

Variance dynamics: Joint evidence from options and high-frequency returns

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ABSTRACT

This paper analyzes the S&P 500 index return variance dynamics and the variance risk premium by combining information in variance swap rates constructed from options and quadratic variation estimators constructed from tick data on S&P 500 index futures. Estimation shows that the index return variance jumps. The jump arrival rate is not constant over time, but is proportional to the variance rate level. The variance jumps are not rare events but arrive frequently. Estimation also identifies a strongly negative variance risk premium, the absolute magnitude of which is proportional to the variance rate level.

JEL classification: C13, C51, G12, G13.

Keywords: Return variance dynamics, variance risk premium, options, variance swap rates, high-frequency returns, market microstructure, realized variance, quadratic variation, time-changed Lévy processes.

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1. Introduction

Return variance on financial securities is stochastic. Understanding return variance dynamics is imperative for derivative pricing, risk management, and asset pricing in general. Yet, how to model and estimate variance dynamics remains a challenging task mainly because return variance is not directly observable as security prices do. Recent advances in two frontiers of finance greatly enhance the identification of return variance dynamics and its pricing. One frontier is the derivatives market. Researchers show that under very general settings, return variance swap rate, which equals the risk-neutral expected value of return variance over a fixed horizon, can be well approximated by the value of a specific portfolio of options across different strikes at the same maturity.¹ In line with this theoretical development, on September 22, 2003, the Chicago Board of Options Exchange (CBOE) launched a new volatility index, the VIX, and back-calculated this index back to 1990. This index approximates the 30-day variance swap rate on the S&P 500 index (Carr and Wu (2006)). Thus, the risk-neutral expected value of the 30-day variance becomes an approximately observable series.

The other frontier is the market microstructure. The increasing availability of high-frequency data spurs a rapid development of new theories on computing realized variance from high-frequency returns. If the true price of a security can be sampled frequently, the sum of the squared returns over a sample period converges in the limit to the return quadratic variation for that period. More recently, researchers realize that microstructure noises can bias the estimate of return quadratic variation. They propose various methods to account for the microstructure effects.²

Variance swap rates constructed from options and quadratic variation estimators constructed from high-frequency returns make the return variance an almost observable quantity, up to a risk-neutral projection in the former and up to a random-jump related error term in the latter. Directly using these series can make inferences on variance dynamics more accurate and less reliant on the underlying return dynamics specification. In this paper, I exploit the recent advances in both frontiers and estimate the variance dynamics and variance risk premium on S&P 500 index from a joint analysis of over 15 years of daily time series on the

¹Theoretical works on the replication of variance swap contracts include Carr and Madan (1998), Demeterfi, Derman, Kamal, and Zou (1999), Britten-Jones and Neuberger (2000), Jiang and Tian (2005), and Carr and Wu (2009).

²Examples include Andreou and Ghysels (2002), Ait-Sahalia, Mykland, and Zhang (2005), Andersen, Bollerslev, and Meddahi (2005), Oomen (2005), Zhang, Mykland, and Ait-Sahalia (2005), Bandi and Russell (2006), Hansen and Lunde (2006), Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008), and Podolskij and Vetter (2009).

VIX index and several quadratic variation estimators constructed from tick data on S&P 500 index futures.

Starting with a stochastically time-changed Lévy process for the S&P 500 index return, this paper uses the stochastic time change to capture the index return variance dynamics. By specifying the time change via the instantaneous variance rate, the paper identifies the variance rate dynamics using the variance swap rate and the quadratic variation estimators, without ever specifying, and hence without jointly testing, the Lévy process that defines the return innovations.

This paper analyzes the variance rate dynamics and pricing within the flexible affine framework of Filipović (2001). Within this framework, the paper estimates several variance dynamics specifications. Estimation shows that the variance rate jumps. The jump arrival rate is not constant over time, but is proportional to the variance level. Jumps in the variance are not rare events but arrive frequently. Estimation also identifies a strongly negative variance risk premium, the absolute magnitude of which is proportional to the variance level.

Combining realized variance estimators with variance swap rates, this paper provides insights on the discontinuous movements of the variance rate dynamics, and answers questions on whether the variance jump intensity is proportional to the variance rate level and whether the variance jumps arrive frequently or they are rare events. These questions have not been effectively addressed in the option pricing literature due to identification issues (e.g., Eraker (2004)), nor in the realized variance literature due to the nonparametric nature of the proposed tests (e.g., Barndorff-Nielsen and Shephard (2004)).

In other related works, Jones (2003) studies the variance dynamics using daily series on index returns and CBOE's old volatility index VXO. Chernov (2007) analyzes the link between high-low range volatility estimators and the VXO. In this paper, I use tick data to construct realized variance estimators, which contain much less noise than estimators from daily returns or high-low data. Furthermore, I use the new VIX index instead of the old VXO, because the new VIX index directly approximates the variance swap rate under very general settings but CBOE's construction of the old VXO involves an erroneous day-counting conversion that biases the estimate upward (Carr and Wu (2006)). Also related is a working paper by Bollerslev, Gibson, and Zhou (2004), who construct a risk aversion index using realized variance and the new VIX.

The rest of the paper is structured as follows. The next section establishes the theory that underlies

our variance dynamics estimation. Section 3 describes the data and the estimation procedure. Section 4 discusses the estimation results. Section 5 concludes.

2. Theory

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ represent a stochastic basis, with \mathbb{P} being the physical measure that governs the time series dynamics of the S&P 500 index returns,

$$\ln S_T/S_t = \int_t^T \theta_s ds + L_{\mathcal{T}_{[t,T]}}, \quad (1)$$

where θ_t denotes the instantaneous drift of the index return, L_t denotes a standardized Lévy process with its variance normalized to t , and $\mathcal{T}_{[t,T]}$ denotes a continuous and differentiable stochastic time change that captures the randomness in the *integrated variance* over horizon $[t, T]$,

$$\mathcal{T}_{[t,T]} \equiv \int_t^T v_-(s) ds, \quad (2)$$

where $v(t)$ denotes the *instantaneous variance rate*.

