Centrality of the Supply Chain Network

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One of the central themes of research is to estimate/test various relations.

- There are two broad approaches:
  
  1. **Statistical estimation**
     - Regressions, dynamic statistical relation specifications/estimations (VAR, GARCH, DCC,...)
     - PCA, Trees, neutral network, boosting, ... (Google translate)
     - More data and more computing power $\rightarrow$ better statistical learning
  
  2. **Structural identification**
     - Industry classification: Companies within the same industry tend to behave similarly and tend to move together
     - Linguistic structures: Translation based on understanding (grammar) of the two languages
     - *Supply chain relation*: A shock to the supplier can generate ripple effects via the supply chain.

- One can gain a better understanding of the relations by going back and forth between the two approaches.
What can we learn from the supply chain information?

*If we know that A is the supplier of B, then what?*

- Commonly formulated questions in the supply chain (finance) literature:
  - What happens to the customers (and the customer’s customers) if a supplier company goes under?
  - What does a good performance of a supplier company foretell about its customers?
  - Mostly focus on the lead-lag relations between supplier-customer pairs (stock returns, earnings etc)

- Statistical analysis of the supply chain:
  - Identify *pairs*, and verify whether the relation is as expected.
But it’s a chain (network), not a stand-alone pair

A is the supplier of B, C, D, the customer of E, F, ... Pairs do not exist in isolation

- *What can we learn, what do we want to learn, from this network?*
Questions on the network

- Are there suppliers that are more important to the economy than others? In what sense/metric?
  - Shocks to which company generate the strongest ripple effect? Why?
  - Which companies are the most central to the network?
  - How do we define the relative importance of a company within this network?

- What roles do these central companies play in the aggregate economy?
  - What types of companies tend to be the central ones in the network?
  - How do they behave differently?
  - Do changes in these central companies foretell the transition of the business cycle, or structural changes in the economy? How?

- What I try to explore in this paper:
  - Explore metrics of relative importance (centrality) for each company within the supply chain network.
  - Explore what types of companies tend to be “central” and how “central” companies interact with the aggregate economy.
The supply chain data

- Historical firm-level supplier-customer relationship information is from FactSet Revere.

- The SEC has a mandate (rule SFAS 131) that calls for companies to disclose their customers if their revenue exposure is 10% or greater.

- Some companies choose to disclose more customer and supplier information if they believe the information will help improve their attractiveness and draw more attention from investors.

- FactSet collects the information from the company’s regulatory disclosure reports, annual reports, and other primary sources.

- I confine the analysis to constituents of the S&P Composite 1500 index.

- From 2004 to 2014, I take snapshots of the database at the beginning of each year, and merge the information with price and accounting data from Bloomberg.

- One can sample the information more frequently, but not sure about the practical feasibility of more frequent information updates.
The supplier network matrix

Based on the FactSet data, I construct a supplier network matrix \( \{A_{i,j}\}_{i=1}^N \) at the beginning of each year on the S&P 1500 companies.

- \( A_{i,j} = 1 \) if company \( i \) is the supplier of company \( j \); 0 otherwise.
- Each column \( j \) represents an index of suppliers for company \( j \).
- Each row \( i \) represents an index of customers that supplier \( i \) supplies to.

The matrix does not distinguish the relative significance of each supplier.

- FactSet database has some sales exposure information, but is far too sparse to be useful.
- Other metrics: irreplaceability, product type, value added, profit margin, sales...

The matrix is incomplete, and has potential biases:

- A S&P1500 company can have important suppliers outside the 1500 list — pruned network.
- There are potentially unidentified suppliers within the 1500 list.
- FactSet uses cross-validation, which may create some bias/noise — A may list B as an important customer, but A may not be a significant supplier to B.
The supply chain network is a *directed network*, similar to the *hyperlink network*...

- One can draw an arrow from a customer to each of its suppliers.
- The network matrix $A$ is positive, but not symmetric.
- A *Peers Network* would be an undirected network, with a positive symmetric network matrix.

The network literature has proposed a long list of *centrality* measures to define the relative importance of a vertex (a company) within the whole network.

