with quasi-maximum likelihood method.

Parameters	Financial-A		Corporate-A		Financial-BBB		Corporate-BBB	
β_1	0.0341	(0.0097)	-0.0607	(0.0070)	-0.0198	(0.0138)	-0.0572	(0.0086)
β_2	-0.0117	(0.0088)	0.0109	(0.0062)	0.0009	(0.0150)	-0.0159	(0.0106)
β_3	0.0275	(0.0072)	0.0324	(0.0059)	0.0032	(0.0153)	-0.0012	(0.0108)
κ_{yx}^1	-0.0063	(0.0178)	0.0279	(0.0133)	-0.0016	(0.0260)	-0.0442	(0.0156)
κ_{yx}^2	0.0034	(0.0036)	-0.0056	(0.0022)	-0.0119	(0.0034)	-0.0066	(0.0025)
κ_{yx}^3	0.0091	(0.0007)	0.0062	(0.0004)	0.0218	(0.0028)	-0.0040	(0.0020)
κ_y	0.0000	(0.0022)	0.0186	(0.0019)	0.1161	(0.0050)	0.1173	(0.0028)
σ_y	0.0012	(0.0000)	0.0007	(0.0000)	0.0017	(0.0001)	0.0015	(0.0001)
θ_y	0.0031	(0.0006)	0.0028	(0.0003)	0.0064	(0.0008)	0.0090	(0.0006)
γ_y	-1.1642	(0.0839)	-2.1864	(0.1611)	-1.7724	(0.1356)	-1.5849	(0.1100)

Table VII. Two-factor default arrival dynamics and the term structure of credit spreads

Entries report the parameter estimates and and the standard errors (in parentheses) that determine the two-factor default arrival dynamics and the term structure of credit spreads. Estimation is based on CDS spreads at each of the four industry and credit rating classes with quasi-maximum likelihood method.

Parameters	Financial-A		Corporate-A		Financial-BBB		Corporate-BBB	
β_1	-0.0253	(0.0122)	0.0123	(0.0054)	0.0330	(0.0183)	0.0759	(0.0135)
β_2	0.0772	(0.0098)	-0.0392	(0.0053)	0.0325	(0.0184)	-0.0293	(0.0138)
β_3	-0.0346	(0.0091)	0.0007	(0.0052)	-0.0110	(0.0154)	-0.0004	(0.0095)
$\kappa_{yx,1}^1$	-0.0594	(0.0172)	0.0532	(0.0087)	-0.0448	(0.0317)	0.0031	(0.0237)
$\kappa_{yx,2}^1$	0.0565	(0.0174)	-0.0370	(0.0123)	0.0007	(0.0336)	-0.0177	(0.0340)
$\kappa_{yx,1}^2$	0.0166	(0.0048)	0.0205	(0.0030)	0.0088	(0.0062)	0.0220	(0.0051)
$\kappa_{yx,2}^2$	0.0143	(0.0032)	-0.0225	(0.0020)	0.0455	(0.0065)	0.0013	(0.0057)
$\kappa_{yx,1}^3$	-0.0000	(0.0092)	-0.0111	(0.0038)	-0.0018	(0.0093)	-0.0009	(0.0068)
$\kappa_{yx,2}^3$	0.0068	(0.0017)	-0.0012	(0.0014)	0.0232	(0.0041)	0.0113	(0.0041)
$\kappa_{y,1}$	0.2691	(0.0126)	0.2390	(0.0113)	0.1789	(0.0114)	0.1333	(0.0081)
$\kappa_{y,2}$	0.0000	(0.0045)	0.0630	(0.0061)	0.0237	(0.0089)	0.0077	(0.0074)
σ_y	0.0020	(0.0001)	0.0011	(0.0000)	0.0025	(0.0001)	0.0019	(0.0001)
θ_y	0.0048	(0.0005)	0.0032	(0.0003)	0.0070	(0.0008)	0.0084	(0.0005)
γ_y	-0.5178	(0.0350)	-1.0273	(0.0482)	-0.3327	(0.0457)	-0.3857	(0.0507)

Table VIII. Summary statistics of pricing errors on libor, swap, and CDS spreads from joint estimation

Entries report the summary statistics of the pricing errors on the eurodollar libor, swap rates, and the average cds spreads under a joint five-factor interest-rate and credit-spread model. We define the pricing error as the difference between the observed time series and the model-implied fair values, in basis points. The columns titled Mean, Std, Auto, Max, and VR denote, respectively, the sample mean, the standard deviation, the first-order daily autocorrelation, the maximum absolute error, and the explained percentage variance, defined as one minus the ratio of pricing error variance to the variance of the original series, in percentages.

