Option P&L Attribution and Pricing

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September 20, 2019
The classic bottom-up approach of derivative pricing

- The approach is analogous to that in classic physics
  - Identify the smallest common denominator (particle)
  - Construct everything bottom up with the smallest common particle

- The analogy in classic derivative pricing
  - *The common denominator*
    - The risk-neutral dynamics (or the physical dynamics and pricing) of a set of common driving factor; the pricing of Arrow-Debreu securities
  - *The bottom-up construction*
    - Take expectation of future payoffs of all derivative securities on the same dynamics; price the payoffs as a basket of Arrow-Debreu securities

- The ambition of a classic quant: *Build one model that prices everything!*
  - Specify the dynamics of the short-rate (or pricing kernel) → Price bonds of *all maturities*
  - Specify the joint factor structure of pricing kernels of all countries → Price bonds of *all maturities, all currencies* and the exchange rates
  - Specify the stock price dynamics → Price *all options* on the stock
  - Specify the factor structure on stock dynamics → Price *all* options on *all* stocks and stock indexes, maybe even bonds...
Why the urge to go to the bottom of everything?

- Human nature or animal instinct?
  - Just like a groundhog who cannot help but keep digging!

- For derivative pricing, the approach offers *cross-sectional consistency*
  - The common denominator specification provides a *single yardstick* (pricing menu) for valuing all derivatives built on top of it.
  - The valuations on different contracts are consistent with one another, in the sense that they are all derived from the same yardstick.
  - Even if the yardstick is wrong, the valuations remain consistent with one another.
    - *They are just consistently wrong.*

- *A foolish consistency is the hobgoblin of little minds, adored by little statesmen and philosophers and divines.*
  — Ralph Waldo Emerson
Drawbacks of the classic bottom-up approach

- It is difficult to get everything under one blanket.
  - Pricing long-dated contracts requires unrealistically long projections.
  - Short-term variations of long-dated contracts often look incompatible with long-run (stationarity) assumptions.
- It is not always desirable to *chain* everything together
  - Pooling can be useful for information extraction, but can be limiting for individual contract pricing/investment, which needs domain expertise.
    - *Jack of all trades, master of none.*
  - *Error-contagion:* Disturbance on one contract affects everything else.
- The net result is the disengagement between \(Q\)-quants and \(P\)-quants:
  - \(Q\): Pricing guarantees cross-sectional consistency, but with little regard to
  - \(P\): Time-series variation, daily P&L attribution, risk management, and investment decisions

*\(P\)-type analysis (data scientists, ML) are in vogue now, can we build pricing on top of statistical P&L analysis?*
A new top-down decentralized perspective

- What do investors want?
  - Different types of investors have different domain expertise and focus on different segments of the markets.
    - They need a model that can lever their domain expertise
  - Short-term investors do not care much about the *terminal* payoffs: All she worries about is the P&L on *her particular investment* over the *short investment horizon*.
    - Short-term risk exposures are more of the focus than terminal payoffs
  - Long-term investors must also worry about their daily P&L fluctuation due to marking to market and margin trading

- We develop a new framework that generates *decentralized* pricing implications based on *short-term* investment risk on a *particular* contract.
  - The *short-term* focus allows the investor to make near-term risk projections without worrying about long-run dynamics.
  - The *particular contract* focus allows the investor to make top-down projections without generating implications on other contracts.

- *Do the next right thing.* Don’t worry about the unknown future, or conformity with others.
Value representation of an option contract

Imagine we have a position in a single vanilla option contract.

- We represent the option via the BMS pricing equation, \( B(t, S_t, I_t; K, T) \)
  - \((K, T)\) capture the contract characteristics.
  - The value of the contract can vary with calendar time \(t\), the underlying security price \(S_t\), and the option’s BMS implied volatility \(I_t\).