The objective of this paper is to estimate the instantaneous variance rate dynamics $v(t)$ without ever specifying the return drift process θ_t or the return innovation (the standardized Lévy process L_t). The estimation is based on observations on two quantities. The first is the volatility index VIX squared, which approximates the 30-day variance swap rate on the S&P 500 index and hence the expected value of the 30-day quadratic variation (QV) under a risk-neutral measure \mathbb{Q} ,

$$VIX_{t,h}^2 \doteq \frac{1}{h} \mathbb{E}_t^{\mathbb{Q}} [QV_{[t,t+h]}] = \frac{1}{h} \mathbb{E}_t^{\mathbb{Q}} [\mathcal{T}_{[t,t+h]}^{\mathbb{Q}}], \quad h = 30/365, \quad (3)$$

where the second equality replaces the quadratic variation with its risk-neutral expected value over random jumps, i.e., the risk-neutral integrated variance $\mathcal{T}_{[t,t+h]}^{\mathbb{Q}}$. The first equality is an approximation. Carr and Wu (2009) show that approximation errors can arise from discrete strikes and discontinuous movements in the S&P 500 index. Nevertheless, under commonly used models and reasonable model parameters, they use numerical analysis to show that the approximation errors from both sources are small. In this paper, I treat

the approximation errors as negligible and infer the variance rate from the VIX squared under affine model specifications.

The second quantity is an estimator on the *daily quadratic variation*, $QV_{[t,t+\delta]}$, with $\delta = 1/365$ denoting the length of one day. The quadratic variation is defined as the limit in probability of the sum of return squared as the sampling interval approaches zero,

$$QV_{[t,t+\delta]} \equiv \lim_{\Delta \rightarrow 0} \sum_{j=1}^{[\delta/\Delta]} (\ln S_{t+j\Delta}/S_{t+(j-1)\Delta})^2, \quad (4)$$

where $[\delta/\Delta]$ denotes the number of observations in a day given sampling interval Δ . Since the integrated variance represents an expectation of the quadratic variation over the random jump realization, we can write,

$$QV_{[t,t+\delta]} = \mathcal{I}_{t,t+\delta} + e_{[t,t+\delta]}, \quad (5)$$

where the zero-mean error term $e_{[t,t+\delta]}$ is induced by the random jumps in the index return. In particular, if we let $\mu_t(dx)$ denote the counting measure of jumps in the index return and let $\nu_t(dx)$ denote the corresponding compensator, which measures the arrival rate of jumps of size x , the error term becomes

$$e_{[t,t+\delta]} = \int_t^{t+\delta} \int_{\mathbb{R}^0} x^2 (\mu_s(dx) - \nu_s(dx)) ds, \quad (6)$$

where \mathbb{R}^0 denotes the real line excluding zero.

Given daily quadratic variation estimators (\widehat{QV}_t) from high-frequency return data, we can rewrite equation (5) as

$$\widehat{QV}_t = v_t \delta \phi + e_t, \quad (7)$$

where I approximate the integral in (5) by assuming constant variance rate within a day t with δ denoting the length of one day. In this approximation, I include an additional scaling coefficient ϕ to further adjust scale mismatches between the quadratic variation estimators and the annualized variance rate derived from the variance swap rate from equation (3). The mismatches can come from a number of sources. For example,

the quadratic variation estimators are computed based on tick data on index futures during the day trading session from 9:30am to 3:15pm Chicago time. Thus, the estimators do not capture after-hour trading activities and overnight information flows that can move the index futures. Further, the instantaneous variance rate derived from the variance swap rate can differ from the variance rate level under the statistical measure if the index level jumps and the jump risk is priced.

The error term e_t in equation (7) can come either from the random return jump realizations as defined in (6), or from noises originated in the particular quadratic variation estimator (Meddahi (2003)). I assume an AR(1) dynamics for the error term to accommodate potential serial dependence,

$$e_{t+1} = \phi_e e_t + \varepsilon_{t+1}, \quad (8)$$

where the daily time step is normalized to one for notational clarity and ε denotes an iid zero-mean random error term. A non-zero autocorrelation ϕ_e can be induced by persistence of the return jump arrival rates and/or microstructure noises in the quadratic variation estimator.

The distribution of ε depends on both the jump structure as defined in (6) and the distribution of the random noise in the quadratic variation estimator. If we make the simplifying assumption that ε is normally distributed with variance V_e , we can write the daily log likelihood contribution from the quadratic variation estimator as,

$$l(e_{t+1}|e_t) = -\frac{1}{2} \left[\ln(2\pi) + \ln(V_e) + (e_{t+1} - \phi_e e_t)^2 / V_e \right]. \quad (9)$$

When the distribution of ε deviates from the normal assumption, (9) represents a valid quasi-likelihood for inferences on the conditional mean of the quadratic estimator, $v_t \delta \varphi$. In particular, the inferences on $v_t \delta \varphi$ do not depend on the distribution of ε nor on its variance V_e , and standard likelihood ratio tests are equally applicable to the quasi-likelihood (McCullagh and Nelder (1983), pp.168–172). In estimating a constant variance from a noisy quadratic variation estimator, Ait-Sahalia, Mykland, and Zhang (2005) show that even if the noise distribution is misspecified, the variance estimator obtained by maximizing the misspecified log-likelihood function remains consistent and the asymptotic variance of the estimator is the same as that from the true likelihood. Xiu (2009) further shows that in the presence of stochastic volatility and market microstructure noise, the integrated variance estimator remains consistent, efficient, and robust as a quasi-

maximum likelihood estimator under the misspecified assumptions. Hence, inferences on the variance rate dynamics remain valid whether equation (9) is regarded as the true log likelihood or quasi-likelihood on the quadratic variation estimator.