Maturity	Libor/swap rates					CDS spreads				
	Mean	Std	Auto	Max	VR	Mean	Std	Auto	Max	VR
1	-0.01	0.65	0.04	2.46	100.00	0.24	0.56	0.76	2.79	99.45
2	0.20	1.34	0.60	6.03	99.99	-0.55	0.44	0.69	4.07	99.64
3	-0.28	0.73	0.68	3.47	100.00	-0.60	0.50	0.73	3.24	99.53
5	0.20	1.24	0.87	6.14	99.97	0.81	0.65	0.84	2.95	98.89
7	-0.06	0.65	0.83	2.68	99.99	-0.25	0.79	0.85	3.72	97.46
10	-0.03	1.18	0.89	3.87	99.93	0.05	0.91	0.82	3.65	96.69
Average	0.00	0.96	0.65	4.11	99.98	-0.05	0.64	0.78	3.40	98.61

Table IX. Joint dynamics of interest rate and credit risks

Entries report the parameter estimates and standard errors (in parentheses) that determine the joint dynamics and term structure of the benchmark libor interest rates and the aggregate CDS spreads. Estimation is based on five interest rate series(12-month libor and swap rates of two, three, five, seven, and ten years) and five CDS spread series at one,two, three, five, seven andten years.

Rates	κ_r	s_κ	θ_r	σ_r	γ_r
	0.0067 (0.0011)	16.8289 (1.4434)	0.0571 (0.0009)	0.0206 (0.0005)	-0.4239 (0.0237)
Credit	$\kappa_{y,1}$	$\kappa_{y,2}$	θ_y	σ_y	γ_y
	0.1813 (0.0164)	0.1647 (0.0275)	0.0031 (0.0002)	0.0006 (0.0000)	-0.7858 (0.0459)
Interactions	β	$\kappa_{yx,1}$	$\kappa_{yx,2}$	$\kappa_{xy,1}$	$\kappa_{xy,2}$
	0.0599 (0.0059)	-0.0227 (0.0046)	0.0410 (0.0016)	-4.4772 (0.6935)	-10.0956 (2.4083)
	-0.0136 (0.0063)	0.0341 (0.0017)	-0.0142 (0.0002)	2.5033 (0.1976)	8.2421 (0.5183)
	0.0023 (0.0029)	0.0147 (0.0014)	0.0147 (0.0014)	0.9713 (0.1674)	-2.5709 (0.5491)

Table X. Sub-sampel parameter estimates on the joint dynamics of interest rate and credit risks

Entries report the sub-sample parameter estimates and standard errors (in parentheses) that determine the joint dynamics and term structure of the benchmark libor interest rates and aggregate CDS spreads.

Sample	May 2003 – July 2005					July 2005 – October 2007				
Rates	κ_r	s_κ	θ_r	σ_r	γ_r	κ_r	s_κ	θ_r	σ_r	γ_r
	0.0069 (0.0029)	15.3572 (3.3414)	0.0559 (0.0022)	0.0182 (0.0010)	-0.3739 (0.0436)	0.0060 (0.0007)	17.9024 (1.1247)	0.1111 (0.0051)	0.0226 (0.0006)	-0.3618 (0.0398)
Credit	$\kappa_{y,1}$	$\kappa_{y,2}$	θ_y	σ_y	γ_y	$\kappa_{y,1}$	$\kappa_{y,2}$	θ_y	σ_y	γ_y
	0.1214 (0.0228)	0.0003 (0.0553)	0.0016 (0.0004)	0.0008 (0.0001)	-0.7041 (0.1610)	0.0557 (0.0143)	0.0000 (0.0137)	0.0000 (0.0004)	0.0004 (0.0000)	-1.3197 (0.1564)
Interactions	β	$\kappa_{yx,1}$	$\kappa_{yx,2}$	$\kappa_{xy,1}$	$\kappa_{xy,2}$	β	$\kappa_{yx,1}$	$\kappa_{yx,2}$	$\kappa_{xy,1}$	$\kappa_{xy,2}$
	-0.0230 (0.0085)	0.0142 (0.0064)	0.0275 (0.0036)	-5.5422 (0.9138)	-18.6046 (3.4997)	0.0435 (0.0095)	0.0216 (0.0065)	0.0401 (0.0017)	-4.1610 (0.4804)	-5.0398 (1.4270)
	0.0142 (0.0106)	0.0256 (0.0016)	-0.0139 (0.0007)	1.9579 (0.3803)	6.0498 (0.6548)	-0.0522 (0.0082)	0.0269 (0.0009)	-0.0057 (0.0001)	2.2132 (0.1180)	7.6024 (0.3520)
	0.0144 (0.0057)	0.0064 (0.0035)	0.0064 (0.0035)	0.0484 (0.2638)	-3.4561 (0.8591)	-0.0027 (0.0027)	0.0122 (0.0009)	0.0122 (0.0009)	1.0296 (0.2444)	-7.9298 (0.8190)

Table XI. Parameter estimates on the general Gaussian-affine specification

Entries report parameter estimates and standard errors (in parentheses) on a general three-factor Gaussian-affine specification on the term structure of interest rates.