- Appropriate representation/transformation is important for highlighting information/risk source, stabilizing quotation...
  - The at-the-money implied volatility term structure reflects the return variance expectation.
  - The implied volatility smile/skew shape across strike reveals the non-normality of the underlying distribution.
  - As long as the option price does not allow arbitrage against the underlying and cash, there existence of a positive \(I_t\) to match the price. (Hodges, 96)

- Different transformation can highlight different insights
  - BMS is a common/nice choice, but one can explore others...
With the BMS representation, we can perform a short-term P&L attribution analysis on the option investment:

\[ dB = \left[ B_t dt + B_S dS_t + B_I dl_t \right] \]
\[ + \left[ \frac{1}{2} B_{SS} (dS_t)^2 + \frac{1}{2} B_{II} (dl_t)^2 + B_{IS} (dS_t dl_t) \right] + \text{HigherOrderTerms}, \]

- The expansion can stop at second order for diffusive moves.
- Higher order terms can become significant when the security price/implied volatility can jump by a random amount.
  - It is difficult/messy to perform integrated analysis of both small and large moves.
    - Diffusive moves can be handled effectively via dynamic hedging (frequent updating of a few instruments)
    - Hedging random jumps need careful positioning of many derivative contracts.
  - The effects of large jumps are better analyzed separately with scenario analysis/stress tests.
Risk-neutral valuation of the investment return

- Take risk-neutral expectation on the investment P&L, and assume zero rate, 

\[ 0 = \mathbb{E}_t \left[ \frac{dB}{dt} \right] = B_t + B_I l_t \mu_t + \frac{1}{2} B_{SS} S_t^2 \sigma_t^2 + \frac{1}{2} B_{II} l_t^2 \omega_t^2 + B_{IS} l_t S_t \gamma_t \]

(1)

- \((\mu_t, \sigma_t^2, \omega_t^2, \gamma_t)\) are the conditional mean and variance/covariances:

\[
\mu_t \equiv \mathbb{E}_t \left[ \frac{dl_t}{l_t} \right] / dt, \quad \sigma_t^2 \equiv \mathbb{E}_t \left[ \left( \frac{dS_t}{S_t} \right)^2 \right] / dt, \\
\omega_t^2 \equiv \mathbb{E}_t \left[ \left( \frac{dl_t}{l_t} \right)^2 \right] / dt, \quad \gamma_t \equiv \mathbb{E}_t \left[ \left( \frac{dS_t}{S_t}, \frac{dl_t}{l_t} \right) \right] / dt.
\]

- (1) can be regarded as a pricing equation: The value of the option must satisfy this equality to exclude dynamic arbitrage.

- At one maturity, vega and theta are co-linear, we expect dollar gamma, vanna, volga to explain the cross-sectional variation of theta decay:

\[-B_t = \frac{1}{2} B_{SS} S_t^2 \sigma_t^2 + \frac{1}{2} B_{II} l_t^2 \omega_t^2 + B_{IS} l_t S_t \gamma_t\]

- In absence of vol move, theta decay is compensated by gamma gain.

- IV move induces extra expected P&Ls from vanna and volga exposures.
Risk-neutral valuation in BMS implied volatility

Start with the pricing relation

\[-B_t = B_I l_t \mu_t + \frac{1}{2} B_{SS} S_t^2 \sigma_t^2 + \frac{1}{2} B_{II} l_t^2 \omega_t^2 + B_{IS} l_t S_t \gamma_t\]

Plug in the partial derivatives and rearrange, we obtain a simple valuation equation on the option implied volatility:

\[l_t^2 = \sigma_t^2 + 2 \tau \mu_t l_t^2 + 2 \gamma_t z_+ + \omega_t^2 z_+ z_-\]

\[z_\pm \equiv (\ln K/F_t \pm \frac{1}{2} l_t^2 \tau)\] — convexity-adjusted moneyness
Top-down mean-variance option pricing

