Estimating risk-return relations with analysts price targets

Liuren Wu
Baruch College, Zicklin School of Business, One Bernard Baruch Way, New York, NY 10010, USA

ABSTRACT

Asset pricing tests often replace ex ante return expectation with ex post realization. The large deviation between the two drastically weakens the power of these tests. This paper proposes to use analysts consensus price target for a stock as the market expectation of the stock’s future price to directly construct the stock’s expected excess return. Analyzing the expected excess return behavior both over time and across different stocks shows that classic asset pricing theory works much better on ex ante return expectations than on ex post realizations. The analysis also provides new insights on the pricing of common equity risk factors.

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1. Introduction

Asset pricing theories generate implications on the relation between the expected excess return of a financial security and its risk. Empirical asset pricing tests often replace the ex ante return expectation with ex post return realization. Realizations, however, can differ greatly and persistently from the expectation. The deviations can come from large surprises, expectation biases, or expectations of certain large, rare events that have not materialized yet in the test sample period (i.e., the peso problem). Regardless of the particular source, the large deviations can drastically weaken the power of the empirical tests (Lundblad, 2007). This lack of testing power contributes to the lack of empirical support for classic asset pricing theories.

This paper proposes to test asset pricing implications using direct constructions of ex ante market expectation instead of using ex post return realization, thus mitigating the impact of ex post surprise on the estimated risk-return relation. Focusing on the U.S. equity market, the paper uses analysts consensus price target for a stock as the market expectation of the stock’s future price and constructs the stock’s ex ante expected excess return, or equity risk premium, as the log deviation between the price target and the stock price minus the one-year financing cost. Analyzing the equity risk premium behavior both over time and across different stocks shows that classic asset pricing theories work much better on ex ante return expectations than on ex post return realizations.

Ex ante risk premium expectation can be constructed from several different channels, all of which can, in principle, be applied to replace ex post return realizations in asset pricing tests. For example, a large accounting literature derives the implied cost of capital (ICC) from current stock prices, various valuation model assumptions, and cash flow forecasts.1 Pastor et al. (2008) and Lee et al. (2009) take the ICC approach to examine the intertemporal and international risk-return relations, respectively. Campello et al. (2008) construct expected equity returns using corporate bond yields by recognizing that bonds and stocks are contingent claims written on the same asset. More recently, several studies explore the idea of extracting risk premiums from option prices.2 The main issue that prevents these implied approaches from broader adoption in testing asset pricing models is that they often involve many assumptions that can significantly alter the results. For example, different combinations of valuation approaches and cashflow assumptions can generate many different sets of ICC estimates.3 Extracting risk premium from options or other contingent claims such as bonds also necessitates strong assumptions on

1 See, for example, Claus and Thomas (2001), William et al. (2001), Easton (2007), Hou et al. (2012), Fama and French (2002), and Duarte and Rosa (2015).
2 Prominent examples include, among others, Bakshi et al. (2008), Bakshi and Wu (2010), Santa-Clara and Yan (2010), Backus et al. (2011), Duan and Zhang (2014), Ross (2015), and Carr and Wu (2016).
3 Several studies strive to evaluate the performances of alternative estimates, e.g., Botosan and Plumlee (2005), Easton and Monahan (2005), Guay et al. (2011), and Lee et al. (2015).
price dynamics. Compared to these implied studies, this paper proposes a particularly simple approach for constructing the risk premium by directly relying on analysts price targets. In coming up with price targets, different analysts may have used different modeling approaches and cash flow forecasts. Directly using the price target consensus allows the paper to rely completely on average market expectation to construct the equity risk premium.

The paper analyzes the relation between the expected excess return and various risk measures using a sample of U.S. stocks from 2003 to 2014. Aggregating the expected excess return across the stock universe generates a time series of the aggregate equity market risk premium. The value-weighted equity market risk premium averages at 10.1% with a median of 8.8%, comparable to the sample average and median of the ex post realized one-year excess return at 8.4% and 10.4%, respectively. The main difference is that the ex post excess return exhibits much larger standard deviation at 17.7%, more than three times the standard deviation of the ex ante market risk premium at 5.3%. The ex ante expectation and the ex post realization do show positive correlation, with a sample cross-correlation estimate of 25.4%, but the sharp difference in the time-series variation of the two series highlights the inherent limitations of traditional risk-return relation tests using ex post return realizations.

The classic intertemporal asset pricing model of Merton (1973) predicts a positive relation between the risk premium on a security and the conditional covariance of the security’s return with the market portfolio return, with the proportionality coefficient measuring the relative risk aversion of a representative agent of the economy. Many empirical studies test the time-series implication of the model on the market portfolio. These studies often regress the ex post return of the market portfolio on some conditional variance estimator of the market return, and generate mostly insignificant or even negative slope coefficient estimates. This paper estimates the same relation using the ex ante equity risk premium construction, and generates positive and strongly significant relative risk aversion coefficient estimates.

Merton’s (1973) intertemporal asset pricing model also generates cross-sectional implications between the expected excess return on each individual stock and the stock’s covariance, or beta, with the market portfolio. The slope coefficient on the covariance has the same relative risk aversion interpretation. The coefficient on the beta relation represents an estimate for the equity market risk premium. Performing both types of cross-sectional regressions on the ex ante equity risk premium generates positive and strongly significant slope coefficient estimates. Using one-year historical return to construct the covariance estimator, the cross-sectional regressions generate an sample average of the relative risk aversion estimate at 3.76. Cross-sectional regressions on the one-year historical return beta generate an average market risk premium estimate of 7.4%.

The strong significance of the estimated relations with the ex ante equity risk premium provides a unique opportunity to investigate further on different risk measures and risk factors. First, given the well-known noise in the beta estimates, the paper proposes to reduce the noise by averaging the stock-market return correlation estimates within the same industry, with the assumption that companies within the same industry share similar co-movements with the market. The within-industry smoothing enhances the statistical significance of the cross-sectional regression coefficient estimates and also raises the average regression $R^2$ from 6.7% to 7.6%. Further replacing historical volatility estimator with option implied volatility generates even stronger results, raising the average cross-sectional regression $R^2$ estimate to 11.6%.

Second, the paper examines the pricing of commonly identified equity risk factors, including the size and book-to-market factors by Fama and French (1993, 1995, 1996), and the momentum factor by Jegadeesh and Titman (2001). Cross-sectional regressions of the ex ante equity risk premium on betas of these factors generates an average risk premium estimate of 3.4% on the size beta, −1% on the book-to-market risk factor, and 1.9% on the momentum risk factor. All the average risk premium estimates are statistically significant. Nevertheless, adding these factor exposures to the cross-sectional regression does not diminish the significance of the market beta, which generates an average risk premium of 6.2%. Therefore, while the other risk factors can be important considerations, the market portfolio beta remains the strongest consideration in ex ante market expectations.

Third, the paper examines the cross-sectional relation between the expected excess return and a long list of firm risk characteristics. Common valuation metrics, including cash yield, earnings yield, earnings growth rate, return on asset, as well as book-to-market ratio, all show strongly positive relation with the expected excess return. By contrast, the past one-year momentum, defined as the past 12-month to one-month cumulative return, shows a negative cross-sectional relation with the expected excess return. The expected excess return also shows strong positive correlation with the option implied volatility level, but its relations with the implied volatility slope measures across money- and maturity are weaker. Finally, the paper constructs a credit risk measure for each firm based on a simple implementation of the Merton (1974) structural model, and identifies a strong positive correlation between a firm’s credit risk and its expected equity excess return.

In other related literature, Söderlind (2009) re-examines the average equity risk premium puzzle by extracting expected equity returns from the Livingston survey and expected return volatility from options data. He finds that the expected excess return from the survey tends to be lower than the ex post realization while the volatility implied from options tends to be higher than the realized volatility. Both findings make the average magnitude of the equity risk premium less of a puzzle. The idea of using surveys is similar to the idea of using analysts price targets; nevertheless, as the Livingston survey is conducted twice a year on the aggregate market, the sparsity of the data limits its application to more extensive asset pricing tests.

The remainder of the paper is organized as follows. Section 2 describes the data sources and the equity risk premium construction methodology. Section 3 summarizes the equity risk premium behavior and the estimation results on various risk-return relations. Section 4 concludes.

2. Data sources and equity risk premium construction

The analysis examines 12 years of data from January 2003 to December 2014. The sample includes stocks in the S&P Composite 1500 index, which covers about 90% of the U.S. market capitalization and contains three leading indices: the S&P 500 index, the S&P MidCap 400 index, and the S&P SmallCap 600 Index. This criterion for sample choice excludes companies with very small market capitalization.