κ			$\kappa + \lambda_2$			b_r	λ_1	a_r
0.657 (0.049)	0 —	0 —	0.102 (0.004)	0 —	0 —	0.0000 (0.0000)	-0.279 (0.005)	$\begin{bmatrix} 0.0012 \\ (0.0001) \end{bmatrix}$
0.075 (0.077)	0.031 (0.002)	0 —	-0.534 (0.024)	0.392 (0.006)	0 —	0.0000 (0.0000)	-0.438 (0.031)	
-0.335 (0.081)	-1.856 (0.078)	1.108 (0.068)	-1.001 (0.045)	-1.706 (0.054)	1.301 (0.024)	0.0076 (0.0002)	-1.467 (0.061)	

Table XII. Summary statistics of pricing errors on the libor and swap rates from the general three-factor Gaussian affine specification

Entries report the summary statistics of the pricing errors on the eurodollar libor and swap rates under the general three-factor Gaussian affine specification. We define the pricing error as the difference between the observed interest-rate quotes and the model-implied fair values in basis points. The columns titled Mean, Std, Auto, Max, and VR denote, respectively, the sample mean, the standard deviation, the first-order autocorrelation, the maximum absolute error, and the explained percentage variance, defined as one minus the ratio of pricing error variance to interest-rate variance, in percentages.

Maturity	Mean	Std	Auto	Max	VR
1	0.05	1.63	0.00	6.37	99.99
2	0.21	1.95	0.55	8.99	99.98
3	-0.30	1.14	0.50	6.02	99.99
5	0.19	1.33	0.87	7.62	99.97
7	-0.10	0.89	0.86	4.29	99.98
10	0.04	1.34	0.85	4.49	99.91
Average	0.01	1.38	0.61	6.30	99.97