- The (risk-neutral) *mean-variance* risk assumption on the security return and the implied volatility return for an option contract,

\[
\begin{bmatrix}
R^{S}_{t+1} 
\equiv & \frac{\Delta S_{t+1}}{S_t} \\
R^{I}_{t+1} 
\equiv & \frac{\Delta I_{t+1}}{I_t}
\end{bmatrix} \sim N \left( \begin{bmatrix}
0 \\
\mu_t
\end{bmatrix}, \begin{bmatrix}
\sigma_t^2 & \gamma_t \\
\gamma_t & \omega_t^2
\end{bmatrix} \right).
\]

- Determines the fair value of this one option contract in a simple form,

\[
I_t^2 = \sigma_t^2 + 2\tau \mu_t I_t^2 + 2\gamma_t z_+ + \omega_t^2 z_+ z_-.
\]

Where the mean-variance estimates come from and how they vary in the future/past are irrelevant for the current valuation of this contract.

- To price any other option contracts, specify/estimate their own mean-variance risk structure.

- To compare the pricing of different contracts, compare their mean-variance risk structure.

- Since Markowitz, mean-variance analysis has a long successful history in finance for both pricing and investing, and now it extends to derivatives pricing and investments.
Mean-variance pricing

- Capital Asset Pricing Model (CAPM): Expected excess return on a security is proportional to its beta.
  - Key is in identification of market risk premium and beta.
- Arbitrage Pricing Theory (APT): The expected excess return on a security is spanned by the expected excess returns on several underlying factors...
  - key is in identification of the risk factors, the risk premiums, and the risk exposures of each security (FF, BARRA, ...)
- Mean-variance option pricing: $l_t^2 = \sigma_t^2 + 2\tau \mu_t l_t^2 + 2\gamma_t z_+ + \omega_t^2 z_+ z_-$. The pricing of implied volatility risk determines the difference between $(\mu_t, \gamma_t, \omega_t)$ and the statistical counterparts.
- Pricing theories analogous to CAPM/APT, with empirical works analogous to FF/BARRA, can be used to determine the pricing of implied volatility risks across different option contracts and securities.
- Common factor structures on $(\mu_t, \gamma_t, \omega_t)$ across contracts and underlyings can link implied volatility surfaces across names.
- We have only built the skeleton framework. All underlying pricing theories and empirical works are waiting to be built and explored...
Common factor structures on an implied volatility surface

\[ I_t^2 = \left[ 2\tau \mu_t I_t^2 + \sigma_t^2 \right] + \left[ 2\gamma_t z_+ + \omega_t^2 z_+ z_- \right] \]

- **Carr&Wu (JFE, 2016):** One common-factor governs the short-term movements of the whole surface
  \[ dI(K, T)/I(K, T) = e^{\eta_t(T-t)}(m_t dt + w_t dZ) \] for all \((K, T)\)
  - That allows them to characterize the whole implied volatility surface at any given time with five states \((m_t, w_t, \eta_t, \rho_t, \sigma_t)\) with no additional model parameters.

- PCA often identifies 3 major sources of variation on the surface:
  - The overall volatility level
  - Term structure variation (short v. long-dated contracts)
  - Implied volatility smile/skew variation along moneyness (OTM put v. straddle v. OTM call)

- We explore how our framework can be applied to any particular segment of the surface.
At-the-money implied variance term structure

- We define “at-the-money” as contracts with \( z_+ = 0 \), or \( k = -\frac{1}{2} l_t^2 \tau \).
- Such contracts have zero volga and vanna.
- The implied volatility level only depends on its expected move \( (\mu_t) \), but not its variance/covariance:
  \[
  A_t^2 = 2\tau \mu_t A_t^2 + \sigma_t^2.
  \]

Applications:

- **1 contract**: Infer risk-neutral drift from the slope against instantaneous variance:
  \[
  \mu_t = \frac{A_t^2 - \sigma_t^2}{2A_t^2 \tau}.
  \]
- **2 contracts**: Infer *locally common* drift from the slope of nearby at-the-money contracts:
  \[
  \mu_t = \frac{A_t^2(\tau_2) - A_t^2(\tau_1)}{2(A_t^2(\tau_2)\tau_2 - A_t^2(\tau_1)\tau_1)}.
  \]

