Crash-O-Phobia: *A Domestic Fear or a Worldwide Concern?*

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From a large options data set on major equity indexes across the world, the authors find that worldwide, implied volatilities of options on equity indexes exhibit strikingly similar behaviors. When plotted against moneyness, implied volatilities show a heavily skewed smirk pattern, implying that out-of-the-money put options are more expensive than the corresponding out-of-the-money call options and that the risk-neutral return distribution for these indexes is heavily negatively skewed. Furthermore, as the option maturity increases from one month to five years, the implied volatility smirk does not flatten out but steepens, indicating that the risk-neutral distribution of equity index returns becomes even more negatively skewed at longer horizons. The average term structure of the implied volatility level is relatively flat, and the standard deviation of the implied volatility declines as maturity increases. Although fairly persistent, the implied volatility series are stationary. Finally, principal component analysis reveals that equity index volatility movements across the world share one global component.

Since the U.S. stock market crash of 1987, the U.S. equity index options have shown a strong regularity along the strike price dimension. Out-of-the-money put options are much more expensive than the corresponding out-of-the-money call options. When the Black and Scholes [1973] implied volatilities on U.S. equity index options are plotted against some measure of moneyness, the plot exhibits a "smirk" pattern. This phenomenon has been repeatedly documented in the literature, (e.g., Aït-Sahalia and Lo [1998]; Rubinstein [1994]; Jackwerth and Rubinstein [1996]). Rubinstein [1994] refers to it as "crash-o-phobia," alluding to the strong excess demand for put options on the S&P 500 index to hedge against market crashes in the United States.

More recently, Carr and Wu [2003] analyze the maturity pattern of the implied volatility smirk on S&P 500 index options and find that when plotted against a standardized measure of moneyness, the smirk remains steep as maturity increases up to the observable horizon of two years. From a modeling perspective, this maturity pattern implies that the conditional risk-neutral distribution for equity index returns remains heavily non-normal (skewed) as the conditioning horizon increases. This maturity pattern also implies that there are probably very sizable risk premiums for downside jumps such as market crashes that increase with the investing horizon up to two years.

In this article, we ask two questions: 1) Is crash-o-phobia apparent in other equity index markets, or is it merely a U.S. phenomenon? 2) Does crash-o-phobia disappear for investing horizons beyond two years? This article addresses these questions by investigating the behavior of implied volatilities of options written on major world equity indexes.

We obtain a large data set of daily over-the-counter implied volatility quotes at fixed moneyness and maturities on several major equity indexes. The data set is based on over-the-counter options on 12 underlying major equity indexes across the world: AEX (Holland), ALO (Australia), CAC (France), DAX (Germany), FTSE (the United Kingdom), HSI (Hong Kong), IBEX (Spain), MIB (Italy), NKY (Japan), OMX (Sweden), SMI (Switzerland), and SPX (the United States). Each series spans 2,520 business days over the 10-year period from May 31, 1995, to May 31, 2005. Each date contains implied volatilities on options of 8 fixed maturities, from 1 month to 5 years. At each maturity, the data set contains quotes on 5 fixed moneyness levels, defined as percentages of the spot index level at 80, 90, 100, 110, and 120%. Thus, for each equity index, we have observations across the 3 most important dimensions of the options price behavior: the moneyness (strike), the time to maturity, and the calendar time.

We analyze the option price behavior along each of these dimensions. We first document the average shape of the implied volatility curve across moneyness. We find that for all 12 equity indexes, volatility displays strikingly similar moneyness patterns. The implied volatility curves across moneyness all show downward-sloping smirk shapes. Even more striking, for any given maturity, the slopes of the implied volatility smirk appear close to one another for different equity markets, suggesting similar degrees of asymmetry for the risk-neutral return distributions of the different equity indexes. If the smirks are caused by fears of future market crashes, these fears are obviously not limited to the U.S. stock market but pervade markets all around the world.