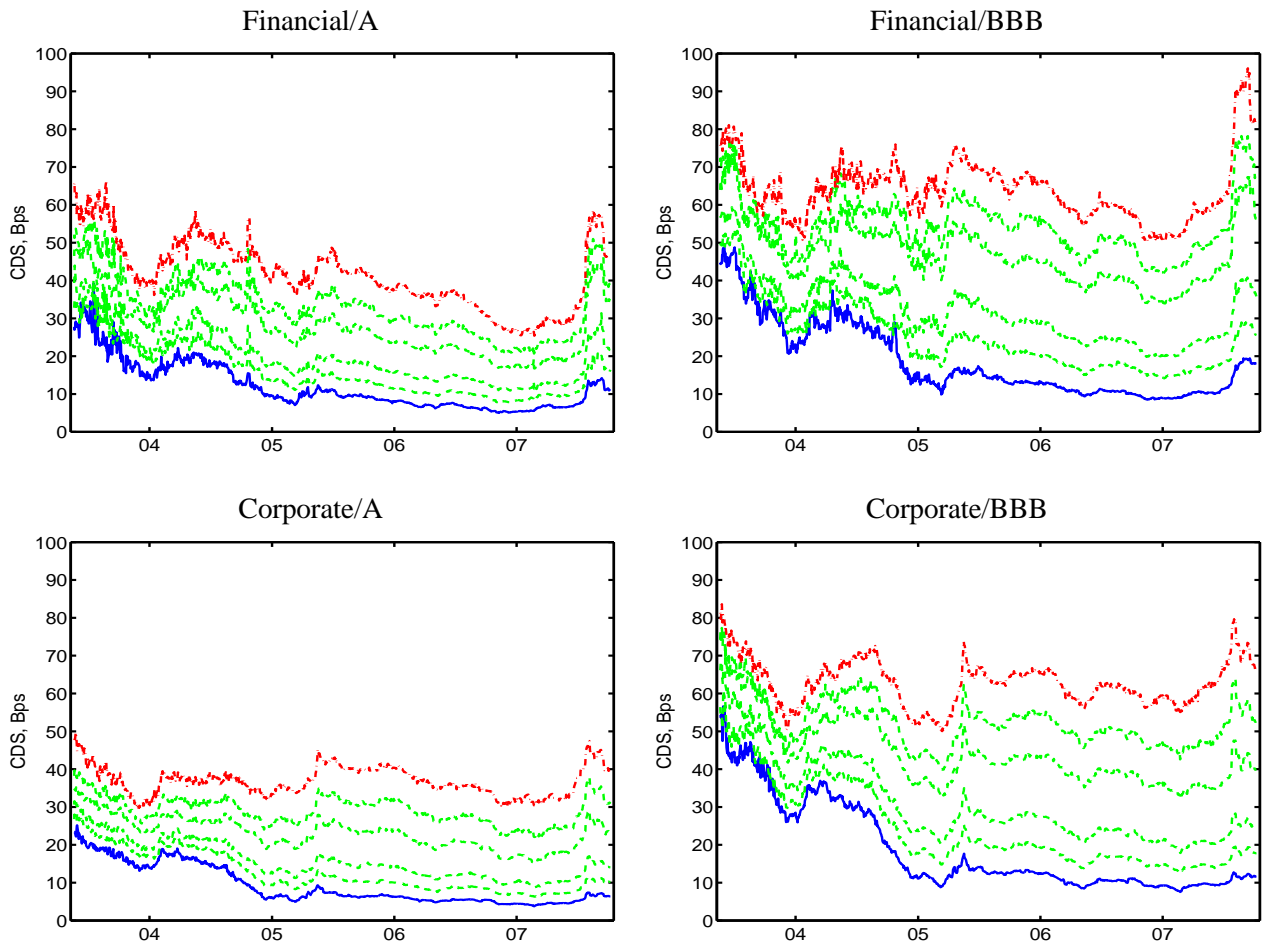


Figure 1. Time series of average CDS spreads.

Each panel represents one industry sector and one rating class. The six lines in each panel plot the time-series of the average quotes on CDS spreads at six fixed maturities. The solid lines represent the one-year CDS series and the dash-dotted lines represent the 10-year CDS series. CDS series at intermediate maturities are in dashed lines.

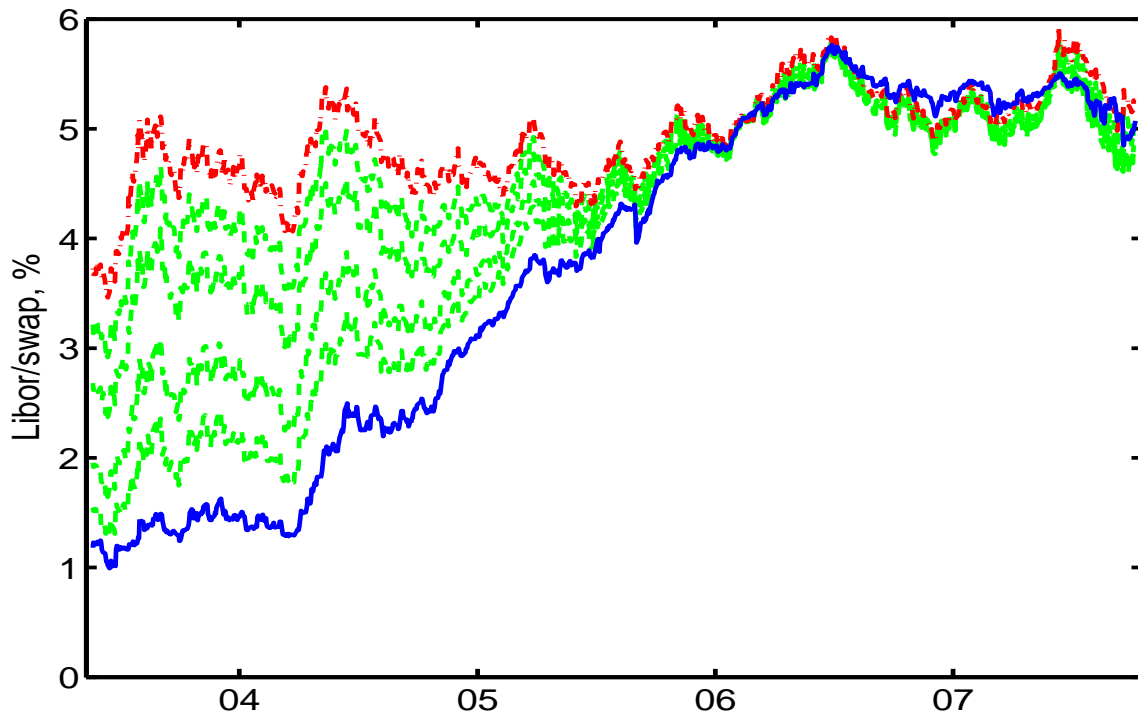


Figure 2. Time series of libor and swap rates.

Lines plot the time series of the eurodollar libor and swap rates. The solid line denotes the 12-month libor series, the dashed-dotted line denotes the 10-year swap rate series, and the remaining dashed line denote swap rates with maturities from two to seven years.