I adapt some of these centrality concepts as measures of the relative importance of a supplier to the overall economy.

1. Degree centrality
2. Eigenvector centrality
3. PageRank
4. Kleinberg supplier authority v customer hub centrality

I analyze the relevance and applicable scenarios of each measure.
Degree centrality: The number of direct customers

- Intuitively, a supplier company is more important to the economy if it is the supplier of many companies.

- A simple measure of centrality for a supplier, the **degree centrality**, can be defined as the number of companies that this company supplies to.

- From the perspective of the network topology, if one draws an arrow from a company to each of its suppliers, the degree centrality measures the degree of a vertex, or the number of arrows pointed to a particular supplier.

- From the supplier matrix $A$, one can compute the degree centrality vector, $c$, as the simple sum of each row,

$$ c_i = \sum_j A_{i,j}, \quad \text{or in matrix form,} \quad c = Ae \quad (1) $$

where $e$ denotes a vector of ones.

- All centrality measures are relative: One can normalize the metric to require, for example, $e^T c = N$ or $e^T c = 1$. 
Top 10 suppliers with the most customers

2004

Number of customers

GE  HPQ  IBM  INFA  INTC  IWOW  MSFT  ORCL  ROK  WBSN

0  10  20  30  40  50  60  70

2014

Number of customers

BFAM  BRK/B  CR  GE  IBM  MSCC  MSFT  NPO  ORCL  SXI

0  10  20  30  40  50  60  70

Top suppliers have 30-70 customers
Eigenvector centrality: The importance of customers as suppliers

- Each customer can be a supplier to other companies. Its importance increases if it is an important supplier itself to many other companies.
- Shocks to a supplier are more likely to cause a stronger propagation (ripple effect) if it is the supplier of important suppliers.

**Eigenvector centrality** gives each supplier a score proportional to the sum of the scores of its customers:

\[
c_i = \frac{1}{\lambda} \sum_j A_{i,j} c_j.
\]

(2)

\(N\) equations, \((N + 1)\) unknowns, up to a scale.

- In matrix notation, we have, \(\lambda c = Ac\).
- The solution is proportional to the right leading eigenvector of matrix \(A\).
- Since all elements of \(A\) are non-negative, the Perron-Frobenius theorem states that the leading eigenvector has strictly positive components.
A customer of many suppliers contributes less to each of these suppliers

- If one company has a high eigenvector centrality, all its suppliers will have high eigenvector centrality.
- In many cases it means less if a company is only one of the many suppliers (esp. if they are substitutes).

**PageRank centrality** discounts the centrality of each customer by its number of suppliers.

\[
c_i = \frac{1}{\lambda} \sum_j A_{i,j} \frac{c_j}{k_j}, \quad k_j = \max \left( 1, \sum_i A_{i,j} \right).
\] (3)

- Brin and Page (1998) proposed the idea in their modeling and ranking of the internet hyperlinks.
- Can be solved as the leading eigenvector of a scaled matrix:
  \[
  \lambda c = AK^{-1}c.
  \]
Kleinberg centrality

- Suppliers and customers can be important for different reasons.
  - A customer is important to the economy if it needs many different parts supplied by different suppliers and hence acts as a hub of different (non-substitute) suppliers.
  - This goes to the opposite direction of PageRank discounting, for different reasoning.
  - A supplier is important if it supplies to many such important hubs.

- Kleinberg (1999) proposes to construct two types of centralities, labeled as the **authority (supplier) centrality** and the **hub (customer) centrality**, to quantify each vertex’s prominence in the two roles.

- Define **supplier centrality** $s$ as the sum of the customer importance, and **customer centrality** $c$ as the sum of its supplier importance:

  $$ s_i = \alpha \sum_j A_{i,j} c_j, \quad c_j = \beta \sum_i A_{i,j} s_i. \quad (4) $$

- Combining the two and in matrix notation, $AA^T s = \lambda s$, $A^T A c = \lambda c$. The solutions are the leading eigenvectors of $AA^T$ and $A^T A$, resp.
Differences among the four centrality measures

- Supplier importance is the average importance of its customers.

