Statistical Arbitrage Based on No-Arbitrage Models

Liuren Wu

Zicklin School of Business, Baruch College

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Review: Valuation and investment in primary securities

- The securities have direct claims to future cash flows.

- Valuation is based on forecasts of future cash flows and risk:
  - DCF (Discounted Cash Flow Method): Discount forecasted future cash flow with a discount rate that is commensurate with the forecasted risk.

- Investment: Buy if market price is lower than model value; sell otherwise.

- Both valuation and investment depend crucially on forecasts of future cash flows (growth rates) and risks (beta, credit risk).
Compare: Derivative securities

- Payoffs are linked directly to the price of an “underlying” security.

- Valuation is mostly based on replication/hedging arguments.
  - Find a portfolio that includes the underlying security, and possibly other related derivatives, to replicate the payoff of the target derivative security, or to hedge away the risk in the derivative payoff.
  - Since the hedged portfolio is riskfree, the payoff of the portfolio can be discounted by the riskfree rate.
  - Models of this type are called “no-arbitrage” models.

- Key: No forecasts are involved. Valuation is based on cross-sectional comparison.
  - It is not about whether the underlying security price will go up or down (given growth rate or risk forecasts), but about the relative pricing relation between the underlying and the derivatives under all possible scenarios.
Readings behind the technical jargons: $P$ v. $Q$

- **$P$**: Actual probabilities that earnings will be high or low.
  - Estimated based on historical data and other insights about the company.
  - Valuation is all about getting the forecasts right and assigning the appropriate price for the forecasted risk.

- **$Q$**: “Risk-neutral” probabilities that we can use to aggregate expected future payoffs and discount them back with risk-free rate, regardless of how risky the cash flow is.
  - It is related to real-time scenarios, but it has nothing to do with real-time probability.
  - Since the intention is to hedge away risk under all scenarios and discount back with risk-free rate, we do not really care about the actual probability of each scenario happening. We just care about what all the possible scenarios are and whether our hedging works under all scenarios.
  - $Q$ is not about getting close to the actual probability, but about being fair *relative to* the prices of securities that you use for hedging.
A Micky Mouse example

Consider a non-dividend paying stock in a world with zero riskfree interest rate. Currently, the market price for the stock is $100. What should be the forward price for the stock with one year maturity?

- The forward price is $100.
  - Standard forward pricing argument says that the forward price should be equal to the cost of buying the stock and carrying it over to maturity.
  - The buying cost is $100, with no storage or interest cost.

- How should you value the forward differently if you have inside information that the company will be bought tomorrow and the stock price is going to double?
  - Shorting a forward at $100 is still safe for you if you can buy the stock at $100 to hedge.
Investing in derivative securities without insights

- If you can really forecast the cashflow (with inside information), you probably do not care much about hedging or no-arbitrage modeling.
  - You just lift the market and try not getting caught for inside trading.

- But if you do not have insights on cash flows (earnings growth etc) and still want to invest in derivatives, the focus does not need to be on forecasting, but on cross-sectional consistency.
  - The no-arbitrage pricing models can be useful.
Example: No-arbitrage dynamic term structure models

Basic idea:

- Interest rates across different maturities are related.

- A dynamic term structure model provides a (smooth) functional form for this relation that excludes arbitrage.
  - The model usually consists of specifications of risk-neutral factor dynamics ($X$) and the short rate as a function of the factors, e.g., $r_t = a_r + b_r^T X_t$.

- Nothing about the forecasts: The “risk-neutral dynamics” are estimated to match historical term structure shapes.

- A model is well-specified if it can fit most of the term structure shapes reasonably well.
A 3-factor affine model

with adjustments for discrete Fed policy changes:

Pricing errors on USD swap rates in bps

<table>
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<tr>
<th>Maturity</th>
<th>Mean</th>
<th>MAE</th>
<th>Std</th>
<th>Auto</th>
<th>Max</th>
<th>$R^2$</th>
</tr>
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<tbody>
<tr>
<td>2 y</td>
<td>0.80</td>
<td>2.70</td>
<td>3.27</td>
<td>0.76</td>
<td>12.42</td>
<td>99.96</td>
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<tr>
<td>3 y</td>
<td>0.06</td>
<td>1.56</td>
<td>1.94</td>
<td>0.70</td>
<td>7.53</td>
<td>99.98</td>
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<tr>
<td>5 y</td>
<td>-0.09</td>
<td>0.68</td>
<td>0.92</td>
<td>0.49</td>
<td>5.37</td>
<td>99.99</td>
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<tr>
<td>7 y</td>
<td>0.08</td>
<td>0.71</td>
<td>0.93</td>
<td>0.52</td>
<td>7.53</td>
<td>99.99</td>
</tr>
<tr>
<td>10 y</td>
<td>-0.14</td>
<td>0.84</td>
<td>1.20</td>
<td>0.46</td>
<td>8.14</td>
<td>99.99</td>
</tr>
<tr>
<td>15 y</td>
<td>0.40</td>
<td>2.20</td>
<td>2.84</td>
<td>0.69</td>
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<tr>
<td>30 y</td>
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<td>4.51</td>
<td>5.71</td>
<td>0.81</td>
<td>22.00</td>
<td>99.55</td>
</tr>
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</table>

- Superb pricing performance: R-squared is greater than 99%. Maximum pricing errors is 22bps.
- Pricing errors are transient compared to swap rates (0.99): Average half life of the pricing errors is 3 weeks. The average half life for swap rates is 1.5 years.
Investing in interest rate swaps based on dynamic term structure models

- If you can forecast interest rate movements,
  - Long swap if you think rates will go down.
  - Forget about dynamic term structure model: It does not help your interest rate forecasting.