For any fixed maturity, the implied volatility smirk reflects the shape of the conditional risk-neutral return distribution over this maturity. The way the smirk evolves along the maturity dimension further reveals how the distribution evolves with the conditioning horizon. We find that for all 12 equity indexes, as the time to maturity increases up to 5 years, the implied volatility smirk does not flatten out but steepens. This maturity pattern suggests that the asymmetry of the conditional risk-neutral return distribution does not diminish but actually increases as the conditioning horizon increases to 5 years.

We also examine the time series properties of the implied volatility series. For each time to maturity and calendar time, we summarize the information in the implied volatility smirk using three quantities: the level, the slope, and the curvature of the smirk. We estimate the three quantities via a second-order polynomial fit on the smirk. We then document the time series properties of the three quantities. We find that for most indexes, the average term structure of the implied volatility level is relatively flat and the standard deviation of the implied volatility level declines monotonically as the maturity increases. The stability of the mean term structure and the declining standard deviation with maturity are evidence of stationarity for the volatility process. We also measure the autocorrelation of the implied volatility level and find that the implied volatility level becomes more persistent with increasing maturities, indicating the potential existence of multiple volatility factors and/or non-linear volatility dynamics.

For all equity indexes, the mean slope of the smirk becomes more negative as the maturity increases, confirming our earlier observation that the smirk steepens with maturity. This steepening occurs mainly for maturities up to one year, after which the term structure of the smirk slope flattens out. The curvature of the smirk is usually very small for most indexes.

Finally, principal component analysis on the daily changes in the volatility level and slope shows that just one global factor explains close to 40% of the variation in the daily volatility level movements. The smirk slope coefficients also show comovements, but the proportion of idiosyncratic movements is larger.

The structure of the article is as follows. The first section describes the data set. The following three sections analyze the option price behavior along the dimensions of moneyness, maturity, and calendar time, respectively. After that, the comovements of the implied volatility smirks of different indexes are documented, and the final section concludes.

DATA DESCRIPTION

Traditionally, options on stocks and stock indexes were mostly traded on the options exchanges. In the United States, the S&P 500 index options are listed on the Chicago Board of Options Exchange. Since early 1990s, an over-the-counter market has developed for equity index options. This market has expanded rapidly over the past decade, mainly to accommodate the growth in exotic derivatives contracts such as variance swaps and long-term reinsurance contracts.

We obtain our data set of implied volatilities from a major dealer in the over-the-counter market, Credit Suisse First Boston. The data set consists of implied volatilities for European-style options underlying 12 major equity indexes across the world. Exhibit 1 lists the symbol, full name, and the country of origin for each of the 12 indexes.

The quotes are daily on option contracts with fixed time to maturity and moneyness, from May 31, 1995, to May 31, 2005, spanning 2,520 business days over the past 10 years. At each date and for each index, the implied

EXHIBIT 1
Index List

Symbol	Name	Country
AEX	Amsterdam Exchanges Index	Holland
ALO	The Australian All Ordinaries Index	Australia
CAC	The CAC-40 Index	France
DAX	The German Stock Index	Germany
FTSE	The FTSE 100 Index	United Kingdom
HSI	The Hang Seng Index	Hong Kong
IBEX	The IBEX 35 Index	Spain
MIB	The Italian MIB Index	Italy
NKY	The Nikkei-225 Stock Average	Japan
OMX	The Swedish Market Index	Sweden
SMI	The Swiss Market Indexx	Switzerland
SPX	The S&P 500 Index	United States

Entries report the list of stock indexes that underlie the option implied volatility quotes used in this article. Source: Credit Suisse First Boston. Used with permission, all rights reserved. CSFB does not make any warranty of accuracy of the data or of the views expressed herein and did not participate in the preparation or review of this article.

volatility quotes are available at 8 fixed time-to-maturity levels: 1 month, 3 months, 6 months, 1 year, 2 years, 3 years, 4 years, and 5 years. Each maturity contains quotes at 5 fixed moneyness levels, defined as strike prices as percentages of the spot index level at 80, 90, 100, 110, and 120%.