- The four centrality measures differ in the customer importance definition:
  1. Degree centrality: All customers are equally important.
  2. Eigenvector centrality: Important suppliers are important customers: customer importance = supplier importance.
  3. PageRank centrality: Discounts the importance of a customer by its number of suppliers: customer importance = supplier importance/Number of suppliers.
  4. Kleinberg centrality: A customer is more important if it has more important suppliers: customer importance = sum of the importance of its suppliers.
Numerical issues with eigenvector centrality

- Supplier centrality avoids degenerations in Eigenvector centrality:
  - If a company is not the supplier to any company, it will lead to a row of zeros in the supplier matrix. The eigenvector centrality is zero.
  - Imagine another company, which is a supplier to thousands of such companies — The eigenvector centrality will remain zero.
  - This can go on and on...Supplier centrality avoids this issue via a different definition of customer centrality.
  - In practical implementation, PageRank employs an ad hoc fix: Add a small quantity to all elements of \( \mathbf{c} = \alpha \mathbf{A} \mathbf{c} + \beta \mathbf{e} \).
    - \( \beta \) can also differ from company to company and can be used as a prior obtained from other sources of information (e.g., sales, size, etc)
    - Adding how much is a bit arbitrary.

- The separate definitions of supplier/customer importance probably make more sense in most practical situations.
  - It would be even better if one can treat substitute suppliers differently from non-substitute suppliers.
Degree centrality is most uniform; eigenvector centrality is most concentrated.

Important customers are more concentrated than important suppliers.
The three supplier centrality measures are highly correlated — They capture similar behavior.

Supplier and customer centralities show small negative correlation.

This approach can indeed separate two types of companies.
Central company characteristics and behaviors

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Panel C. Supplier authority centrality

Panel D. Customer hub centrality

- Central authorities (suppliers) tend to have higher volatility, but lower correlation with market.
- Central hubs tend to have lower volatility, but higher correlation with market.
- Highly connected firms tend to have lower returns?
Top 10 suppliers with the most number of customers

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Mostly tech companies in earlier years, some industrial goods later, and one service.
### Top 10 suppliers with the highest eigenvector centralities

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Mostly tech companies.
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Mostly tech earlier years. 2014 has 6 financials (real estate), 0 tech.
## Top 10 companies with the highest customer hub centralities

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Many stores: BestBuy, Costco, Home depot, Lowe’s, Sears, Target, Walmart
Portfolios are formed equal weighting with top 10 central companies:

<table>
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<tr>
<th>Year</th>
<th>$\beta$</th>
<th>$\rho$</th>
<th>Quarterly total return 3-month</th>
<th>Quarterly total return 6-month</th>
<th>Quarterly excess return 3-month</th>
<th>Quarterly excess return 6-month</th>
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<td>Degree</td>
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</table>

- $\beta$: Regression against the S&P 500 index: Central supplier portfolios seem to be more volatile.
- $\rho$: correlation with the index

Predictive correlations with the index over 3/6-month horizons: Excess returns on central supplier portfolios predict future stock returns. Excess returns on central customer portfolios do not.
Concluding remarks and future works

- It *should be* useful to look into the topology of the supply chain network to better understand its aggregate characteristics.
  - The performance of important suppliers lead the economic cycle.
  - Other unexplored questions:
    - Sector rotation in the business cycle
    - Are important suppliers “too central to fail”?  

- The topology should also be useful for firm-level analysis
  - A shock to any companies first propagates to *all* its customers
  - then to all the customers of its customers, ...
  - The impulse-response can be analyzed through the network matrix $A$ and its powers, $A^2$, $A^4$, ...
Much work can be done on refining the accuracy of the network matrix

- Expand the matrix to be less pruned — Too much pruning can distort the topology...
- Accommodate/Include other informations (such as fraction of sales/COGS) to refine relative significance of each customer/supplier
- Use product-level information to differentiate substitute suppliers w. non-substitute suppliers
- Combine network centrality with other centrality/importance measures: 
  Katz centrality: \( c = \alpha Ac + \beta c_0 \),
  with \( c_0 \) being the prior constructed from other criteria.