2.1. Affine variance rate dynamics and variance swap pricing

To derive the likelihood on the instantaneous variance rate, I consider a general one-factor affine specification (Filipović (2001)), under which the variance rate is governed by the stochastic differential equation,

$$dv_t = (a - \kappa v_t) dt + \sqrt{\omega v_t} dW_t + \int_{\mathbb{R}_0^+} x [\mu(dx) - (v_c(dx) + v_t v_p(dx))] dt, \quad (10)$$

where a, ω, κ are all positive constants, \mathbb{R}_0^+ denotes the positive half line excluding zero, and $v_c(dx)$ and $v_p(dx)$ denote two nonnegative Borel measures that define the arrival rate of jumps in the variance rate, with $v_c(dx)$ describing a jump component with a constant arrival rate and $v_p(dx)$ describing a jump component with the arrival rate proportional to the variance rate level. The two Borel measures must satisfy the following condition:

$$\int_{\mathbb{R}_0^+} (x 1_{x < 1}) v_c(x) dx + \int_{\mathbb{R}_0^+} (x^2 1_{x < 1}) v_p(x) dx < \infty. \quad (11)$$

where $1_{x < 1}$ is an indicator function that is valued at one when $x < 1$ and zero otherwise.

To obtain the conditional likelihood on the variance rate, I start with the characteristic function,

$$\phi_{v_t}(u) \equiv \mathbb{E}_t [e^{iu v_{t+\delta}}], \quad u \in \mathbb{R}, \quad (12)$$

where δ refers to the daily time interval. The affine dynamics in equation (10) dictates that the characteristic function is exponential affine in v_t ,

$$\phi_{v_t}(u) = \exp(-b(\delta) v_t - c(\delta)), \quad (13)$$

where the coefficients $[b(\delta), c(\delta)]$ solve the following ordinary differential equations:

$$\begin{aligned} b'(\delta) &= -\kappa b(\delta) - \frac{1}{2}\omega b(\delta)^2 + \int_{\mathbb{R}_0^+} \left(1 - e^{-xb(\delta)} - b(\delta)(x1_{x<1})\right) v_p(dx), \\ c'(\delta) &= ab(\delta) + \int_{\mathbb{R}_0^+} \left(1 - e^{-xb(\delta)}\right) v_c(dx), \end{aligned} \quad (14)$$

with the initial boundary conditions: $c(0) = 0$ and $b(0) = -iu$. The ordinary differential equations can be solved using standard numerical methods. Given the characteristic function, the conditional density of $v_{t+\delta}$ conditional on the observation v_t can be computed via fast Fourier inversion.

To infer the variance rate from the variance swap rate according to equation (3), I further specify an analogous variance rate dynamics under the risk-neutral measure \mathbb{Q} ,

$$dv_t = (\tilde{a} - \tilde{\kappa}v_t)dt + \sqrt{\omega v_t}dW_t + \int_{\mathbb{R}_0^+} x[\mu(dx) - (\tilde{v}_c(dx) + v_t\tilde{v}_p(dx))]dt, \quad (15)$$

where the tildes on a, κ, v_c , and v_p denote their corresponding quantities under measure \mathbb{Q} . Given the affine \mathbb{Q} -dynamics, we can write the variance swap rate as an affine function of the instantaneous variance rate,³

$$SW_{t,h} = \frac{1}{h}\mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{t+h} v_-(s)ds\right] = b(h)v_t + c(h), \quad (16)$$

where the coefficients $[b(h), c(h)]$ are given by,

$$b(h) = \frac{1 - e^{-\tilde{\kappa}_J h}}{\tilde{\kappa}_J h}, \quad c(h) = \frac{\tilde{a}_J}{\tilde{\kappa}_J} \left(1 - \frac{1 - e^{-\tilde{\kappa}_J h}}{\tilde{\kappa}_J h}\right), \quad (17)$$

with \tilde{a}_J and $\tilde{\kappa}_J$ being the jump-adjusted risk-neutral drift coefficients $\tilde{\kappa}_J = \tilde{\kappa} - \nabla\psi_p(0)$ and $\tilde{a}_J = \tilde{a} - \nabla\psi_c(0)$, where $\nabla\psi_c(0)$ and $\nabla\psi_p(0)$ denote the gradients of the characteristic exponents of $v_c(x)$ and $v_p(x)$, respectively, evaluated at zero. The variance swap rate depends only on the risk-neutral drift specification of the variance rate, but not on the innovation specification. Hence, we leave $\tilde{v}_c(dx)$ and $\tilde{v}_p(dx)$ unspecified.

³See Egloff, Leippold, and Wu (2009) for the derivation of variance swap pricing under general affine specifications.

2.2. Variance rate jump specification

The coefficients on the characteristic function of the instantaneous variance rate in (14) depend on the specification of the Borel measures for the two jump components. The contributions of the jump components are related to their respective characteristic exponents $\psi_j(u)$, defined as

$$\psi_j(u) \equiv \int_{\mathbb{R}_0^+} (1 - e^{iux} + iu(x1_{x<1})) \nu_j(dx), \quad j = c, p, \quad (18)$$

where the term with the truncation function $(x1_{x<1})$ is needed for infinite variation jump components so that the integral is well defined (Bertoin (1996)).

In this paper, I consider a general class of jump specifications, with the jump arrival rate controlled by an exponentially dampened power law:

$$\nu_c(dx) = \lambda_c e^{-\frac{x}{v_c}} x^{-\alpha_c-1} dx, \quad \nu_p(dx) = \lambda_p e^{-\frac{x}{v_p}} x^{-\alpha_p-1} dx, \quad \alpha_c < 1, \alpha_p < 2. \quad (19)$$

Intuitively, the proportional coefficients (λ_c, λ_p) control the overall arrival rate, the exponential scaling coefficients (v_c, v_p) control the arrival of large jumps, and the power coefficients (α_c, α_p) control the arrival of small jumps. Depending on the value of the power coefficient α , the jump process shows different behaviors. I consider three values for α , at -1 , 0 , and 1 , respectively.