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4 Li et al. (2013), among others, also find positive predicting power for aggregate implied cost of capital estimates on future market returns.

5 Several studies report negative risk-return relation estimates. Examples include Campbell (1987), Breen et al. (1989), Turner et al. (1989), Nelson (1991), Gloslen et al. (1993), Whitelaw (1994), and Harvey (2001). Many others fail to identify a statistically significant intertemporal relation, e.g., French et al. (1987), Gojal and Santa-Clara (2003), Bali et al. (2005), Chan et al. (1992), Baille and DeCenarro (1990), Campbell and Hentschel (1992), and Gloslen et al. (1993). Harrison and Zhang (1999) find a significantly positive risk and return relation at one-year horizon, but they do not find a significant relation at shorter holding periods such as one month.
Company-specific data are from two major sources: Bloomberg and OptionMetrics. Bloomberg provides stock price series, accounting fundamentals, and analysts consensus forecasts on cash flow per share, earnings per share, long-run earnings growth rates, return-on-assets, and price targets. OptionMetrics provides the stock price series, total return series, historical return volatility estimators with different windows, and interpolated option implied volatility estimators at different maturities and deltas.

At any given date, for a stock to be included in the analysis, it must satisfy the following filtering criteria: (1) data are available for the expected risk premium and realized excess return construction; (2) the stock prices from the two data sources match; (3) the stock price level during the past year is higher than $5; and (4) the quarterly average daily trading volume is higher than $100,000. Cross-validation through the two price sources minimizes data error. The price level and volume filtering further ensures that the estimated risk-return relations are not overly affected by highly illiquid stocks. The filtering generates 3,134,703 daily viable observations. The number of chosen firms range from a minimum of 786 to a maximum of 1,318 per day.

At each date and for each given stock, the ex ante expected risk premium is computed as the log deviation between the analysts consensus price target and the stock's closing price at that date, minus the one-year U.S. dollar libor rate as a proxy for the one-year financing cost of the investment. For comparison, ex post realized stock excess returns are also computed over the next month, the next quarter, and the next year, where the financing cost is proxied by the U.S. dollar libor rate of the corresponding maturity.

To compute loadings on common equity risk factors, the paper obtains daily return time series on Fama and French (1993) factors and the momentum factor from Professor French's online data library. To examine how the risk-return relation varies with business cycles and economic activities, the paper obtains the NBER recession indicator and the Chicago Fed National Activity (CFNAI) diffusion index from the Federal Reserve Bank of St. Louis.

### 3. The equity risk premium behavior

Table 1 compares the summary statistics of the ex ante equity risk premium with the corresponding ex post one-year realized excess return. Panel A reports the sample average and percentiles over the pooled sample. The pooled average risk premium is 10.4%, similar in magnitude to the average ex post realized excess return at 11%. The medians are also similar: 8.7% for the ex ante risk premium and 9.8% for the ex post excess return.

One concern for using analyst forecasts or other types of surveys is whether such survey estimates reflect true market expectations. In particular, a large literature discusses how analysts may have incentives to deliberately bias their forecasts upward. 6 Such an average bias would have distorted the estimate on the average equity risk premium (Easton and Sommers, 2007), but the bias does not significantly affect the slope estimate of a risk-return relation so long as it is not strongly correlated with the risk measures used in the regression. In particular, a constant bias will not affect the estimated slope of the risk-return relation. A proportional bias can change the magnitude of the slope estimate but not its sign. The summary statistics in Table 1 shows that there does not exist an obvious average bias in this particular data sample.

Despite the similar average levels, the excess returns vary over a much wider range from −73.2% at the 1st percentile to 124.0% at the 99th percentile, compared to a much narrower range for the equity risk premium from −19.4% at the 1st percentile to 53.8% at the 99th percentile.

Panel B of Table 1 reports the standard deviation estimates on the pooled sample, which is at 13.9% for the ex ante risk premium, but almost three times as large at 37.5% for the ex post excess return. The panel also computes the cross-sectional standard deviation at each date and reports the time-series average of the cross-sectional standard deviation estimates (CS). The average cross-sectional deviation is 12.1% for the ex ante risk premium and 30.4% for the ex post excess return. The last row of Panel B reports the cross-sectional average of the time-series standard deviation estimates for each stock (TS), provided that the stock has more than one year of daily data available. The average time-series standard deviation is 11.6% for the ex ante risk premium and 30% for the ex post excess return. The standard deviation estimates show that the equity risk premium varies strongly both over time and across different companies. Compared to the risk premium variation, the ex post realized excess returns vary almost three times as much in standard deviation terms, highlighting the tremendous amount of random noise in the realization.

To understand whether the ex ante equity risk premium predicts the ex post excess return, Panel C of Table 1 reports the forecasting correlation between the two. Over the pooled sample, the forecasting correlation is 9.8%. The average cross-sectional forecasting correlation is 2.8%. The average time-series forecasting correlation per each stock is stronger at 22.7%. Overall, the risk premium constructed from price targets predicts future excess returns in the right direction both cross-sectionally and over time. The predictability is weak by nature. It is exactly because of this weak

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6 Earlier evidence for an average positive bias in analysts forecasts includes Abarbanell (1991), Brown et al. (1985), and Sticierl (1990). More recent researches examine the drivers underlying the average bias (e.g., Brown et al., 2014; Cowen et al., 2006; Lim, 2001; Lundqvist et al., 2007).

Table 1 Summary statistics of ex ante equity risk premium and ex post realized excess return. Entries report the summary statistics of ex ante equity risk premium and ex post realized excess returns over the next year. At each date, the ex ante risk premium for each stock is computed as the log deviation between analyst consensus price target and the closing stock price at that date, minus the one-year U.S. dollar libor rate as the financing cost. The ex post realized excess return is computed as the stock realized return over the next year minus the libor rate. The statistics are computed on daily estimates from January 2003 to December 2014. Panel A reports the pooled sample average (“Mean”) and percentile values. Panel B reports the standard deviation estimates on the pooled sample (“Pooled”), as well as the time-series average of the daily cross-sectional standard deviation estimates (CS) and the cross-sectional average of the time-series standard deviation estimates per each stock (TS). Panel C reports the forecasting correlation between the ex ante risk premium and the ex post excess return on the pooled sample, as well as the time-series averages of the cross-sectional correlation (CS) and cross-sectional averages of the time-series correlation per stock (TS). The time-series statistics are computed on all stocks with over one year worth of daily observations.

<table>
<thead>
<tr>
<th></th>
<th>Ex ante equity risk premium</th>
<th>Ex post realized excess return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Mean and percentiles</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.104</td>
<td>0.110</td>
</tr>
<tr>
<td>1%</td>
<td>−0.194</td>
<td>−0.732</td>
</tr>
<tr>
<td>10%</td>
<td>−0.043</td>
<td>−0.328</td>
</tr>
<tr>
<td>25%</td>
<td>0.019</td>
<td>−0.102</td>
</tr>
<tr>
<td>50%</td>
<td>0.087</td>
<td>0.098</td>
</tr>
<tr>
<td>75%</td>
<td>0.172</td>
<td>0.296</td>
</tr>
<tr>
<td>90%</td>
<td>0.273</td>
<td>0.530</td>
</tr>
<tr>
<td>99%</td>
<td>0.538</td>
<td>1.240</td>
</tr>
<tr>
<td>B. Standard deviation estimates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pooled</td>
<td>0.139</td>
<td>0.375</td>
</tr>
<tr>
<td>CS</td>
<td>0.121</td>
<td>0.304</td>
</tr>
<tr>
<td>TS</td>
<td>0.116</td>
<td>0.300</td>
</tr>
<tr>
<td>C. Correlations between expectation and realization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pooled</td>
<td>0.098</td>
<td></td>
</tr>
<tr>
<td>CS</td>
<td>0.028</td>
<td></td>
</tr>
<tr>
<td>TS</td>
<td>0.227</td>
<td></td>
</tr>
</tbody>
</table>
predictability that makes the ex post realized excess return an extremely noisy proxy for the ex ante risk premium in estimating risk-return relations.