- If you cannot forecast interest rate movements (it is hard), use the dynamic term structure model not for forecasting, but as a *decomposition* tool:
  \[ y_t = f(X_t) + e_t \]
  - What the model captures \((f(X_t))\) is the persistent component, which is difficult to forecast.
  - What the model misses (the pricing error \(e\)) is the more transient and hence more predictable component.

- Form swap-rate portfolios that
  - neutralize their first-order dependence on the persistent factors.
  - only vary with the transient residual movements.
Static arbitrage trading based on no-arbitrage dynamic term structure models

- For a three-factor model, we can form a 4-swap rate portfolio that has zero exposure to the factors.
  - The portfolio should have duration close to zero
  - No systematic interest rate risk exposure.
  - The fair value of the portfolio should be relatively flat over time.

- The variation of the portfolio’s market value is mainly induced by short-term liquidity shocks...

- Long/short the swap portfolio based on its deviation from the fair model value.
  - Provide liquidity to where the market needs it and receives a premium from doing so.
The time-series of 10-year USD swap rates

Hedged (left) v. unhedged (right)

It is much easier to predict the hedged portfolio (left panel) than the unhedged swap contract (right panel).
Back-testing results from a simple investment strategy
95-00: In sample. Holding each investment for 4 weeks.
Caveats

- Convergence takes time: We take a 4-week horizon.
- Accurate hedging is vital for the success of the strategy. The model needs to be estimated with dynamic consistency:
  - Parameters are held constant. Only state variables vary.
  - Appropriate model design is important: parsimony, stability, adjustment for some calendar effects.
  - Daily fitting of a simpler model (with daily varying parameters) is dangerous.
- Spread trading (one factor) generates low Sharpe ratios.
- Butterfly trading (2 factors) is also not guaranteed to succeed.
Another example: Trading the linkages between sovereign CDS and currency options

- When a sovereign country’s default concern (over its foreign debt) increases, the country’s currency tend to depreciate, and currency volatility tend to rise.
  - “Money as stock” corporate analogy.
- Observation: Sovereign credit default swap spreads tend to move positively with currency’s
  - option implied volatilities (ATMV): A measure of the return volatility.
  - risk reversals (RR): A measure of distributional asymmetry.
Co-movements between CDS and ATMV/RR

Mexico

CDS Spread, %

Implied Volatility Factor, %

Brazil

Risk Reversal Factor, %

CDS Spread, %
A no-arbitrage model that prices both CDS and currency options

• Model specification:
  - At normal times, the currency price (dollar price of a local currency, say peso) follows a diffusive process with stochastic volatility.
  - When the country defaults on its foreign debt, the currency price jumps by a large amount.
  - The arrival rate of sovereign default is also stochastic and correlated with the currency return volatility.

• Under these model specifications, we can price both CDS and currency options via no-arbitrage arguments. The pricing equations is tractable. Numerical implementation is fast.

• Estimate the model with dynamic consistency: Each day, three things vary: (i) Currency price (both diffusive moves and jumps), (ii) currency volatility, and (iii) default arrival rate.

• All model parameters are fixed over time.
The hedged portfolio of CDS and currency options

Suppose we start with an option contract on the currency. We need four other instruments to hedge the risk exposure of the option position:

1. The underlying currency to hedge infinitesimal movements in exchange rate
2. A risk reversal (out of money option) to hedge the impact of default on the currency value.
3. A straddle (at-the-money option) to hedge the currency volatility movement.
4. A CDS contract to hedge the default arrival rate variation.

The portfolio needs to be rebalanced over time to maintain neutral to the risk factors.

- The value of hedged portfolio is much more transient than volatilities or cds spreads.
Back-testing results

Brazil

Mexcio

Portion Residuals

Portfolio Residuals

Portfolio Residuals

Portfolio Residuals

Cumulative P&L

Cumulative P&L

Feb04 Sep04 Mar05 Oct05 May06

Feb04 Sep04 Mar05 Oct05 May06

Feb04 Sep04 Mar05 Oct05 May06

Feb04 Sep04 Mar05 Oct05 May06

Feb04 Sep04 Mar05 Oct05 May06

Feb04 Sep04 Mar05 Oct05 May06

Feb04 Sep04 Mar05 Oct05 May06

Feb04 Sep04 Mar05 Oct05 May06
Similar linkages between corporate CDS and stock options

All series are standardized to have similar scales in the plots.
Each bar represents one hedged portfolio. Each hedged portfolio includes 5 instruments: two CDS contracts, two options at two maturities, and the underlying stock.
Bottom line

• If you have a working crystal ball, others’ risks become your opportunities.
  • Forget about no-arbitrage models; lift the market.

• No-arbitrage type models become useful when
  • You cannot forecast the future accurately: Risk persists.
  • Hedge risk exposures.
  • Perform statistical arbitrage trading on derivative products that profit from short-term market dislocations.

• Caveats
  • When hedging is off, risk can overwhelm profit opportunities.
  • Accurate hedging requires modeling of all risk dimensions.
    • Interest rates do not just move in parallel, but also experience systematic moves in slopes and curvatures.
    • Capital structure arbitrage: Volatility and default rates are not static, but vary strongly over time in unpredictable ways.