When $\alpha_j = -1$ for $j = c, p$, the power component in (19) disappears. The jump structure generates a finite number of jumps within any finite time interval, with the mean jump arrival rate given by $\int_0^\infty \nu_j(dx) = \lambda_j v_j$. The characteristic exponent is given by

$$\psi_j(u) = \lambda_j \int_0^\infty (1 - e^{iux}) e^{-\frac{x}{v_j}} dx = \frac{-iu\lambda_j v_j^2}{1 - iuv_j}. \quad (20)$$

The jump-adjusted drift terms become $a_J = a + \lambda_c v_c^2$ and $\kappa_J = \kappa + \lambda_p v_p^2$.

When $\alpha_j = 0$ for $j = c, p$, the sample path exhibits infinite activity but finite variation. The jump structure generates an infinite number of jumps within any finite time interval. The integral of the Borel measure $\nu_j(dx)$ is no longer finite, but the integral $\int_0^\infty (x1_{x<1}) \nu_j(dx) < \infty$ is finite. The characteristic

exponent is

$$\Psi_j(u) = \lambda_j \int_0^\infty (1 - e^{iux}) e^{-\frac{x}{v_j}} x^{-1} dx = \lambda_j \ln(1 - iuv_j). \quad (21)$$

The jump-adjusted drift terms become $a_J = a + \lambda_c v_c$ and $\kappa_J = \kappa + \lambda_p v_p$.

When $\alpha_p = 1$, the sample paths of the jump show infinite variation, with $\int_0^\infty (x 1_{x < 1}) v_j(dx)$ no longer finite. The sample paths retains finite quadratic variation as $\int_0^\infty (x^2 1_{x < 1}) v_j(dx) < \infty$. Hence, it can only be used to model $v_p(dx)$. The characteristic exponent is (Wu (2006)),

$$\Psi_p(u) = \lambda_p \int_0^\infty (1 - e^{iux} + iu(x 1_{x < 1})) e^{-\frac{x}{v_p}} x^{-2} dx = -\lambda_p (1/v_p - iu) \ln(1 - iuv_p) - iu\lambda_p (1 + \mathcal{E}(1/v_p)),$$

where $\mathcal{E}(x)$ denotes the exponential integral function, $\mathcal{E}(x) \equiv \int_x^\infty e^{-z} z^{-1} dz$. The jump-adjusted drift term becomes $\kappa_J = \kappa + \lambda_p \mathcal{E}(1/v_p)$.

3. Data and estimation

The estimation involves two sets of data. The first is CBOE's new VIX series constructed from European options on S&P 500 index. The VIX squared approximates the 30-day variance swap rate on the S&P 500 index. The daily closing time series of VIX can be downloaded from the CBOE website. The sample used for estimation is daily from January 2, 1990 to December 31, 2004, a total of 3,787 observations.

The second set of data are quadratic variation estimators that I construct from tick data on S&P 500 index futures. The data are purchased from Tick Data, Inc. for the matching sample period. The tick data cover the day session from 9:30 am to 3:15 pm Chicago time. From the tick data, I construct three daily quadratic variation estimators. The first is the *raw return realized variance* (RRRV), which is constructed by simply summing the squared returns,

$$RRRV_t = \sum_{j=1}^m r_{t,j}^2, \quad (22)$$

where $r_{t,j}$ denotes the futures return at day t and the j th second and m denotes the number of seconds in day t . The tick data are expanded to second-by-second equidistant intervals to obtain second by second returns.

When more than one observations are available within a second interval, the last one is used. When there are no transactions, the previously available price is carried over. In the latter case, the second-by-second return is zero. The raw variance estimators are used in early works, e.g., Andersen, Bollerslev, Diebold, and Ebens (2001) and Andersen, Bollerslev, Diebold, and Labys (2003). To avoid the impact of microstructure noise, these authors often choose a lower sampling frequency.

The second estimator is the autoregressive bias corrected realized variance (ACRV) proposed by Hansen and Lunde (2006):

$$ACRV_t = \sum_{j=1}^m r_{t,j}^2 + 2 \sum_{h=1}^q \frac{m}{m-h} \sum_{i=1}^{m-h} r_{t,i} r_{t,i+h} = RRRV_t + 2 \sum_{h=1}^q \frac{m}{m-h} \sum_{i=1}^{m-h} r_{t,i} r_{t,i+h}. \quad (23)$$

The third estimator is the two-scale realized variance estimator (TSRV) proposed by Zhang, Mykland, and Aït-Sahalia (2005), which involves calculating the realized variance at two sampling frequencies. Let $RRRV_t^1$ denote the raw return realized variance estimated at the highest frequency of the available data, which is second by second in this paper, and let $RRRV_t^q$ denote the raw return realized variance based on data sampled every q seconds. By starting at different points, one can compute q different estimators for $RRRV_t^q$ based on q different subsamples. Let \overline{RRRV}_t^q denote the average of the q subsample $RRRV_t^q$ estimators. The two-scale estimator combines the average subsample estimators with the raw return estimator at the highest frequency to cancel out the effect of microstructure noise,

$$TSRV_t = \left(\overline{RRRV}_t^q - \frac{1}{q} RRRV_t^1 \right) / \left(1 - \frac{1}{q} \right). \quad (24)$$

Based on a comparison of the sample mean and the sample variance of the ACRV estimators at different window sizes and the TSRV estimators at different second scales, I choose $q = 50$ seconds as the window size for the ACRV estimator and $q = 50$ seconds as the subscale for the TSRV estimator when they are used in the variance rate dynamics estimation.