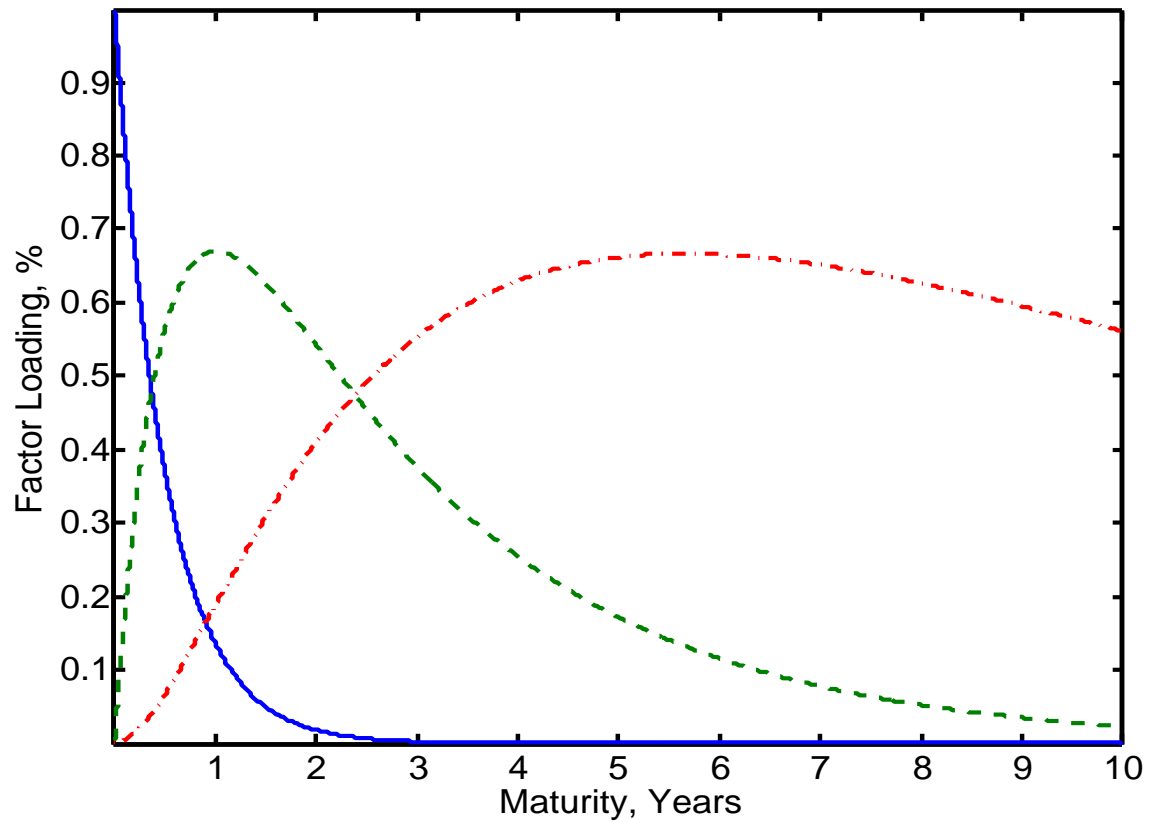


Figure 3. Contemporaneous response of the forward rate curve to different frequency components. The three lines denote the contemporaneous response of the instantaneous forward rate curve to the three frequency components in the benchmark interest-rate movements. The solid line denotes the response to the highest frequency. The dashed line denotes the response to the intermediate frequency. The dash-dotted line denotes the response to the lowest frequency.