3.1. The time series behavior of the aggregate equity market risk premium

Aggregating the risk premium estimates across different stocks at each date generates an aggregate equity market risk premium (MRP) measure. Table 2 performs this aggregation with both equal weighting (Panel A) and value weighting based on the market capitalization of each stock (Panel B). Corresponding to the summary statistics for each aggregate market risk premium, the table also computes the statistics for the aggregate ex post excess return over the next year (MER). The statistics from the two panels are similar. The expected equity risk premium averages at 10.3% for the equal-weighted portfolio and 10.1% for the value-weighted portfolio. The median estimates are smaller at 9% for the equal-weighted portfolio and 8.8% for the value-weighted portfolio. The corresponding average one-year ex post excess return averages higher at 11.2% for the equal-weighted portfolio, but lower at 8.4% for the value-weighted portfolio. Their medians are at 13.6% for the equal-weighted portfolio and 10.4% for the value-weighted portfolio.

While the average risk premium estimates are similar between expectation and realization, their standard deviation estimates are quite different. For the equal-weighted portfolio, the expected risk premium has a standard deviation of 6.6%, whereas the standard deviation for the realized one-year excess return is three times as large at 21.4%. Similarly, for the value-weighted portfolio, the standard deviation estimate is 5.3% for the expected risk premium, and 17.7% for the realized one-year excess return. The minimum and maximum statistics tell a similar story: Whereas the expected risk premium moves between −0.7% and 47.8% for the equal-weighted portfolio and between 1.4% and 39% for the value-weighted portfolio, the one-year realized excess return moves in a much wider range, from −52.8% to 103.2% for the equal-weighted portfolio and from −48.8% to 68.4% for the value-weighted portfolio. The much wider variation for the realized excess return suggests that although the realization is close to expectation on average in the very long run, there can be very large deviations at each moment in time.

To understand how the ex ante aggregate market risk premium relates to the ex post market realized excess return, Table 3 performs the following forecasting regression,

\[ \text{MER}_{t+1} = \alpha + \beta \text{MRP}_t + \epsilon_{t+1}. \]  

where the ex post future market excess return is regressed on the ex ante market risk premium. The table reports the regression coefficient estimates and the \( R^2 \) estimates. In parentheses are the Newey and West (1987) standard errors of the coefficient estimates, which are computed with a lag of 252 days to adjust for the overlapping sample. The forecasting regressions generate a 5.42% \( R^2 \) for the equal-weighted portfolio and 6.45% \( R^2 \) for the value-weighted portfolio. For the equal-weighted portfolio, the intercept estimate is positive at 3.7%, although not statistically significant. The slope estimate is 0.726, significantly different from zero at 10% confidence level. For the value-weighted portfolio, the intercept estimate is close to zero. The slope estimate is strongly positive at 0.821, and one cannot reject the null hypothesis of \( \beta = 1 \). Therefore, over the sample period, at least for the value-weighted portfolio, the market risk premium constructed bottom up from analysts price targets represents a reasonably unbiased predictor of future market realized excess returns.

**Table 2**

Summary statistics of ex ante aggregate equity market risk premium and ex post equity market excess return. Entries report the summary statistics of ex ante expected aggregate equity market risk premium and the corresponding ex post market excess return over the next year. Panel A aggregates the risk premium and excess return with equal weighting across stocks. Panel B performs the aggregation with value weighting based on each stock’s market capitalization. The aggregation is over the universe of the selected sample. The statistics are computed on daily estimates from January 2003 to December 2014.

<table>
<thead>
<tr>
<th>A. Equal weighting</th>
<th>B. Value weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expectation (MRP)</strong></td>
<td><strong>Realization (MER)</strong></td>
</tr>
<tr>
<td>Mean</td>
<td>0.103</td>
</tr>
<tr>
<td>Median</td>
<td>0.098</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.066</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.466</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.048</td>
</tr>
<tr>
<td>Minimum</td>
<td>−0.007</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.478</td>
</tr>
</tbody>
</table>

**Table 3**

Predict ex post realized market excess return with ex ante market risk premium. Entries report the coefficient estimates, the Newey-West standard errors (in parentheses), and the \( R^2 \)-squared for the forecasting regression that regresses ex post realized market excess returns on ex ante market risk premium estimates. Panel A aggregates the market risk premium and excess return with equal weighting across stocks. Panel B performs the aggregation with value weighting based on each stock’s market capitalization. The regressions are performed on daily observations from January 2003 to December 2014. The Newey–West standard errors are computed with a lag of 252 days.

<table>
<thead>
<tr>
<th>A. Equal weighting</th>
<th>B. Value weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.037</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.726</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>5.42%</td>
</tr>
</tbody>
</table>
A. Equal weighting

B. Value weighting

Fig. 1. Time variation in expected and realized equity market risk premium. Lines plot the time series of ex ante market risk premium (solid lines) and ex post realized market excess return (dashed lines). In constructing the market aggregates, Panel A applies equal weighting while Panel B applies value weighting to the stock universe.

with analysts consensus price targets. Other sources of surveys can also be used to construct the market risk premium. For example, the Livingston Survey provides forecasts of future S&P 500 index levels at horizons of 6 and 12 months twice a year in June and December of each year. Lakonishok (1980) investigates the historical accuracy of this survey prediction. Söderlind (2009) uses this survey to construct equity market risk premium estimators. The sparsity of the Livingston Survey limits its application for performing extensive asset pricing tests; nevertheless, it is interesting to examine whether the bottom-up market risk premium estimates constructed from analysts price targets on individual companies move in line with economists forecasts on the aggregate economy.

From the Livingston Survey, I first compute the one-year expected capital gain on the S&P 500 index twice a year over the overlapping sample period. The survey provides forecasts from 0-month to 12-month. The expected capital gain is computed as the log percentage difference between the 12-month forecast and the 0-month forecast. I then construct a market risk premium estimator by adjusting the expected capital gain for the dividend yield of the index and the financing cost. The dividend yield information on the index is obtained from OptionMetrics.

The 12-year sample period span 24 surveys from June 2003 to December 2014. Fig. 2 overlays the two bottom-up market risk premium time series (solid line for equal-weighting and dashed line for value weighting) with the risk premium constructed from the Livingston survey, represented in circles placed at the end of the survey month and linked by a dotted line. The two sources of estimates show common variation. The estimates from both sources are low in 2007 and high 2009. The bottom-up estimates show more variation, partly reflecting the higher resolution in the daily updating frequency.

Over the common sample, I map the risk premium estimates constructed from the Livingston Survey to the end-of-the-month bottom-up estimates and compute their correlation. The correlation estimates are strongly positive, 40% with the equal-weighted risk premium and 46% with the value-weighted risk premium. By treating the survey numbers as average estimates over the survey month, I also map them to the monthly averages of the daily bottom-up estimates. The monthly smoothing leads to even higher correlation estimates at 49% with the equal-weighted portfolio and 56% with the value-weighted portfolio. Panel B of Fig. 2 overlays the monthly smoothed bottom-up estimates with the risk premium constructed from the Livingston Survey. The strong co-movements between the two sources of estimates provide some cross-validation on the bottom-up estimates.

3.2. The intertemporal risk-return relation on the aggregate market

In his seminal paper, Merton (1973) derives an intertemporal capital asset pricing model that predicts the following equilibrium relation between the expected excess return and the expected risk on a financial security $i$, 

$$\mu_i - r = \gamma \sigma_{im}. \quad (2)$$

where $\mu_i$ denotes the expected return on the security, $r$ denotes the riskfree rate, $\gamma$ denotes the average relative risk aversion of market investors, and $\sigma_{im}$ denotes the return covariance between the financial security $i$ and the market portfolio $m$.

The model has both time-series and cross-sectional implications. Many empirical studies focus on the time-series implication of the model on the market portfolio,

$$\mu_m - r = \gamma \sigma_{m}^2. \quad (3)$$

where the expected excess return on the market portfolio ($\mu_m - r$), or market risk premium, co-moves positively with the conditional return variance of the market portfolio ($\sigma_{m}^2$), with the slope coefficient measuring the average relative risk aversion of market investors.

Several empirical difficulties arise from attempts to estimate the positive relation in (3). First, the conditional variance is not observable. A historical variance estimator with a short window is likely to be noisy and thus induces the errors-in-variable problem, while a long window can overly smooth the time-series variation of the conditional variance. Unless the market risk experiences dramatic variation during the sample period, the identification can be weak. Second, using realized returns to replace the return expectation in the estimation brings a large amount of noise to the dependent variable, which can drastically reduce the $R^2$ of the regression and possibly the statistical significance of the estimated coefficient. The

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3 When the investment opportunity of the economy is stochastic, the relation includes a second term induced by the intertemporal hedging demand and capturing the covariance with the state variables that govern the stochastic investment opportunity.
sometimes persistent deviation between market expectation and ex post realization induces further bias in the estimated relation. The net result is that estimating the intertemporal risk-return relation often leads to insignificant and sometimes even negative slope coefficient estimates, casting doubt on the validity of the classic theory.