The variance rate dynamics are estimated with (quasi-)maximum likelihood method based on the time series of VIX and one of the three quadratic variation estimators. The estimation proceeds as follows. First, given model parameters, the instantaneous variance rate, v_t , is inferred from the observation on the VIX according to the following affine relation,

$$VIX_t^2 = b(h)v_t + c(h), \quad (25)$$

where $h = 30/365$ denotes the 30-day maturity of the variance swap rate and the coefficients $[b(h), c(h)]$ are functions of the risk-neutral drift parameters $(\tilde{\alpha}_J, \tilde{\kappa}_J)$ of the variance rate, as given in (17). Second, given the extracted time series on the instantaneous variance rates v_t , the daily likelihood function for v_{t+1} conditional on v_t , $l(v_{t+1}|v_t)$, can be computed by applying fast Fourier transform on the characteristic function in equation (13). Third, given the instantaneous variance rate v_t , the error term in the quadratic variation estimator, e_t , is derived from equation (7). The daily (quasi-)likelihood function on the error term, $l(e_{t+1}|e_t)$, is constructed according to equation (9). Finally, the model parameters, Θ , are estimated by maximizing the sum of daily likelihood functions on v_t and e_t :

$$\Theta = \arg \min_{\Theta} \mathcal{L}(\Theta), \quad \text{with} \quad \mathcal{L}(\Theta) = \sum_{t=1}^{N-1} l(v_{t+1}|v_t) + l(e_{t+1}|e_t), \quad (26)$$

where $N = 3,787$ denotes the number of business days in the 15-year sample period.

With each quadratic variation estimator, I estimate eight models that differ in the jump specifications. Among the eight models, two models assume constant jump arrival rates ($v_p(dx) = 0$) with the power coefficients $\alpha_c = -1$ and 0 , respectively; three models assume proportional jump arrival rates ($v_c(dx) = 0$) with $\alpha_p = -1, 0$, and 1 , respectively; and three nesting models that include both a constant and a proportional jump model, with $\alpha_c = \alpha_p = -1$, $\alpha_c = \alpha_p = 0$, and $\alpha_c = -1, \alpha_p = 1$, respectively. For each model, I repeat the estimation three times, each time using a different quadratic variation estimator.

4. Return variance dynamics and pricing

Tables 1 to 3 report the parameter estimates, t -statistics, and the maximized likelihood values for the eight model specifications and the three quadratic variation estimators. Each table summarizes the results on one quadratic variation estimator.

4.1. Return variance jumps, with the arrival rate proportional to the variance rate level

Under each jump type specification (α) and for each quadratic variation estimator, I estimate a model with only constant jump arrivals and a model with jump arrival rates proportional to the instantaneous variance rate. In all scenarios, the maximized likelihood values are larger for the proportional jump specification

than for the constant jump specification, suggesting that the proportional jump arrival specification captures the index return variance dynamics better.

To measure the statistical significance of the performance difference, I construct a likelihood ratio test for non-nested models according to Vuong (1989), $\mathcal{V}_{pc} = \sqrt{N-1}\mu_{lr}/\sigma_{lr}$, where μ_{lr} and σ_{lr} denote the sample mean and standard deviation of the daily likelihood ratios between the two models p and c , with p denoting the model with proportional jumps and c the model with constant jumps. Under the null hypothesis that the two models are identical in performance, the statistic \mathcal{V}_{pc} has a standard normal distribution asymptotically. Table 4 reports the estimates on \mathcal{V}_{pc} under two different jump type specifications ($\alpha = -1, 0$) and three different quadratic variation estimators (RRRV, ACRV, and TSRV). In calculating the statistics, the standard deviation estimator is adjusted for serial dependence according to Newey and West (1987), with the number of lags optimally chosen according to Andrews (1991) based on an AR(1) specification on the series. The estimates for the statistics are large and positive in all scenarios, suggesting that the proportional jump specification significantly outperforms the constant jump specification.

The paper also estimates models that allow both components, with the same jump type for both components: $\alpha_c = \alpha_p = -1$ and $\alpha_c = \alpha_p = 0$. When the proportional jump exhibits infinite variation with $\alpha_p = 1$, the finite activity jump ($\alpha_c = -1$) is used for the constant jump component, since the sample paths of the constant jump component must show finite variation. To test whether a constant jump component is still needed when there exists a proportional jump component, I perform a standard likelihood ratio test between the nesting model with both jump components and the corresponding restricted version with only the proportional jump component, $\mathcal{LR}_{bp} = 2(\mathcal{L}_b - \mathcal{L}_p)$, where the subscript b denotes a model with both constant and proportional jump components and p denotes the corresponding model with only the proportional jump component. The statistic \mathcal{LR}_{bp} has a chi-square distribution with two degrees of freedom since the nesting model has two additional free parameters. Table 5 reports the likelihood ratio estimates and the corresponding p -values in parentheses. The test statistics suggest that a constant jump component is needed in addition to the proportional jump component when the chosen jump specification shows finite activity ($\alpha_p = -1$). The need becomes marginal when the jump specification shows infinite activity but finite variation ($\alpha_p = 0$). Finally, when an infinite variation jump specification is chosen ($\alpha_p = 1$) for the proportional jump component, a constant jump component is no longer necessary, as the model with a purely proportional jump

component can no longer be rejected.

Also indicative are the t -statistics of the parameter estimates on the two jump components. In all scenarios, the estimates for the two parameters that govern the proportional jump component (λ_p, ν_p) are highly significant. The estimates for the constant jump parameters (λ_c, ν_c) are also statistically significant when proportional jumps are not allowed. However, once a proportional jump component is allowed, the estimates for the constant jump parameters become insignificant. Compared with the chi-square tests, the power of the t -tests seem lower. Nevertheless, the results from the two types of tests are largely consistent with each other. As α_p increases from -1 to 0 and then to 1 , the t -values on the constant jump parameters (λ_c, ν_c) become smaller, so are the p values from the chi-square tests.

Taken together, the estimation results show that the index return variance rate jumps, and the jump arrival rate is not constant over time, but is proportional to the instantaneous variance rate level. Once a proportional jump component is incorporated, it is largely unnecessary to include another jump component with constant arrival rates.