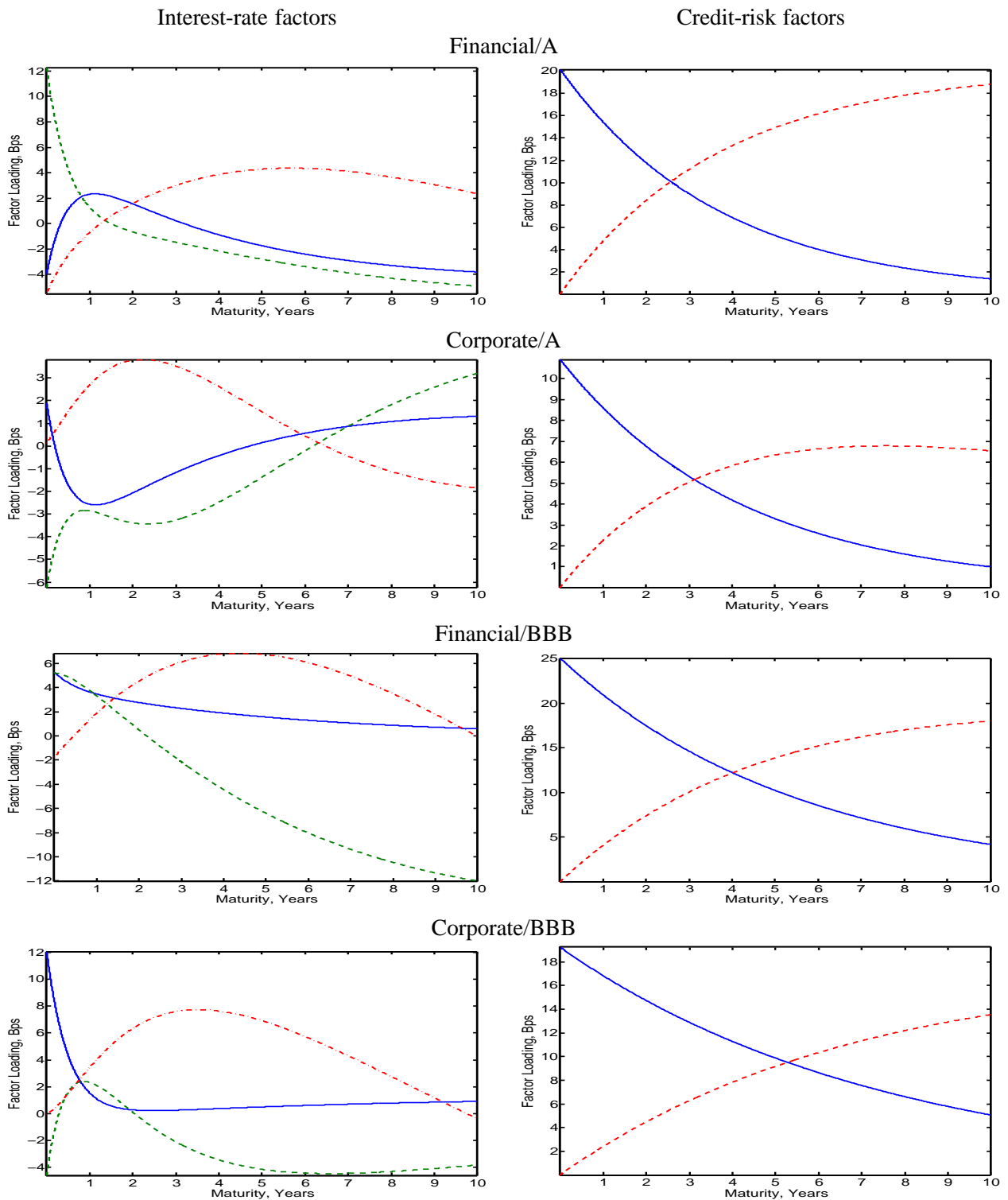


Figure 4. Credit spread responses to one standard deviation moves in the interest-rate and credit-risk factors. Panels on the left side denote the contemporaneous response of the forward default rate to one standard deviation shocks from the three interest-rate factors. Panels on the right side denote the response of the forward default rate to one standard deviation moves from the two credit-risk factors. Within each panel, the frequencies of the factors decline from the solid line to the dashed line and then to the dash-dotted line.

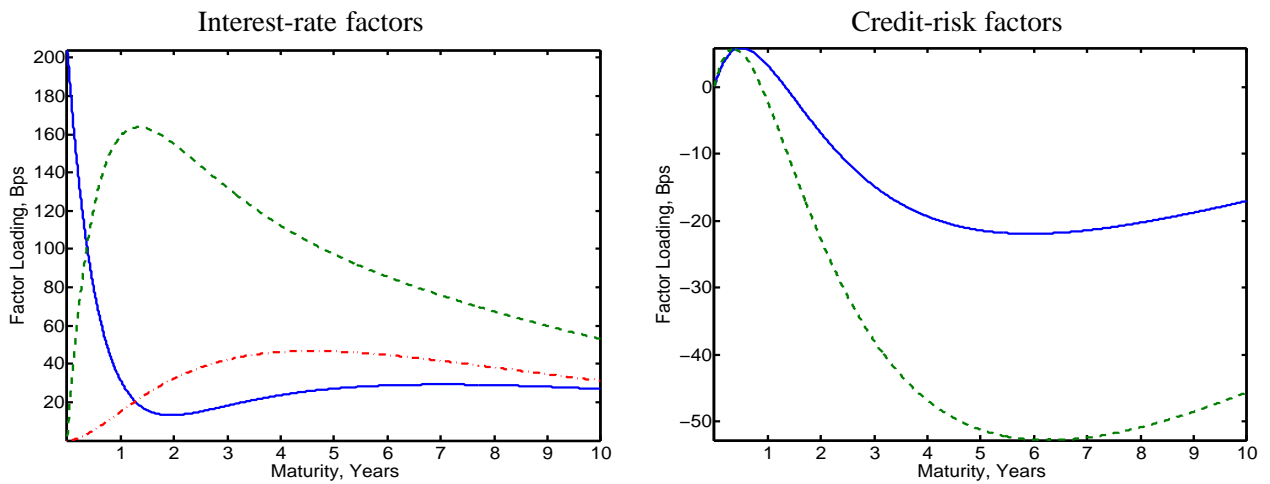


Figure 5. Contemporaneous responses of forward interest-rate curve to interest-rate and credit risk shocks. The three lines in the left panel plot the responses of the instantaneous forward rate curve to one standard deviation movements in the three interest-rate factors. The two lines in the right panel plot the responses to one standard deviation movements in the two credit-risk factors. In each panel, the frequency declines from the solid line, to the dashed line, and then to the dash-dotted line.