- **Locally constant** drift leads to locally linear term structure — Drift estimates can be tied to local linear (nonparametric) regression fitting of the term structure.
The implied volatility smile

- To highlight the implied volatility smile at a certain maturity, we can vega hedge the option with the at-the-money contract of the same maturity, assuming they strongly co-move.
- Take the ATM implied variance $A_t^2$ as given and focus on the implied variance deviation of other contracts from the ATM variance level.
- Assume proportional drift at the same maturity: $\mu_t l_t^2 = \mu_t A_t^2$.
- Plug in the at-the-money implied variance to highlight the “implied volatility smile” effect at the single maturity,

$$l_t^2 - A_t^2 = 2\gamma_t z_+ + \omega_t^2 z_+ z_-,$$

(3)

- The smile slope is determined by the covariance rate $\gamma_t$ of the contract.
- The smile curvature is determined by the variance rate $\omega_t^2$.

Application: Assuming \textit{locally common} proportional implied volatility movements within \textit{a particular moneyness range}, we can identify the common moment conditions $(\omega_t^2, \gamma_t)$ by regressing $l_t^2 - A_t^2$ against $(2z_+, z_+ z_-)$. 
Application: Extrapolate observed smile to long maturity

Exchange-traded contracts are short-dated, but many OTC deals are long term. *How to extrapolate exchange quotes to price long-dated OTC deals?*

- **The classic approach**
  - Calibrate a standard stochastic vol model to observed quotes (say up to 2 years), price long-dated options (say 10 years) with the model parameter estimates.
  - The atm vol level will flatten out to the long-run mean (roughly matching the 2-year ATM vol level)
  - The *implied volatility smile/skew will flatten* due to central limit theorem (and the fact that volatility converges to its long-run mean).

- **Our pricing approach:**
  - If the atm vol is flat extrapolated from 2 to 10 years, the 2-year and 10-year atm variance will *vary by the same amount* — same \( (\gamma_t, \omega_t^2) \).
    - Empirically, we do observe long-dated contracts vary much more than suggested by estimated mean reversion processes.
  - The smile/skew shape must be extrapolated from 2 to 10 years as well!

Different perspectives often lead to different *seemingly innocuous* asymptotic assumptions.
Empirical analysis

We use exchange-traded SPX options to

- Distinguish between changes in floating implied vol series and *implied volatility changes of a fixed option contract*
  - Traditional models are more related to the dynamics of the former, our approach is based on the moments of the latter.

- In according with our effort for *decentralization*, introduce the concept of *local commonality*, and contrast with traditional global factor structures

- Compare historical (TS) estimates with cross-sectional option-implied (CS) mean-variance moment conditions
  - Expected implied volatility change v. the at-the-money term structure
  - Variance/covariance estimates v. the implied volatility smile
  - Trade the difference as risk premium (*risk-return tradeoff*), and compare with *statistical arbitrage trading*
Constructing floating series of IV levels and changes

- The exchange-listed options have fixed strike and expiry.
- The standard approach is to interpolate the implied volatilities of these contracts to obtain floating series at fixed time to maturities and moneyness
  - OptionMetrics provides floating implied vol series at 1,2,3,6,12 months and different deltas.
  - OTC market often provide indicative quotes on floating time-to-maturity and relative strike grids.
  - Analyzing variations of these floating series provides insights for traditional option pricing modeling
- Our model depends on the implied volatility variation of a fixed contract, making it necessary to construct implied volatility changes of fixed contracts
  - At each date, construct log implied volatility change over the next business date on each option contract \( i \), \( R_{t+1}^i \equiv \ln(l_{t+1}^i/l_t^i) \).
  - Interpolate (via Gaussian kernel smoothing) the changes to floating time to maturity (1,2,3,6,12 months) and moneyness points: \( x \equiv z_+ / l_t \sqrt{\tau} = 0, \pm 0.5, \pm 1, \pm 1.5, \pm 2 \).
Our theory links an option's implied volatility level to its own first and second risk-neutral conditional moments ($\mu_t, \gamma_t, \omega_t$).