4.2. Variance jumps arrive frequently

For a proportional jump component, the behavior of the sample paths of the jump component is determined by the power coefficient α_p in the Borel measure $\nu_p(dx)$. The sample paths exhibit finite activity, and generate a finite number of jumps within any finite interval when the coefficient $\alpha_p < 0$. The sample paths show infinite activity but finite variation when $0 \leq \alpha_p < 1$ and infinite variation when $1 \leq \alpha < 2$.

The estimation results in Tables 1 to 3 show that in all cases the maximized likelihood values increase monotonically as the power coefficient α_p increases from -1 to 0 and then to 1 , suggesting that high-frequency jumps are better suited to capture the discontinuous component of the variance dynamics. To test the statistical significance of the jump differences, the paper resorts to Vuong (1989)'s likelihood ratio test statistic for nonnested models, \mathcal{V}'_α , and construct the test between the infinite-variation jump specification ($\alpha_p = 1$) and the other two jump specifications ($\alpha = -1$ and 0 , respectively). Given the conclusion from the previous subsection, the test is conducted on the models with only proportional jumps. The estimates for the statistics are reported in Table 6. The statistics are all positive, but the magnitudes are moderate at

around one, thus only showing marginal significance at around 80 percent confidence level for one-sided tests and 60 percent level for two-sided tests. Overall, the estimation provides weak evidence that jumps in the variance rate dynamics arrive frequently.

4.3. *Variance risk premia are negative, with the magnitude proportional to the variance level*

Using data from both the options market and high-frequency returns, the estimation identifies the drift parameters under both the statistical measure and the risk-neutral measure. The differences between the two sets of parameters determine the variance risk premium. The constant portion of the variance risk premium is $CRP = a_J - \tilde{a}_J$. The proportional portion of the variance risk premium is $PRP = \tilde{\kappa}_J - \kappa_J$.

Tables 1 to 3 also report the two risk premium components and their t -statistics corresponding to each set of parameter estimates. The estimates for the constant risk premium component (CRP) are positive, but small and mostly insignificant. In contrast, the estimates for the proportional risk premium component (PRP) are negative, large in absolute magnitude, and highly significant. The strongly negative risk premium that is proportional to the variance rate level can come from a negative and proportional market price of the diffusion risk or from a negative market price on the proportional jump risk, or both. The negative sign of the risk premium is in line with the empirical findings in Bakshi and Kapadia (2003), Bollerslev, Gibson, and Zhou (2004), and Carr and Wu (2009).

Compared to the literature, the estimates for the statistical mean reversion speed κ are two-three times larger in this paper. By directly inverting the VIX squared to obtain the variance rate $v(t)$, the potential measurement errors in the VIX index are directly carried over to the variance rate series. These transient measurement errors, together with the daily estimation frequency, can potentially reduce the estimated persistence of the variance rate dynamics. A direction for future research is to incorporate variance swap rates across several different maturities to reduce the impact of the measurement errors (Egloff, Leippold, and Wu (2009)).

5. Conclusion

This paper studies the S&P 500 index return variance dynamics and variance risk premium using information from both the options market and high-frequency returns. Estimation shows that the index return variance rate jumps, and that the jump arrival rate is not constant over time, but is proportional to the variance level. Jumps in the variance rate dynamics are not rare events, but arrive frequently. Estimation also identifies a strongly negative variance risk premium, the absolute magnitude of which is proportional to the instantaneous variance rate level.

Option pricing and market microstructure represent two frontiers in finance that are experiencing rapid development in terms of both industry expansions and academic advancement. This article represents a nascent effort in bringing these two frontiers together. One direction for future research is to lever the recent development in the two frontiers to study the dynamic interactions of return variance and variance risk premiums on a large cross section of stocks and stock indexes.

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Table 1

Model parameter estimates on variance dynamics using RRRV estimators.

Θ	$\alpha_c = \alpha_p = -1$			$\alpha_c = \alpha_p = 0$			$\alpha_c = -1, \alpha_p = 1$	
$\tilde{\alpha}_J$	0.2094 (24.29)	0.2094 (227.01)	0.2093 (97.55)	0.2095 (31.71)	0.2090 (146.55)	0.2092 (149.15)	0.2089 (172.32)	0.2099 (8.69)
$\tilde{\kappa}_J$	0.0038 (0.08)	0.0038 (0.49)	0.0036 (0.01)	0.0010 (0.00)	0.0010 (0.44)	0.0010 (0.01)	0.0003 (0.00)	0.0003 (0.00)
a	0.1893 (6.66)	0.2509 (9.32)	0.2666 (10.20)	0.1621 (5.39)	0.2572 (9.38)	0.2700 (6.51)	0.2767 (10.32)	0.3021 (0.00)
κ	12.7990 (13.41)	26.7110 (14.71)	29.5967 (14.35)	12.1606 (12.59)	27.6823 (13.69)	33.5412 (12.39)	10.3803 (10.52)	95.6644 (0.56)
ω	0.1920 (36.24)	0.1435 (26.43)	0.1330 (21.87)	0.1922 (33.27)	0.1462 (26.42)	0.1355 (23.30)	0.1286 (19.10)	0.1223 (19.09)
λ_c	254.1445 (7.39)	— —	23.2130 (1.35)	9.2714 (5.36)	— —	0.2040 (0.01)	— —	95.5480 (0.56)
v_c	0.0133 (7.24)	— —	0.0667 (1.60)	0.0313 (5.63)	— —	0.0028 (0.01)	— —	0.0000 (0.08)
λ_p	— —	19.5190 (11.40)	21.8630 (11.33)	— —	59.8609 (5.67)	100.1524 (5.32)	8.1537 (8.90)	9.0945 (8.60)
v_p	— —	0.1552 (13.89)	0.1346 (11.81)	— —	0.3482 (8.32)	0.2574 (9.86)	1.1049 (5.32)	0.9557 (5.79)
ϕ_e	0.4606 (112.25)	0.4603 (111.99)	0.4596 (118.43)	0.4603 (116.46)	0.4605 (112.09)	0.4591 (111.39)	0.4596 (117.88)	0.4610 (113.71)
V_e	8.0118 (237.93)	8.0122 (238.78)	8.0158 (242.09)	8.0360 (240.14)	8.0111 (238.73)	8.0189 (238.49)	8.0170 (241.89)	8.0439 (229.68)
φ	1.1752 (96.13)	1.1751 (96.12)	1.1748 (51.97)	1.1734 (56.26)	1.1750 (96.52)	1.1746 (59.10)	1.1746 (60.20)	1.1734 (81.67)
<i>CRP</i>	0.0252 (0.67)	0.0415 (1.50)	0.1605 (1.30)	0.2431 (5.01)	0.0482 (1.70)	0.0614 (2.04)	0.0678 (2.45)	0.0922 (0.00)
<i>PRP</i>	-12.7953 (-13.43)	-27.1776 (-15.00)	-29.9895 (-14.43)	-12.1596 (-11.47)	-48.5229 (-13.00)	-59.3227 (-11.11)	-12.4830 (-11.82)	-97.5111 (-89.55)
\mathcal{L}/N	0.2677	0.2961	0.2986	0.2668	0.2970	0.2976	0.2988	0.2989