As a reference to the standard literature, I regress the ex post realized excess returns for the value-weighted portfolio ($\text{MER}_{t+h}$) over different horizons ($h$) against several conditional variance estimators ($\sigma^2_h$).

$$\text{MER}_{t+h} = \alpha + \gamma \sigma^2_{m.t} + e_{t+h}. \tag{4}$$

Given the similar behaviors between equal-weighted and value-weighted portfolios, the analysis henceforth focuses on the value-weighted market portfolio. Panels A to C in Table 4 report the regression results with ex post annualized excess returns over one, three, and 12-month horizons, respectively. Within each panel, each column uses a different conditional variance estimator, including historical return variance estimators with one, three, and 12 months of daily portfolio returns, as well as at-the-money option implied variance on the S&P 500 index at one, three, and 12 month maturity, respectively. For each specification, the table reports the constant ($\alpha$) and slope ($\gamma$) estimates of the relation, the Newey and West (1987) t-statistics (in parentheses), and the regression $R^2$ estimates.

With one-month realized excess return as a proxy for the equity risk premium, Panel A shows that the regressions generate negative slope estimates in four out of the six cases, against the theory implication that the slope should reflect the average relative risk aversion of market investors. None of the slope estimates are statistically significant. The results are similarly negative in Panel B based on three-month excess returns. Only when using 12-month ex post realized excess return do the slope estimates in Panel C become all positive across the six conditional variance estimators. Still, none of the estimates reach 95%-level statistical significance.

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*OptionMetrics computes implied volatility for puts and calls separately at the same delta. I take the average of the 50-delta put and 50-delta call implied volatility as the at-the-money implied volatility.*

The weak and many times negative finding on ex post excess returns is in line with previous literature findings.

Panel D of Table 4 uses analysts price targets to construct the ex ante equity risk premium for the market portfolio ($\text{MRP}$) and regresses it against the same set of conditional variance estimators,

$$\text{MRP}_t = \alpha + \gamma \sigma^2_{m.t} + e_t. \tag{5}$$

In this case, the relative risk aversion coefficient estimates are all positive and strongly significant. The estimates range from 0.344 when using the 12-month historical variance as the conditional variance estimator to 1.438 when using the 12-month option implied variance as the conditional variance estimator. The $R^2$ estimates of the regressions also become much higher, ranging from 9.8% to 56.7%. These results provide much better support to the classic asset pricing theory, and show that the weak evidence in the literature is less a rejection of the theory, but more reflects the weak power of the estimation approach.

Comparing the results in Panel D across different conditional variance estimators shows that using option-implied variance generates higher $R^2$ estimates than using historical return variance estimators, highlighting the importance of the forward-looking variance information in the options contract. While the analysts price targets reveal market expectation of the excess return, the option prices reflect market expectation of the conditional risk. Matching the two expectations generates the strongest support for the classic asset pricing theory.

When using historical return variance estimators of different windows, the results show that both the $R^2$ estimates and the statistical significance of the slope estimator decline with increasing window length. This pattern suggests that the expected risk premium is very responsive to the most recent variance realization. Smoothing over a longer window can suppress the actual co-movements between risk and the expected return.

The results in Table 4 show that replacing ex post realization with ex ante expectation can drastically reduce the noise in the regression and lend much better support to the classic asset pricing theory. Nevertheless, there remain strong deviations. First, if the constructed equity risk premium and the conditional variance estimator were to reflect true market expectations and if the investment opportunities were constant, the regression should have...
generated a perfect fit. An $R^2$ estimate of over 50% is high, but a large proportion of variation remains unexplained by the variation of the conditional variance estimators. Second, the intercept estimate is large and strong, suggesting that a significant proportion of the aggregate ex ante risk premium constructed from the price targets cannot be explained by the aggregate conditional variance variation. Third, although the slope coefficient estimates become positive. The estimates remain at the low end of what market expects what the average relative risk aversion should be.

These deviations can be regarded as directions for future research on asset pricing models. They also point to potential issues with the particular regression. Eq. (5) represents a fairly narrow interpretation of the intertemporal asset pricing model. While the model has implications on the whole universe of individual stocks, the regression relies solely on the time series variation of the risk in the aggregate market. In reality, risk varies much more cross-sectionally than over time. Thus, estimating the risk-return relation by exploiting the cross-sectional variation can potentially lead to stronger identification. The next section explores the cross-sectional implication.

### 3.3. The cross-sectional risk-return relation

Merton’s (1973) intertemporal capital asset pricing model in Eq. (2) not only has time-series implications on the risk-return relation of the market portfolio, but also has cross-sectional implications between the equity risk premium of each stock and the stock return’s covariance with the market portfolio,

$$
\mu_i - r = \gamma' \sigma_{im}^2.
$$

where the slope coefficient of the cross-sectional relation captures the average relative risk aversion of market investors at that time. Furthermore, combining the market portfolio implication ($\mu_m - r = \gamma' \sigma_m^2$) with Eq. (6), one can substitute out the relative risk aversion coefficient with the market risk premium to arrive at the more commonly-tested version of the capital asset pricing model,

$$
\mu_i - r = \Phi \beta_i,
$$

where the beta of the security is defined as $\beta_i \equiv \sigma_{im}/\sigma_m^2$ and the slope coefficient of the cross-sectional relation measures the market risk premium, $\Phi \equiv \mu_m - r$. This section examines the empirical cross-sectional implications of Eqs. (6) and (7) using different equity risk premium, covariance, and beta estimates.

As a starting reference point, Table 5 follows the standard literature in using ex post one-year excess return ($ER_{t+1}$) as the dependent variable and performing Fama and MacBeth (1973)-type regressions: Each date, ex post excess returns on different stocks are regressed against their covariance ($\sigma_{im, t}$) or beta ($\beta_{it}$) estimators cross-sectionally,

$$
ER_{t+1} = \alpha_1 + \gamma_1 \sigma_{im, t} + \epsilon_{t+1, 1}.
$$

Furthermore, combining the market portfolio implication ($\mu_m - r = \gamma' \sigma_m^2$) with Eq. (6), one can substitute out the relative risk aversion coefficient with the market risk premium to arrive at the more commonly-tested version of the capital asset pricing model,
standard deviation (in parentheses) of cross-sectional regression $R^2$ estimates. Since the covariance and the beta estimator only differ by a scale of the market portfolio return variance, which is a constant in the cross-sectional regression, the two sets of regressions generate identical intercept and $R^2$ estimates, and the estimates for the slope coefficients at each date are linked by market portfolio return variance estimator at that date, $\phi = \gamma \sigma_{mt}^2$.

With the conditional variance $\sigma_{mt}^2$ as the regressor, the average slope coefficient estimates in Panel A are negative across all three conditional covariance estimation windows, against the implication of the asset pricing theory that the slope should reflect the average relative risk aversion of market investors. When the regressor is the conditional beta estimate $\beta_{it}$, the average slope coefficient estimates in Panel B are positive but statistically insignificant. The magnitudes of the slope estimates, which should reflect the average market risk premium ($\phi$), are very small. They, and sometimes even negative, findings are similar to the findings in the literature, casting doubt on the classic asset pricing theory, or the relevance of market beta.

The story, however, changes drastically in Table 6, which uses the ex ante equity risk premium ($RP_{it}$) to replace the ex post excess return ($ER_{it,t+h}$) as the dependent variable for the risk-return relation estimation,

$$RP_{it} = \alpha_t + \gamma_t \sigma_{mt}^2 + e_{it}, \quad (10)$$

$$\beta_{it} = \alpha_t + \phi_t \beta_{it} + e_{it}. \quad (11)$$

In this case, the time-series averages of the slope coefficients are all positive, and the Newey–West $t$-statistics on the sample average are strongly significant. The large and positive $t$-statistics provide strong support to the classic asset pricing theory.

Comparing regressors estimated with different window lengths shows that the covariance and beta estimators with 12-month history generates the strongest results in terms of both the $R^2$ estimates and the $t$-statistics of the slope estimates. Regressing the equity risk premium against the 12-month historical covariance estimator generates relative risk aversion estimates averaging at 3.762. Regressing against the 12-month historical beta estimator generates an average market risk premium of 7.4%.