To reverse engineer the conditional moments from option observations, we propose to make *local commonality* assumptions that implied volatilities of *nearby contracts* move closely together and thus share similar moments.

- The assumption is not meant to be *exact*,
- But it is *robust* to actual dynamics variations/assumptions.

It is a matter of what you trust: model assumption or contract structure
- If you trust a model, you can hedge/combine any contract exactly with any other contract .... contract choice is irrelevant.
- You can fail miserably if the model turns out wrong (and it is always!)
  - Hedging a $100$-strike option with a $101$-strike option has a max loss of $1$ regardless of what model or what happens.

Empirically identified global factor structures are rarely global
- Number of principal components depend on the maturity/strike span
- Local commonality in practice (e.g., OCC): divide the implied vol surface into grids and treat contracts with each grid as “common.”
How local is local?

- Take 3-month at-the-money at the reference point, measure implied vol change correlations of other contracts with this reference contract.

- At the money: Correlations with adjacent maturities (2m & 6m) are > 95%.
- Same-month smile: Correlations with |x| < 1 (one std) are > 93%.
- Correlations with far-away contracts can be as low as 60%.
Extract risk-neutral drift from the local term structure

<table>
<thead>
<tr>
<th>Maturity</th>
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<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
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<tbody>
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<td>Mean</td>
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<td>0.219</td>
<td>0.178</td>
<td>0.104</td>
<td>0.048</td>
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<tr>
<td>Std</td>
<td>0.497</td>
<td>0.367</td>
<td>0.241</td>
<td>0.147</td>
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<tr>
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<td>0.941</td>
<td>0.958</td>
<td>0.969</td>
<td>0.967</td>
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</table>

**Panel A: Summary statistics of risk-neutral drift estimates**

**Panel B: Average difference between $P$ and $Q$ estimates**

| $P$  | 0.133 | 0.096 | 0.101 | 0.102 | 0.084 |
| Diff | -0.111 | -0.123 | -0.077 | -0.002 | 0.036 |

- Risk-neutral rate of change estimates vary greatly over time, from highly negative to highly positive.
- Compared to statistical average estimates, risk-neutral drifts on average are higher at short term, but lower at long term.
  - Risk premium trades: *Short short term, long long term* (Egloff, Leippold, Wu (2010))
- To do: With statistical forecasts, one can invest in calendar spreads with time-varying weights based on the $P - Q$ difference.
Extract variance and covariance from the *local* smile

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<tr>
<th>Maturity</th>
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**Panel A. Covariance Rate Estimates $\gamma_t$**

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<th></th>
<th>Mean</th>
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<td>0.066</td>
<td>0.647</td>
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<td>0.046</td>
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<td>-0.090</td>
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**Panel B. Variance Rate Estimates $\omega^2_t$**

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**Panel C. Regression R-squared Estimates**

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<th>Std</th>
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<th>Std</th>
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<tbody>
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<td>1.000</td>
<td>0.001</td>
<td>1.000</td>
<td>0.001</td>
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</table>

- Super high $R^2$ — Near perfect fit of a local smile
- Skew/covariance are highly correlated, not so for the variance/curvature estimates
Predicting realized variance/covariance rates

\[ RV_{t+1} = \alpha + \beta_1 CS_t + \beta_2 TS_t + e_t \]