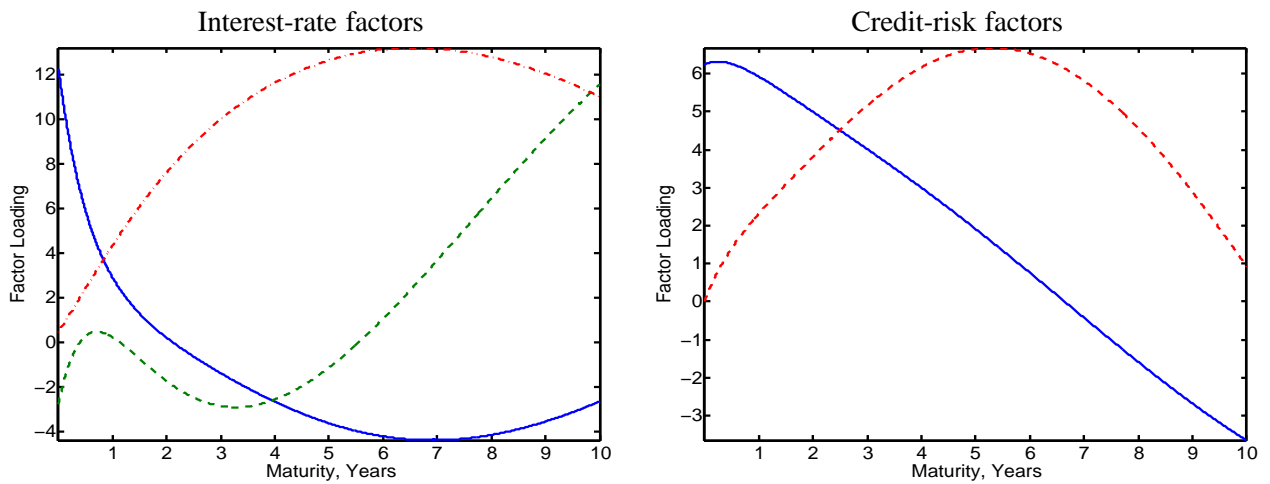


Figure 6. Contemporaneous responses of forward default rate to interest-rate and credit risk shocks. The three lines in the left panel plot the responses of the forward default rate to one standard deviation movements in the three interest-rate factors. The two lines in the right panel plot the responses to one standard deviation movements in the two credit-risk factors. In each panel, the frequency declines from the solid line, to the dashed line, and then to the dash-dotted line.