Since the risk exposures such as covariance or beta are not directly observable but are estimated from historical data, the regressions suffer from errors-in-variable problem. The window length choice for exposure estimation reflects a trade-off between capturing the time-variation of the exposure and reducing measurement error. Shorter window length better captures the time variation of the exposure but suffers from larger measurement error in the estimates. The time-series regressions of market equity risk premium against market portfolio return variance estimators in Table 4 rely on the time-series variation of risk and risk premium for identification. As a result, a shorter window size for the variance estimator leads to stronger identification. By contrast, the cross-sectional regressions in Table 6 rely more on the cross-sectional variation of the risk and risk premium. A longer window for the covariance and beta estimators reduces the measurement noise but has smaller impact on the cross-sectional variation. Accordingly, a longer window for the risk exposure estimators leads to stronger identification of the cross-sectional risk-return relation.

Another way of reducing measurement noise is via cross-sectional averaging, which reduces both measurement noise and, unfortunately, cross-sectional variation. Under the fourth column in Table 6, I perform cross-sectional averaging within each industry on the 12-month correlation estimates between the stock return and the market portfolio return. The underlying assumption of this within-industry smoothing is that companies within the same industry have the same (or similar) return correlation with the market portfolio and that the observed variation on the raw correlation estimates within an industry is mainly driven by noise. The purpose of this within-industry smoothing is to mitigate estimation errors in the covariance and beta estimates. By smoothing the correlation instead of the covariance or beta directly, the measure accommodates cross-sectional variations in the risk level within the industry but smoothes out the variation in the correlation with the market. Via within-industry smoothing, the cross-sectional regressions generate higher $R^2$ estimates. The average $R^2$ estimate increases from 6.7% to 7.6%. The $t$-statistics of the slope estimates in both panels also become higher. The average relative risk aversion in Panel A increases in magnitude from 3.762 to 4.391, and the average market risk premium estimate in Panel B increases from 7.4% to 8.6%.

In addition to the within-industry smoothing of the historical correlation estimates, the last column of Table 6 further replaces the historical volatility estimator for each stock with the 12-month at-the-money option implied volatility on that stock, and replaces the historical volatility estimator for the market portfolio with the 12-month at-the-money option implied volatility on the S&P 500 index. With the forward-looking option-implied volatility as an input to the risk measure construction, the average $R^2$ of the regressions increases further to 11.6%. The $t$-statistics of the average slope coefficient estimate also become much higher. The relative risk aversion estimates average at 4.229 and the market risk premium estimates average at 13.5%.

### 3.4. Time variation of relative risk aversion and market risk premium estimates

The cross-sectional regression generates daily estimates of the market’s relative risk aversion when using covariance as the regressor and the market risk premium when using beta as the regressor. Fig. 3 plots the time series of the daily estimates, relative risk aversion in Panel A and market risk premium in Panel B. Each panel contains three lines corresponding to three different estimators: 12-month historical estimator (HE, solid line), within-

### Table 5

Cross-sectional estimation of intertemporal risk-return relations based on ex post excess returns. Entries report results of cross-sectional intertemporal risk-return relation regressions using ex post one-year excess returns ($ER_{t+1}$) as the dependent variable. Panel A regresses the excess return on the historical covariance estimator with the market portfolio ($\sigma_{mt}$). Panel B regresses the excess return on the historical beta estimator of each stock ($\beta_{it}$). For each panel, the three columns correspond to three different historical rolling windows at 1, 3, and 12 months, respectively, for the historical covariance and beta estimators. Entries report the time-series averages of the coefficient estimates and the Newey–West $t$-statistics (in parentheses), as well as the time-series average and standard deviation (in parentheses) of the cross-sectional $R^2$ estimates.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Historical estimators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Months</td>
<td>1</td>
</tr>
<tr>
<td>$A. \ ER_{t+1} = \alpha + \gamma \sigma_{mt}^2 + e_{t+1}$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td></td>
<td>(0.130)</td>
</tr>
<tr>
<td>$B. \ ER_{t+1} = \alpha + \phi \beta_{it} + e_{t+1}$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td></td>
<td>(0.130)</td>
</tr>
</tbody>
</table>
The wedge and is problem three historical and industry line plots. Fig. 3. Time variation in relative risk aversion and market risk premium estimates. Panel A plots the time series of relative risk aversion estimated from cross-sectional regressions of ex ante equity risk premium on covariance estimators with the market portfolio. Panel B plots the time series of the market risk premium obtained from cross-sectional regressions of ex ante equity risk premium on conditional beta estimators. The three lines in each panel denote three different types of risk estimators: 12-month historical estimator (HE, solid line), within-industry smoothing on the correlation estimates (IS, dashed line), and 12-month at-the-money option-implied volatility replacing the corresponding historical volatility (OI, dash-dotted line).

Table 6
Cross-sectional estimation of intertemporal risk-return relations using ex ante equity risk premium. Entries report results of cross-sectional intertemporal risk-return relation regressions using ex ante equity risk premium as the dependent variable. Panel A regresses the equity risk premium of different stocks on their covariance with the market portfolio (\(\sigma_{it}^{\text{w}}\)). Panel B regresses the equity risk premium of different stocks on their beta with the market portfolio (\(\hat{\beta}_{it}\)). Within each panel, the five columns correspond to five different risk estimators, including historical estimators with historical rolling windows at one, three, and 12 months, within-industry smoothing of the 12-month correlation estimates, and further replacing the historical volatility estimator with the corresponding 12-month at-the-money option implied volatility. Entries report the time-series averages of the coefficient estimates and the Newey-West t-statistics (in parentheses), as well as the time-series average and standard deviation (in parentheses) of the cross-sectional \(R^2\) estimates.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Historical estimators</th>
<th>Industry-smoothing</th>
<th>Option-implied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Months</td>
<td>1</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(\alpha)</td>
<td>(\gamma)</td>
<td>(\phi)</td>
</tr>
<tr>
<td></td>
<td>(\phi)</td>
<td>(\sigma^2)</td>
<td>(\phi)</td>
</tr>
<tr>
<td>A.</td>
<td>(\alpha = \alpha_1 + \gamma_1 \sigma_{it}^{\text{w}} + \varepsilon_{it})</td>
<td>(\alpha_1)</td>
<td>(\gamma_1)</td>
</tr>
<tr>
<td></td>
<td>(\alpha = \phi \beta_{it} + \alpha_1)</td>
<td>(\phi)</td>
<td>(\beta_{it})</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
<td>(1.263)</td>
<td>(0.034)</td>
</tr>
<tr>
<td></td>
<td>((11.80)</td>
<td>(2.270)</td>
<td>((0.052)</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(3.762)</td>
<td>(0.047)</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(6.61)</td>
<td>(0.067)</td>
</tr>
<tr>
<td>B.</td>
<td>(\alpha = \alpha_1 + \gamma_1 \sigma_{it}^{\text{w}} + \varepsilon_{it})</td>
<td>(\alpha_1)</td>
<td>(\gamma_1)</td>
</tr>
<tr>
<td></td>
<td>(\phi = \phi \beta_{it} + \alpha_1)</td>
<td>(\phi)</td>
<td>(\beta_{it})</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
<td>(0.029)</td>
<td>(0.034)</td>
</tr>
<tr>
<td></td>
<td>((11.80)</td>
<td>(0.047)</td>
<td>((0.052)</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.074)</td>
<td>(0.047)</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.067)</td>
<td>(0.067)</td>
</tr>
</tbody>
</table>

industry smoothing of the correlation estimates (IS, dashed line), and 12-month at-the-money option-implied volatility replacing the historical volatility estimator (OI, dash-dotted line). In Panel A, the three time series of relative risk aversion estimates show strong co-movements and share similar magnitudes, except that the dash-dotted line estimated with option-implied volatility shows more temporal stability. In Panel B, the market risk premium estimates tend to be higher when using the option-implied volatility, but the three lines still show strong co-movements. The level differences can partly be driven by the different degrees of errors-in-variable problem in the different types of risk estimators. In addition, there is well-documented evidence on variance risk premium (e.g., Carr and Wu, 2004; Carr and Wu, 2006), which can drive an average wedge between the option-implied and historical-realized volatility. Such differences can affect the average magnitude of the covariance and beta estimators, leading to differences in the relative risk aversion and risk premium estimates. Despite these differences, most of the relative risk aversion and market risk premium estimates stay positive except under rare occasions. The relative risk aversion estimates show a U-shaped sample path, with high estimates at both the start and end of the sample period, but low estimates in the middle of sample around 2008 and 2009. The patterns on the market risk premium time series are less obvious. For example, in 2008, although the relative risk aversion estimates become lower, the market volatility becomes very high. The product of the two remains in the same range as in other sample periods.
To understand how the market risk premium estimates vary with the business cycle, I obtain from the Federal Reserve Bank of St. Louis the NBER recession indicator. The NBER classifies 2008 and the first half of 2019 as a recession. During this recession period, the relative risk aversion estimates average between 0.75 (for the HE estimator) and 1.80 (for the OI estimator). The average risk aversion during the other part of the sample averages between 4.24 and 4.61. The risk aversion coefficients average lower during this recession period than during the other part of the sample. On the other hand, the market risk premium estimates average about the same magnitudes for the different sample periods: from 6.48% (HE) to 13.72% (OI) during the recession period versus from 7.6% to 13.45% during the other part of the sample. The reason is that although the relative risk aversion averages lower during the recession period, the market volatility averages higher, leading to stable risk premium estimates. Nevertheless, one should refrain from drawing too much conclusion from this one particular sample period as the period only includes one recession.