The dealer compiles the quotes at the end of each business day in the respective local markets. As inputs for the calculation, the dealer uses prices of options and possibly other volatility-sensitive instruments such as variance swaps, traded both over the counter and in exchanges. The fixed-grid values are interpolated using a proprietary algorithm. To convert the implied volatility quotes into option prices, the dealer uses the corresponding LIBOR/swap rates as the risk-free interest rates. The expected dividend yield is inferred from the relative pricing of spot and futures on the index.

This over-the-counter data set contains more information than is available in purely exchange-traded options. The design of the over-the-counter options market differs from that of the exchanges market in several important ways. First, the quotes are not expressed directly in terms of option prices but in terms of Black and Scholes [1973] implied volatilities. Given the quote on the implied volatility, the invoice price for the option contract is computed based on the Black-Scholes model, with mutually agreed-upon inputs on the underlying spot and futures index level and interest rates. Second, when a transaction takes place, it involves not only the exchange of the option position but also the corresponding Black-Scholes delta hedging position on the underlying index. Therefore, the over-the-counter transactions are approximately delta neutral and hence are not sensitive to the underlying index movement. As a result, synchronization between implied volatilities quotes and the underlying spot level is not as crucial as in the exchange market. Third, for very large trades, the over-the-counter market also has something similar to the "upstairs" market, where the broker-dealer tries to match buyers directly with sellers so that the broker-dealer does not need to take large inventory.

The derivative nature of the options contract creates issues for market-making that do not exist in the primary security markets. First, option prices are directly linked to the price of the underlying security. Therefore, whenever the underlying spot price changes, the prices of all options underlying this security change with it. On an options exchange, this feature forces the market-maker to update all the options prices whenever the underlying security price moves. Delays in the updating process can potentially put the market-maker in a risky position. Therefore, the market-maker in an options market must have advanced technology that can update quotes on hundreds of options on any security within a very short time. For the same reason, for academic studies, it is important to obtain the option price quotes across all contracts underlying the same security at the same time (or within a very short time) to avoid synchronization issues. Second, when a customer has private information on the underlying security, the customer can simultaneously send buy or sell orders on all the options underlying this security. Thus, even if the market-maker provides a small quote size on each option, the aggregate risk exposure can still be very large as a result of the highly correlated nature of all the options underlying the same security. These concerns force market-makers to post wider bid-ask spreads and smaller quote sizes, thus making the options exchange market less liquid and unattractive for institutional players who want to engage in options trades of large quantities.

The over-the-counter options market addresses most of these concerns by design and thus improves the liquidity and depth of the market. The exchange of the covered position (the option with its delta), rather than a naked option position, significantly reduces the dealer's directional exposure to the underlying security. The quotation convention of using implied volatility rather than prices reduces the dealer's need to constantly update the option prices for every move in the underlying security price. An update becomes necessary only when the dealer thinks that the underlying volatility or other higher moments of the market have changed. The spot price of the underlying currency plays a lesser role given the covered transaction. Finally, for very large transactions, the "upstairs" mechanism further reduces the broker-dealer's exposure to large inventory positions and also reduces the transaction costs for both sides. As a result, the over-the-counter market can handle very large trades with small bid-ask spreads and little market impact, an ideal situation for institutional players. For long-dated options (maturities longer than two years), the exchange market is almost non-existent, but the over-the-counter market remains very active. For example, out-of-the-money options on the S&P 500 index with times to maturity between three and five years trade almost on a daily basis. Furthermore, interested parties can easily obtain tight quotes for these options from various brokers. Therefore, the over-the-counter implied volatility quotes provide important information that is not readily available from the exchange markets.