Notes: Entries report the estimates of the structural parameters and their t -statistics (in parentheses) that govern the S&P 500 index return variance rate dynamics and risk premium. CRP and PRP denote the constant and proportional portion of the variance risk premium, computed from the parameter estimates. The last row reports the average of the maximized daily likelihood value. Estimation is based on daily data on the VIX and daily raw return realized variance (RRRV) estimators constructed using second-by-second returns on S&P 500 index futures. The sample is from January 2, 1990, to December 31, 2004, a total of 3,787 observation for each series.

Table 2

Model parameter estimates on variance dynamics using ACRV estimators.

Θ	$\alpha_c = \alpha_p = -1$			$\alpha_c = \alpha_p = 0$			$\alpha_c = -1, \alpha_p = 1$	
$\tilde{\alpha}_J$	0.2094 (24.62)	0.2094 (242.65)	0.2093 (90.49)	0.2095 (34.01)	0.2090 (147.42)	0.2092 (120.23)	0.2089 (188.91)	0.2099 (8.52)
$\tilde{\kappa}_J$	0.0038 (0.12)	0.0038 (0.16)	0.0036 (0.01)	0.0010 (0.00)	0.0010 (0.09)	0.0010 (0.00)	0.0003 (0.01)	0.0003 (0.00)
a	0.1891 (6.66)	0.2520 (9.34)	0.2664 (10.18)	0.1620 (5.37)	0.2571 (9.38)	0.2700 (5.94)	0.2770 (10.25)	0.3026 (0.02)
κ	12.7829 (13.35)	26.7474 (14.73)	29.5598 (14.35)	12.1603 (12.60)	27.7475 (13.70)	33.5550 (13.46)	10.3992 (10.55)	95.6678 (5.20)
ω	0.1921 (36.25)	0.1436 (26.73)	0.1331 (21.90)	0.1922 (32.74)	0.1459 (26.24)	0.1355 (20.71)	0.1283 (20.09)	0.1212 (18.14)
λ_c	253.4256 (7.57)	—	23.2460 (1.35)	9.2730 (5.67)	—	0.2040 (0.01)	—	95.5514 (5.30)
v_c	0.0133 (7.37)	—	0.0668 (1.62)	0.0313 (6.30)	—	0.0028 (0.04)	—	0.0000 (0.09)
λ_p	—	19.5117 (11.41)	21.8341 (11.32)	—	60.1537 (5.70)	100.1863 (5.79)	8.1719 (9.14)	9.2324 (8.42)
v_p	—	0.1555 (14.16)	0.1348 (12.00)	—	0.3480 (8.48)	0.2573 (10.23)	1.1036 (5.66)	0.9404 (6.33)
ϕ_e	0.3546 (90.96)	0.3544 (90.85)	0.3547 (91.56)	0.3552 (92.15)	0.3543 (90.83)	0.3543 (90.68)	0.3548 (92.00)	0.3535 (90.38)
V_e	6.5667 (282.15)	6.5734 (283.23)	6.5732 (283.96)	6.5667 (285.22)	6.5722 (283.09)	6.5846 (282.42)	6.5745 (285.87)	6.5840 (267.37)
φ	0.9259 (92.75)	0.9254 (95.92)	0.9255 (51.80)	0.9263 (52.80)	0.9254 (95.14)	0.9272 (52.46)	0.9249 (52.15)	0.9268 (50.80)
<i>CRP</i>	0.0245 (0.66)	0.0427 (1.54)	0.1609 (1.34)	0.2431 (5.09)	0.0482 (1.70)	0.0613 (2.12)	0.0681 (2.44)	0.0928 (0.01)
<i>PRP</i>	-12.7792 (-13.39)	-27.2152 (-15.04)	-29.9530 (-14.38)	-12.1593 (-11.97)	-48.6785 (-13.00)	-59.3315 (-12.29)	-12.5026 (-11.98)	-97.4908 (-87.00)
\mathcal{L}/N	0.3679	0.3963	0.3988	0.3670	0.3972	0.3977	0.3989	0.3991

Notes: Entries report the estimates of the structural parameters and their t -statistics (in parentheses) that govern the S&P 500 index return variance rate dynamics and risk premium. CRP and PRP denote the constant and proportional portion of the variance risk premium, computed from the parameter estimates. The last row reports the average of the maximized daily likelihood value. Estimation is based on daily data on the VIX and daily ACRV estimators constructed using second-by-second returns on S&P 500 index futures. The sample is from January 2, 1990 to December 31, 2004, a total of 3,787 observation for each series.

Table 3
Model parameter estimates on variance dynamics using TSRV estimators.