In addition to the binary recession indicator, I obtain the Chicago FED National Activity (CFNAI) diffusion index as a continuous measure of the relative strength of the US economic activity. I also construct a stock market performance measure using the past quarter return on the S&P 500 index. Table 7 regresses the relative risk aversion and market risk premium estimates on these two strength measures, Panel A for the CFNAI diffusion index and Panel B for the stock market performance. Entries report the regression coefficient estimates, Newey–West $t$-statistics for the coefficient estimates, and $R^2$ estimate for each regression. A 252-day lag is used in computing the Newey–West standard errors.

Results in Panel A of Table 7 show that both the relative risk aversion coefficients and the market risk premium estimates positively co-move with the economic strength index, but the co-movement is stronger for the relative risk aversion estimates. The regressions on the three relative risk aversion estimators generate $R^2$ estimates ranging from 17.6% to 18.8%, and the slope coefficient estimates show strong statistical significance. The $R^2$ estimates for the regressions on the market risk premiums are much higher, and the slope coefficient estimates are not statistically significant.

Panel B shows that both the relative risk aversion and market risk premium estimates negatively co-move with the stock market performance over the last quarter, but this time the stronger statistical significance comes from the market risk premium. The slope coefficient estimates from the relative risk aversion regressions are not statistically significant.

### 3.5. Risk premiums on common equity risk factors

The literature has identified other common risk factors in the equity market, such as size, book-to-market by Fama and French (1993) and momentum by Jegadeesh and Titman (2001), mostly based on the average ex post excess return differences on portfolios formed according to rankings of these factor characteristics or factor loadings. This section examines how these risk factors are related to the ex ante equity risk premium.

Daily returns on the market, size, book-to-market, and momentum risk factors are available on Professor French’s online data library. At each date $t$ and for each stock $i$, I estimate its factor loading $(β_{kt})$ for each risk factor $k$ via the following multivariate regression with a one-year rolling window,

$$R_{it} = α_i + \sum_1^3 β_{kt} R_{kt} + ε_{it},$$

where $R_{kt}$ denotes the daily return on the $k$th risk factor.

With the factor loading estimates $β_{kt}$, I then perform a second-stage cross-sectional regression of the ex ante equity risk premiums on the factor loading estimates to identify the factor risk premium,

$$RP_{it} = α_i + \sum_1^3 φ_i β_{kt} R_{kt} + ε_{it}.$$  

The procedure is similar to that described by Fama and MacBeth (1973), except that the second-stage cross-sectional regression in (13) replaces the ex post realized return in the traditional literature with the ex ante equity risk premium.

Table 8 reports the sample average of the risk premium estimates and Newey and West (1987) $t$-statistics for the average risk premium (in parentheses). The two panels represent two specifications. Panel A considers the Fama and French (1993) three-factor model, and Panel B adds momentum as an additional risk factor. Under the three-factor model in Panel A, the risk premium estimates on the market portfolio exposure average at 6.4%. The $t$-statistics show strong statistical significance. The average risk premium on the size factor exposure is at 3.4% and also strongly significant. The average risk premium estimate on the book-to-market factor exposure is small and negative at −0.8%, with lower statistical significance.

Panel B of Table 8 expands the three-factor model to include the momentum factor exposure. The estimates on the market size, and book-to-market exposures are largely the same as those reported in Panel A, suggesting that the momentum factor does not induce much interactions with the other three factors. Exposures to the momentum factor generate an average risk premium estimate of 1.9%. Both the magnitude and statistical significance for the momentum factor risk premium are lower than that on the size or market exposure.

Different from what many researchers have found with ex post excess returns, the analysis using ex ante risk premiums show strong pricing and statistical significance on the market risk, regardless of whether other risk factors are included or not.

### 3.6. Explaining the cross-sectional risk premium variation with firm characteristics

In linking future stock returns to risk factors, the literature considers both loadings to factor returns, as is done in the previous section, and also directly firm characteristics. Table 9 examines the cross-sectional relation between the ex ante equity risk premium and a long list of firm characteristics. The list includes the following dimensions:
Table 8
Risk premium on common equity risk factors. Entries report the time-series averages and Newey-West t-statistics (in parentheses) of the risk premium estimates on the market, size, book-to-market, and momentum factors. Panel A performs the estimation with the 3 Fama–French factors. Panel B also includes the momentum factor exposure in the estimation. The exposures ($\beta_k^t$) on each stock $i$ at each date $t$ for each factor $k$ are estimated by regressing the stock return on the factor return with a one-year rolling window. The last column reports the time-series averages and the standard deviation (in parentheses) of the cross-sectional $R^2$ estimates.

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Market</th>
<th>Size</th>
<th>Book-to-Market</th>
<th>Momentum</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Fama–French three-factor model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.031</td>
<td>0.064</td>
<td>0.034</td>
<td></td>
<td>-0.008</td>
<td>0.097</td>
</tr>
<tr>
<td>t-stats</td>
<td>(3.28)</td>
<td>(5.40)</td>
<td>(6.30)</td>
<td></td>
<td>(-2.35)</td>
<td>(0.086)</td>
</tr>
<tr>
<td>B. Fama–French three-factor + Momentum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.034</td>
<td>0.062</td>
<td>0.034</td>
<td></td>
<td>-0.010</td>
<td>0.019</td>
</tr>
<tr>
<td>t-stats</td>
<td>(3.86)</td>
<td>(5.20)</td>
<td>(6.56)</td>
<td></td>
<td>(-2.79)</td>
<td>(3.07)</td>
</tr>
</tbody>
</table>

Table 9
Explaining the cross-sectional behavior of ex ante equity risk premiums with firm characteristics. The table reports results from explaining the cross-sectional variation of the ex ante equity risk premium with a long list of firm characteristics, including the cash yield (CP), the earnings yield (EP), the book-to-market ratio (BP), long-run earnings growth rate forecast (EC), return on asset (RoA) defined as the ratio of operating income to total assets, momentum (MM) measured as the cumulative return from one year to one month ago, the one-year at-the-money option implied volatility level (IV), the 25-delta call minus the 25-delta put implied volatility skew at one month and one year maturity (SK1m, SK1y), the one-year minus one-month at-the-money implied volatility term spread (TS), the one-year call minus put implied volatility difference (CPD) at 50 delta, and a credit spread measure (MCDS) constructed via a simple implementation of the Merton (1974) model. Panel A reports the time-series average and Newey-West t-statistics (in parentheses) of the pair-wise cross-sectional Pearson linear correlations and Spearman rank correlations between the equity risk premium and each characteristic. Panel B reports the results of a multivariate regression that includes all the characteristics as explanatory variables. For each coefficient, the table reports the time-series average of the coefficient estimate and the Newey-West t-statistics (in parentheses) on the sample average. For $R^2$, the table reports both the sample average and its standard deviation in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>CP</th>
<th>EP</th>
<th>BP</th>
<th>EG</th>
<th>RoA</th>
<th>MM</th>
<th>IV</th>
<th>SK1m</th>
<th>SK1y</th>
<th>TS</th>
<th>CPD</th>
<th>MCDS</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Cross-sectional correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pearson</td>
<td>0.173</td>
<td>0.248</td>
<td>0.120</td>
<td>0.225</td>
<td>0.229</td>
<td>-0.182</td>
<td>0.323</td>
<td>0.027</td>
<td>-0.012</td>
<td>-0.044</td>
<td>0.085</td>
<td>0.159</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6.38)</td>
<td>(9.87)</td>
<td>(6.26)</td>
<td>(13.51)</td>
<td>(12.97)</td>
<td>(-10.20)</td>
<td>(12.74)</td>
<td>(4.96)</td>
<td>(-1.06)</td>
<td>(-2.94)</td>
<td>(21.66)</td>
<td>(8.92)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spearman</td>
<td>0.156</td>
<td>0.262</td>
<td>0.111</td>
<td>0.235</td>
<td>0.242</td>
<td>-0.181</td>
<td>0.314</td>
<td>0.020</td>
<td>-0.055</td>
<td>-0.035</td>
<td>0.094</td>
<td>0.130</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.53)</td>
<td>(9.03)</td>
<td>(6.30)</td>
<td>(12.44)</td>
<td>(12.79)</td>
<td>(-10.01)</td>
<td>(12.28)</td>
<td>(2.35)</td>
<td>(-4.44)</td>
<td>(-2.17)</td>
<td>(30.04)</td>
<td>(7.21)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. Multivariate cross-sectional regression</td>
<td>-0.125</td>
<td>0.062</td>
<td>0.644</td>
<td>0.008</td>
<td>0.412</td>
<td>0.497</td>
<td>-0.066</td>
<td>0.277</td>
<td>0.074</td>
<td>0.031</td>
<td>0.033</td>
<td>0.283</td>
<td>0.420</td>
<td>0.320</td>
</tr>
<tr>
<td>t (10.28)</td>
<td>(4.60)</td>
<td>(9.36)</td>
<td>(0.98)</td>
<td>(15.17)</td>
<td>(10.21)</td>
<td>(-10.67)</td>
<td>(10.11)</td>
<td>(6.74)</td>
<td>(1.20)</td>
<td>(1.62)</td>
<td>(13.63)</td>
<td>(2.87)</td>
<td>(0.11)</td>
<td></td>
</tr>
</tbody>
</table>