THE MONEYNESS PATTERN OF OPTION IMPLIED VOLATILITIES

One of the striking features of S&P 500 index options is that the Black-Scholes implied volatility quotes show a heavily skewed line when plotted against moneyness, indicating that out-of-the-money put options are more expensive than the corresponding out-of-the-money call options. The skewed plots are often referred to as the "implied volatility smirk," in contrast to the more symmetric smile shape observed in currency options. The smirk has become a persistent feature of the U.S. equity index options market since the stock market crash in 1987. Rubinstein [1994] refers to this phenomenon as "crash-o-phobia", alluding to the strong market demand and weak supply for out-of-the-money put options on the S&P 500 index to hedge against market crashes in the United States.

The smirk in implied volatilities corresponds to a heavily skewed risk-neutral distribution for the index return, in sharp contrast to the relatively symmetric return distribution estimated from time series data. In principle, we can attribute the difference between the two distributions to risk premia. Several studies ask whether the much higher premium on the put options is a reasonable compensation for the crash risk (e.g., Jackwerth [2000]; Engle and Rosenberg [2002]; Bliss and Panigirtzoglou [2004]; Wu [2006]). The general consensus is that the market premium for downside index movements is very high even if it does not constitute an arbitrage opportunity.

Using implied volatilities on different world equity indexes, we study whether the high premium on downside risk observed in the U.S. equity index options market is merely a local phenomenon or a common feature worldwide. If it is a unique local phenomenon, we can conclude that the 1987 U.S. stock market crash has indeed scared only U.S. investors into paranoia. On the other hand, if options on major equity indexes around the world all share the same pattern, we need to search for more fundamental reasons for the large premiums charged on out-of-the-money put options on equity indexes.

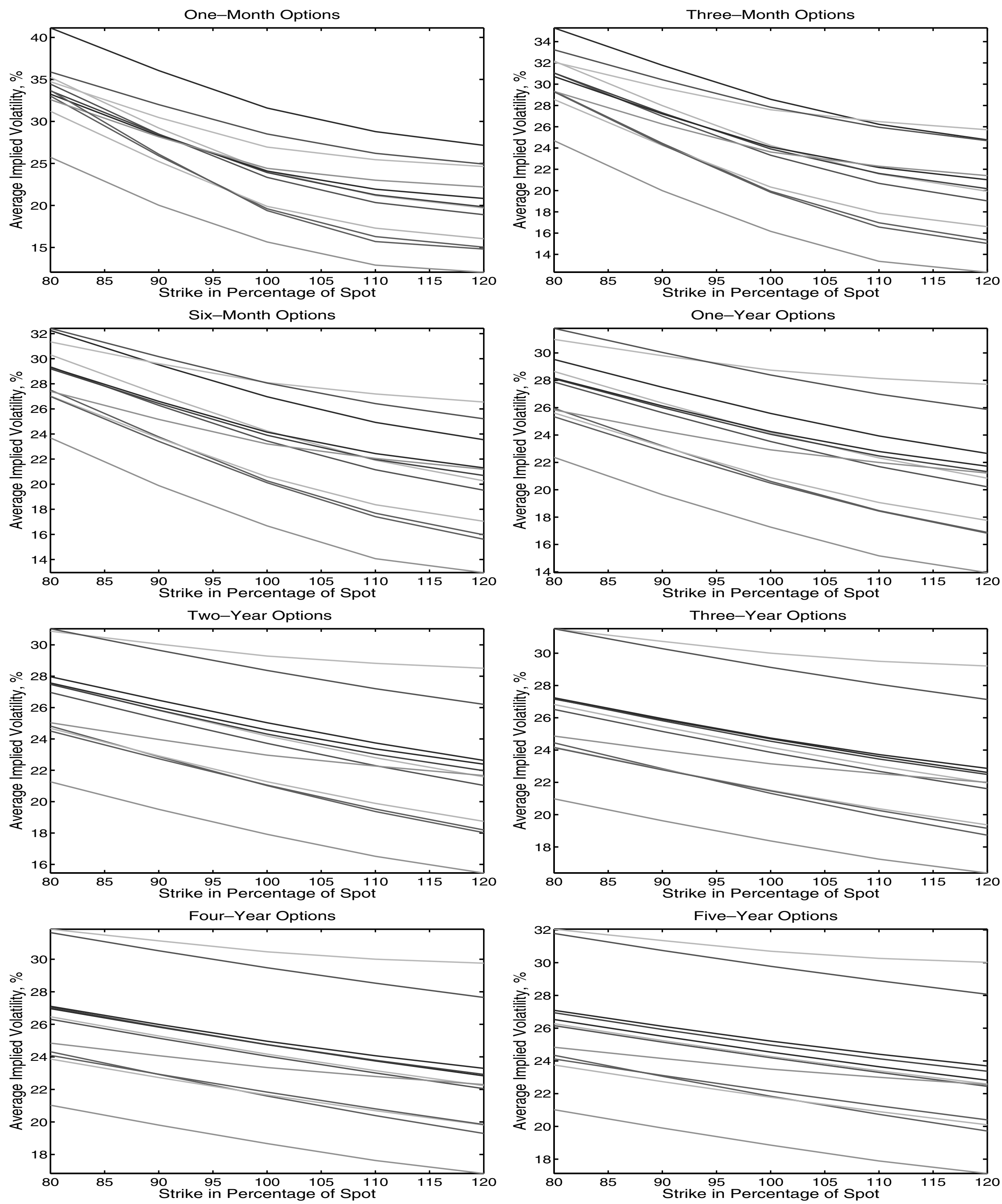
Exhibit 2 plots the sample average of the implied volatility against fixed moneyness levels, defined as the strike price as a percentage of the spot index level. For comparison, we group the plots for different equity indexes into the same panel, one panel for each time to maturity. The eight panels correspond to the eight different maturities from one month to five years.