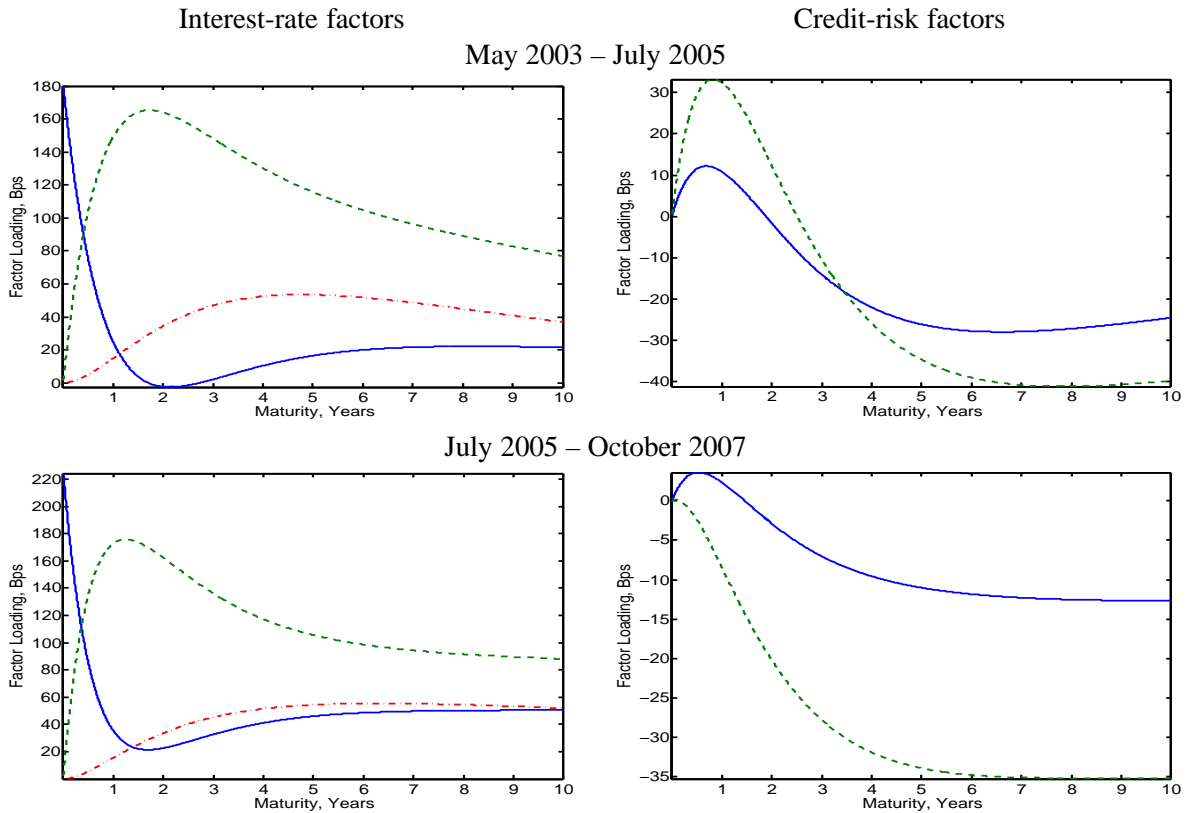


Figure 7. Contemporaneous responses of forward interest-rate curve to interest-rate and credit risk shocks at different sample periods.

The left panels plot the responses of the instantaneous forward rate curve to one standard deviation movements in the three interest-rate factors. The right panels plot the responses to one standard deviation movements in the two credit-risk factors. In each panel, the frequency declines from the solid line, to the dashed line, and then to the dash-dotted line.

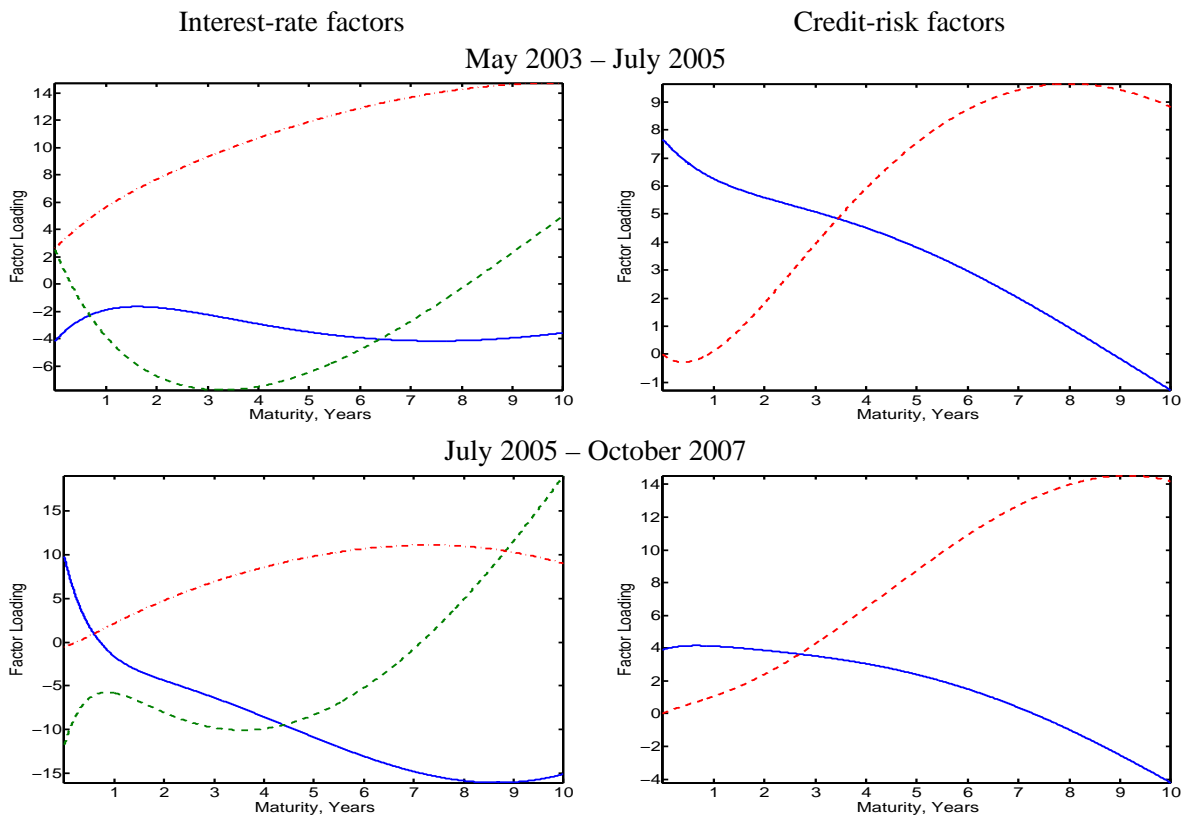


Figure 8. Contemporaneous responses of forward default rate to interest-rate and credit risk shocks at different sample periods.

The left panels plot the responses of the forward default rate to one standard deviation movements in the three interest-rate factors. The right panels plot the responses to one standard deviation movements in the two credit-risk factors. In each panel, the frequency declines from the solid line, to the dashed line, and then to the dash-dotted line.