- Valuation metrics, including the cash yield (CP), the earnings yield (EP), the book-to-market ratio (BP), long-run earnings growth rate forecast (EC), and return on asset (RoA) defined as the ratio of operating income to total assets. As before, cash per share, earnings per share, long-run earnings growth, and operating income are all one-year ahead analyst forecasts.
- Momentum (MM), measured as the cumulative return from one year to one month ago.
- Option implied volatility behavior, including the one-year at-the-money option implied volatility level (IV), the 25-delta call minus the 25-delta put implied volatility skew at one month and one year maturity (SK1m, SK1y), the one-year minus one-month at-the-money implied volatility term spread (TS), and the one-year call put implied volatility difference (CPD) at 50 delta. Put-call parity dictates that the implied volatilities for European puts and calls at the same strike and maturity should be the same to exclude arbitrage. The two can differ for American options due to different early exercise premiums, unaccounted-for borrowing costs, mis-specified dividend projections, or simply short-term supply-demand pricing pressures. The CPD measure captures this deviations. Several studies (e.g., Cremers and Weinbaum, 2010) show that this call put deviation predicts future stock returns.
- Credit risk (MCDS), measured by a credit spread estimate constructed via a simple implementation of the Merton (1974) model using total debt, market capitalization, and one-year 25-delta put implied volatility. The implementation follows Bai and Wu (2016) except with the put option implied volatility to replace the historical return volatility estimator. Since the possibility of default induces negative skewness in the risk-neutral return distribution, the put option implied volatility contains credit information (Carr and Wu, 2010).

Table 9 reports in Panel A the pair-wise cross-sectional correlations between the equity risk premium and each of the characteristics. To account for potential nonlinearity and data outliers, the panel reports both Pearson linear correlation and Spearman rank correlation. The cross-sectional correlations are estimated every day and entries report the time-series averages of the correlation estimates and the Newey and West (1987) t-statistics on the sample average in parentheses.

All the cashflow metrics generate strongly positive average cross-sectional correlations with the equity risk premium. The sample averages range from 12% on BP to 24.8% on EP for Pearson correlation, and from 11.1% on BP to 26.2% on EP for Spearman correlation. The two sets of correlation estimates tell a similar story. The price-target-based equity risk premium has a strong valuation element in it.

Interestingly, the stock momentum has a negative cross-sectional correlation with the equity risk premium on average at around –18% for both correlation measures, even though Table 8 shows that loadings to the momentum risk factor generate a positive risk premium.

When linking the risk premium to the option implied volatility surface, the entries show that the strongest correlation comes from the implied volatility level at 32.3%. The correlations with other shape characteristics are much smaller. The one-month skew has a small positive correlation with the equity risk premium whereas the one-year skew shows a negative correlation. A more negative long-term skew implies more risk and hence asks for a larger
risk premium. By contrast, the short-term skew is likely to contain more information on cash flow than on discount rate. Finally, the call-put implied volatility deviation shows a strong positive relation with the equity risk premium, consistent with the literature finding on future return prediction.

The ex ante equity risk premium shows a positive relation with the credit risk measure (MCDS), suggesting that investors ask for higher expected returns for companies with higher credit risk. Chava and Purnanandam (2010) also find a positive cross-sectional relation between default risk and the expected stock return using the implied cost of capital approach, even though Campbell et al. (2008) find a negative relation between financial distress and ex post realized return.

Panel B includes all the characteristics into a multivariate cross-sectional regression. Entries report the time-series averages of the coefficient estimates and the Newey and West (1987) t-statistics on the sample average of the coefficient (in parentheses). Within the multivariate context, the coefficient estimates on most characteristics are strongly statistically significant and retain the same sign as the corresponding pairwise correlation. The only two exceptions are the one-year skew and term structure, both of which show weak pairwise correlation to begin with and remain weak in the multivariate setting. The multivariate regression has an average $R^2$ estimate of 32% and a Newey-West standard deviation of 11%.

It is worth noting that a growing literature strives to identify new factors that can predict future stock returns. These findings are often dubbed as “asset pricing anomalies”; nevertheless, these studies do not constitute direct tests or estimation of an existing asset pricing model, but rather as an exploration of new factors that can potentially predict future returns and new dimensions that future asset pricing models should strive to capture. Of ten times, predicting future return itself is the objective. For that purpose, the predictive regressions on ex post returns have their own value and cannot fully be replaced by any ex ante risk premium estimates.

3.7. Link price-target-based equity risk premium to other information sources

This paper proposes to construct the ex ante equity risk premium based on analysts price targets, thus circumventing the need to make model choices and cashflow projections in the ICC approach, or the need to make dynamics assumptions in extracting risk premium from option prices. This section examines the statistical linkages between the three approaches on the aggregate market risk premium.

For the ICC approach, if one makes the simplifying assumption of constant cash flow growth on the aggregate market, one can equate the implied cost of equity to the sum of the cash yield and the constant growth rate,

$$\text{MRP} + r = \text{CP} + \text{CG},$$

where the left side of the equation decomposes the cost of equity on the market to market risk premium (MRP) and the riskfree rate ($r$), and the right hand side equates the cost of equity to the sum of cash yield (CP), defined as the ratio of the expected one-year ahead free cash flow per share to the current stock price, and the constant cash flow growth rate (CG).

If one further assumes constant payout ratio on earnings, one can replace the expected cash yield CP with the expected earnings yield (EP) multiplied by a constant payout ratio and replace the cash growth rate CG with the earnings growth rate EG.

$$\text{MRP} + r = \text{EP} + (1 - \text{BP})\text{RG},$$

where RG denotes the assumed constant growth rate for the residual earnings.

Table 10 investigates in Panel A the relation between the price-target-based equity risk premium and the various cashflow projection metrics. The analysis uses the one-year ahead analysts forecasts on free cash flows per share to define the cash yield CP, uses the one-year ahead earnings per share analyst forecasts to define the earnings yield EP, uses the analysts long-term earnings growth forecasts to proxy EG, and uses the book value of equity to market capitalization to define book-to-market ratio BP. Each metric is constructed on the stock level and aggregated with market-capitalization weighting analogous to the aggregate risk premium calculation. The first row reports the correlation of the equity risk premium with each metric. The correlation estimates are strongly positive with CP at 67.6%, and with EP at 73.4%, and moderately positive with BP at 48.1%. The correlation estimate with the long-run earnings growth estimate is somewhat negative at −12.1%.

Panel A also reports estimates on four regression specifications corresponding to different valuation assumptions. Specification I regresses the equity risk premium on the expected cash yield CP and earnings growth EG. The slope coefficient estimates are strongly positive for both variables, with the regression $R^2$ estimated at 59.4%. The positive coefficient estimate on earnings growth estimate suggest that conditional on the cash yield level, growth rate contributes positively to the equity risk premium, even though its unconditional correlation with the equity risk premium is negative.