Θ	$\alpha_c = \alpha_p = -1$			$\alpha_c = \alpha_p = 0$			$\alpha_c = -1, \alpha_p = 1$	
$\tilde{\alpha}_J$	0.2094 (25.29)	0.2094 (212.34)	0.2093 (79.61)	0.2095 (36.14)	0.2090 (146.70)	0.2092 (95.92)	0.2089 (104.43)	0.2099 (8.82)
$\tilde{\kappa}_J$	0.0038 (0.04)	0.0038 (0.43)	0.0036 (0.01)	0.0010 (0.01)	0.0010 (0.24)	0.0010 (0.00)	0.0003 (0.00)	0.0003 (0.01)
a	0.1898 (6.60)	0.2462 (9.10)	0.2673 (10.22)	0.1625 (5.37)	0.2581 (9.38)	0.2702 (5.30)	0.2767 (10.24)	0.2945 (0.00)
κ	12.8925 (13.41)	26.6643 (14.58)	29.6984 (14.35)	12.1595 (12.60)	27.3224 (13.52)	33.5572 (13.38)	10.2673 (10.37)	95.6668 (0.83)
ω	0.1921 (36.34)	0.1436 (26.41)	0.1329 (21.78)	0.1923 (31.86)	0.1473 (26.51)	0.1355 (21.20)	0.1297 (20.70)	0.1245 (17.34)
λ_c	256.8047 (7.42)	— —	23.1298 (1.35)	9.2626 (5.60)	— —	0.2040 (0.01)	— —	95.5505 (0.83)
v_c	0.0135 (7.27)	— —	0.0663 (1.62)	0.0314 (5.88)	— —	0.0028 (0.01)	— —	0.0000 (0.07)
λ_p	— —	19.5483 (11.37)	21.9395 (11.32)	— —	58.8775 (5.62)	100.2270 (5.86)	8.0606 (8.56)	8.9471 (8.20)
v_p	— —	0.1544 (14.01)	0.1342 (11.96)	— —	0.3473 (8.37)	0.2572 (10.20)	1.1063 (4.99)	0.9468 (5.54)
ϕ_e	0.4093 (96.91)	0.4052 (95.56)	0.4055 (98.64)	0.4108 (99.80)	0.4049 (95.35)	0.4062 (95.88)	0.4088 (99.11)	0.4060 (96.90)
V_e	5.8441 (237.47)	5.8365 (238.65)	5.8374 (240.77)	5.8428 (240.19)	5.8422 (238.26)	5.8360 (238.55)	5.8235 (241.06)	5.8282 (227.95)
φ	0.9574 (100.25)	0.9574 (101.85)	0.9573 (52.87)	0.9577 (52.74)	0.9563 (101.34)	0.9573 (57.54)	0.9618 (67.23)	0.9582 (51.62)
<i>CRP</i>	0.0274 (0.72)	0.0368 (1.32)	0.1596 (1.35)	0.2437 (4.95)	0.0491 (1.72)	0.0616 (2.19)	0.0678 (2.41)	0.0847 (0.00)
<i>PRP</i>	-12.8887 (-13.38)	-27.1267 (-14.86)	-30.0900 (-14.32)	-12.1585 (-12.44)	-47.7679 (-12.83)	-59.3379 (-12.24)	-12.3501 (-11.78)	-97.4543 (-12.51)
\mathcal{L}/N	0.4280	0.4564	0.4589	0.4271	0.4572	0.4579	0.4590	0.4592

Notes: Entries report the estimates of the structural parameters and their t -statistics (in parentheses) that govern the S&P 500 index return variance rate dynamics and risk premium. CRP and PRP denote the constant and proportional portion of the variance risk premium, computed from the parameter estimates. The last row reports the average of the maximized daily likelihood value. Estimation is based on daily data on the VIX and daily TSRV estimators constructed using second-by-second returns on S&P 500 index futures. The sample is from January 2, 1990 to December 31, 2004, a total of 3,787 observation for each series.

Table 4

Likelihood ratio tests for the outperformance of proportional jumps over constant jumps.

QV estimators\Jump types	$\alpha_c = \alpha_p = -1$	$\alpha_c = \alpha_p = 0$
RRRV	7.27	7.37
ACRV	7.22	7.34
TSRV	7.29	7.50

Notes: Entries report the likelihood ratio test statistics \mathcal{V}'_{pc} for non-nested models on the statistical significance of difference in performance between models with proportional jump arrivals and their corresponding models with constant jump arrivals. A statistic greater than 1.67 suggests that the proportional jump specification significantly outperforms the constant jump specification under the 95 percent confidence interval.

Table 5

Likelihood ratio tests for the need of a constant jump component.

QV estimators\Jump types	$\alpha_p = -1$	$\alpha_p = 0$	$\alpha_p = 1$
RRRV	18.58 (0.0001)	4.42 (0.1097)	1.25 (0.5354)
ACRV	18.54 (0.0001)	4.33 (0.1145)	1.05 (0.5928)
TSRV	18.79 (0.0001)	4.84 (0.0888)	1.69 (0.4295)

Notes: Entries report the standard likelihood ratio test statistics $\mathcal{L}\mathcal{R}_{bp}$ between the nesting models that include both a constant and a proportion jump component and the corresponding restricted models that only have the proportional jump component. The statistics have a chi-square distribution with two degrees of freedom. The numbers in the parentheses report the p -values of each corresponding statistic.

Table 6

Likelihood ratio tests for the outperformance of infinite-variation over finite-variation jumps.

QV estimators\Jump types	$\alpha_p = 1$ v. $\alpha_p = -1$	$\alpha_p = 1$ v. $\alpha_p = 0$
RRRV	1.00	1.01
ACRV	1.01	1.00
TSRV	0.99	0.97

Notes: Entries report the likelihood ratio test statistics \mathcal{V}'_{α} for non-nested models on the statistical significance of difference in performance between infinite variation jump type ($\alpha_p = 1$) and the other two finite-variation jump types ($\alpha_p = -1$ and 0, respectively) for purely proportional jump arrival specifications. A statistic greater than 1.67 suggests that the infinite variation jump type significantly outperforms the other jump type under the 95 percent confidence interval.