<table>
<thead>
<tr>
<th>Maturity</th>
<th>( \alpha )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( R^2, % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. Covariance Rate ( \gamma_t )</td>
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</tr>
<tr>
<td>1</td>
<td>-0.011 (1.64)</td>
<td>0.120 (1.79)</td>
<td>0.377 (2.97)</td>
<td>20.23</td>
</tr>
<tr>
<td>2</td>
<td>-0.011 (1.65)</td>
<td>0.097 (1.20)</td>
<td>0.433 (3.45)</td>
<td>23.11</td>
</tr>
<tr>
<td>3</td>
<td>-0.012 (1.70)</td>
<td>0.070 (0.76)</td>
<td>0.463 (3.84)</td>
<td>24.18</td>
</tr>
<tr>
<td>6</td>
<td>-0.009 (1.63)</td>
<td>0.056 (0.58)</td>
<td>0.484 (4.21)</td>
<td>25.28</td>
</tr>
<tr>
<td>12</td>
<td>-0.004 (1.11)</td>
<td>0.129 (1.61)</td>
<td>0.466 (4.00)</td>
<td>25.85</td>
</tr>
</tbody>
</table>

| Panel B. Variance Rate \( \omega_t \) |
| 1        | 0.231 (7.02)  | -0.004 (0.15) | 0.19 (3.72) | 3.52 |
| 2        | 0.148 (6.84)  | -0.040 (1.07) | 0.22 (3.86) | 5.40 |
| 3        | 0.116 (6.62)  | -0.091 (1.83) | 0.25 (4.08) | 7.63 |
| 6        | 0.063 (6.47)  | -0.160 (2.16) | 0.29 (3.95) | 10.28 |
| 12       | 0.034 (6.58)  | -0.198 (2.26) | 0.34 (3.30) | 13.46 |

Historical estimators dominate the prediction.
Risk-return tradeoff strategy on the implied volatility smile

- Take delta/vega-neutral spread positions (against the ATM contract) at each maturity
- Alpha source: difference between the observed smile and the smile constructed from the forecasted variance/covariance rates (risk premium).

Annualized information ratio from the investments:

<table>
<thead>
<tr>
<th>τ \ x</th>
<th>-1.0</th>
<th>-0.5</th>
<th>0.5</th>
<th>1.0</th>
<th>All</th>
</tr>
</thead>
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<td>0.36</td>
<td>0.35</td>
<td>0.52</td>
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<tr>
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<td>3.39</td>
<td>3.06</td>
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<td>3.53</td>
<td>3.76</td>
<td>3.59</td>
<td>3.21</td>
<td>3.81</td>
</tr>
</tbody>
</table>

- The trades are not that profitable at short maturity (one month) — Missing contribution from jumps can play a large role at short maturity?
- The trades are very profitable at long maturity — Variance/covariance drives the main delta-hedged P&L of long-dated smiles.
Statistical arbitrage strategy on the implied volatility smile

- Take delta/vega-neutral spread positions (against the ATM contract)
- Alpha source: *pricing error of the CS regression* (reversion of pricing error)

Annualized information ratio from the investments:

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\chi$</th>
<th>-1.0</th>
<th>-0.5</th>
<th>0.5</th>
<th>1.0</th>
<th>All</th>
</tr>
</thead>
<tbody>
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<td>0.28</td>
<td>1.77</td>
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<td>0.68</td>
</tr>
</tbody>
</table>

- Much lower profitability due to concentration of contracts ...
- Can be much more profitable (for market making) when applied to a larger universe of contracts
- The two strategies focus on different aspects and have different applications
Concluding remarks

- Virtually all security classes can be analyzed from two perspectives:
  - **Bottom-up valuation**: DCF for stocks, DTSM for bonds, time-changed Lévy processes for options
    - Long-run projections are needed for pricing current securities.
    - Pricing errors can be regarded as stat arb trading opportunities.
  - **Top-down return analysis**: CAPM/APT-type research for pricing theories, FF/BARRA-type empirical research for stock returns...
    - Focus on current risk-return tradeoff (of different horizons)
    - Identified risk structure can be used to construct robust covariance matrix for mean-variance optimization
    - Identified factor risk premiums can be used to constructed “smart-beta” portfolios

- A lot has been done on the former (bottom-up valuation) on option pricing.
- Much is needed to explore the option pricing implication from the top-down return analysis angle.
- Ultimately, we hope to tie the pricing analysis to risk management and mean-variance investments for an expanded instruments universe that include both primary securities (stocks, bonds) and their options.