Specification II replaces expected cash yield with expected earnings yield EP. The results are similar. The slope coefficient estimates from both regressions are strongly positive and the $R^2$ estimates are both high at around 60%.

Motivated by the residual income valuation model, Specification III regresses the equity risk premium on the expected earnings yield and book-to-market ratio. The regression $R^2$ is higher.
at 70.8%. The coefficient estimates on both earnings yield and the book-to-market ratio are positive and strongly significant. The positive coefficient estimate on the book-to-market ratio implies a negative residual earnings growth rate.

Specification IV adds the earnings growth estimate as an additional explanatory variable to Specification III. The coefficient estimates on earnings yield and book-to-market ratio stay largely the same while the coefficient estimate on the earnings growth is strongly positive. The $R^2$ of the regression increases to 76.6%. The high $R^2$ estimates from these regressions suggest that the price-target based equity risk premium estimates, in aggregate, contain similar information to the implied cost of equity calculation based on cash flow or earnings projections. Nevertheless, the slope coefficient estimates on these specifications are quite different from Eqs. (14) and (15), suggesting that the consensus price targets are computed based on more sophisticated model assumptions than the simple constant growth assumption underlying these equations.

Another source of information for expected equity risk premium is from the stock options market. From the S&P 500 index options prices, one can infer the risk-neutral index return distribution across different conditional horizons (Breeden and Litzenberger, 1978). Furthermore, the level and shape of the option implied volatility surface as compared to the historical realized return variance estimators reveals the risk premium. Based on findings from the existing literature, Panel B of Table 10 constructs three risk premium measures from the S&P 500 index options:

- Variance risk premium (VRP), defined as the difference between the VIX squared and the past one-month realized return variance on the S&P 500 index. Carr and Wu (2008) show that a strip of vanilla options can be used to approximate the variance swap rate under general market conditions and one can use the difference between ex post realized variance and this variance swap rate to measure the ex post return on the variance swap contract. The VIX index is constructed based on the same theoretical principle to approximate the one-month variance swap rate. Bollerslev et al. (2009) propose to use historical realized variance to proxy the ex ante variance expectation to arrive at a variance risk premium (VRP) measure. Since variance risk premium is negative on average, they switch the sign to obtain a positive number on average. Given the strongly negative correlation between index return and variance, the negative of the variance risk premium can partially capture the contribution of return risk premium (Carr and Wu, 2016). In addition to the risk premium component, the VRP measure also reflects market’s expectation of future index return volatility movement: positive when the volatility is expected to increase and negative when it is expected to decline.

- Term risk premium (TRP), defined as the difference between one-year at-the-money implied volatility and one-month at-the-money implied volatility. Negative variance risk premium not only induces a difference between option implied variance and realized variance, but also leads to an average upward sloping implied variance term structure (Egloff et al., 2010). Thus, the average of the term structure slope (TRP) represents another way of capturing the average of the variance risk premium. At each point in time, the TRP includes both a risk premium component and an expectation component: upward sloping when the market expects the variance to increase and downward sloping when the market expects the variance to decline.

- Skew risk premium (SRP), defined as the difference between one-year 25-delta put option implied volatility and one-year 25-delta call option implied volatility. Under a representative economy with Lévy aggregate shocks and power utilities, risk aversion not only induces a return risk premium, but also induces more negative skewness on the risk-neutral return distribution (Polimenis, 2006; Wu, 2006). For S&P 500 index options, the out-of-the-money put option implied volatilities are almost always higher than the corresponding out-of-the-money call option implied volatilities, especially at long option maturities, implying a strongly negative risk-neutral return distribution. The SRP measure captures the skewness of the risk-neutral return distribution, which contains the contribution of the return risk premium.9

Panel B of Table 10 reports in the first row the correlation of the equity risk premium with these three option-implied risk premium measures. Interestingly, the equity risk premium shows negative co-movements with both VRP and TRP. The correlation is weak with VRP at −13.3% but strongly negative at −69.5% with TRP. The negative correlation estimates with VRP and TRP suggest that the equity market risk premium tends to be higher when the volatility term structure is downward sloping. Risk aversion dictates that the implied volatility term structure is on average upward sloping. A downward sloping term structure tends to happen when the stock market falls sharply and market volatility spikes up. To the point where market expectation dominates the shape of the volatility term structure. Thus, it is possible that the variance risk premiums are high during these volatile times, but their presence in the VRP and TRP measure are overshadowed by the expectation component.

Different from VRP and TRP, the skew risk premium SRP generates a strongly positive correlation with the equity risk premium at 73%. Since the SRP mainly captures the contribution of large but rare negative events, it reflects more of market expectation or fear of large negative events.

Given the correlation, Specification V regresses the equity risk premium on the term risk premium and the skew risk premium, which generates a high $R^2$ estimate of 71.4%. The slope coefficient estimates remain negative for term risk premium and positive for skew risk premium. Adding variance risk premium to the regression in Specification VI does not raise the $R^2$ estimate significantly and the slope coefficient estimate on the variance risk premium is not statistically significant.

It is interesting to observe that although option pricing and fundamental equity valuation take on drastically different approaches, the aggregate market risk premiums inferred from the two markets share strong co-movements. The exact linkages between them depend on the details of the cashflow projections and risk dynamics specifications. The beauty of using consensus price targets to construct the risk premium is to arrive at a market consensus risk premium without superimposing the author’s personal view on model choices, cash flow projections, and/or dynamics assumptions.

4. Concluding remarks

Asset pricing theories generate implications on the relation between the expected risk and the expected excess return on a security or portfolio. Most empirical tests replace the ex ante return expectation with ex post return realizations. Unfortunately, ex post realizations can deviate greatly and persistently from the ex ante expectation. The large and sometimes persistent deviations can drastically weaken the power of the empirical tests, making it extremely difficult to verify the theoretical implications. The implied cost of capital approach can reduce the noise of the ex post realization, but demands a myriad of assumptions on cash flow.

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9 Empirically, several studies find that various measures of the option implied volatility skew predict future stock returns. Examples include Xing et al. (2010), Chang et al. (2013), and Conrad et al. (2013).
projections and valuation model choices. Extracting ex ante risk premium from option prices is another possibility, but it also depends on strong dynamics assumptions.

This paper proposes to use analysts’ consensus price target for a stock as the investor expectation of its future price and to define the expected excess return, or equity risk premium, as the log deviation between the consensus price target and the current stock price minus the one-year financing cost. Through this simple construction, the paper generates ex ante risk premium estimates based on market consensus, without superimposing the author’s personal view on model choices, cash flow projections, and/or dynamics specifications.

The analysis shows that classic asset pricing theories work much better on ex ante expectations than generally found on ex post realized returns. In aggregate time series, the equity market risk premium shows strongly positive relations with conditional market portfolio variance estimators. The relation is particularly strong when equity index option-implied volatilities are used as the conditional variance expectation. Cross-sectionally, regressions of the ex ante equity risk premium on conditional covariances with the market portfolio generate positive and strongly significant average relative risk aversion coefficient estimates; regressions of the risk premium on market beta estimates generate strongly positive average market risk premium estimates. Results from these cross-sectional regressions become even stronger when the historical correlation estimates between the stock return and the market portfolio return are averaged within each industry to mitigate errors-in-variable problems in the regression, and even more so when historical volatility estimators are further replaced by forward-looking option-implied volatilities. Finally, expanding the regressions to include loadings on other commonly identified equity risk factors does not in any way diminish the strength and significance of the risk premium on the market beta.

Historically, researchers have perennially been puzzled by the behavior of risk premiums across a wide variety of asset classes, including forward risk premium puzzle in currencies, term risk premium puzzle in interest rates, and the long list of puzzles or anomalies identified in the equity market. Since most of these puzzles are found in studies with ex post excess returns replacing ex ante risk premiums, a potential source of these puzzles can simply be the large and sometimes persistent deviations between ex ante expectation and ex post realization. Since professional surveys are available on many macroeconomic and financial series, using these survey forecasts to construct ex ante expectations can potentially go a long way in verifying or clarifying many of these risk premium puzzles (as in Söderlind, 2009 for equity risk premium puzzle), and in estimating many economic and asset pricing models that involve expectations (as in Kim and Orphanides, 2012 for enhancing the risk premium identification in term structure model estimation.) This can be a promising ground for future research.

References