A striking pattern emerges. For all indexes and at all maturities, the average implied volatility plots against moneyness are all skewed to the left, implying that out-of-the-money put options are more expensive than the corresponding out-of-the-money call options. The main difference among different equity indexes is the level of the implied volatility. The slopes, and hence the degrees of the skew in the volatility smirk plot, appear strikingly close from one equity index to another.

These plots indicate that the implied volatility smirk is not a purely domestic phenomenon limited to the United States but rather a worldwide phenomenon apparent in all major equity markets. World investors demand, and are comfortable paying, much higher premiums for out-of-the-money put options than for the corresponding out-of-the-money call options regardless of the underlying index or its geographical location. One potential explanation for this global phenomenon is that downside movements in any index are likely to be highly correlated with those in other markets as a result of worldwide contagion. Since the downside risk cannot be readily

EXHIBIT 2

Implied Volatility Smirk on Major Equity Indexes



Lines represent the sample averages of the implied volatility quotes plotted against the fixed moneyness levels defined as strike prices as percentages of the spot level. Different panels are for options at different maturities. Data are daily from May 31, 1995, to May 31, 2005, spanning 2,520 business days for each series. The 12 lines in each panel represent the 12 equity indexes listed in Exhibit 1.

diversified by investing in different equity markets, and in aggregation the markets are long these indexes, insurance for downside movements on these indexes commands a higher premium than does insurance for upside movements. Furthermore, given the aggregate long position on the indexes, investors tend to view upside movements of the index not as risks but rather as opportunities. Based on standard definitions of risk preferences and on U.S. stock index options data, Jackwerth [2000] and Engle and Rosenberg [2002] find that investors become actually risk loving rather than risk averse at some point of the return distribution. Wu [2006] proposes a pricing kernel that explicitly allows different pricing for downside and upside jumps in the equity index. Estimation on the S&P 500 index returns and options shows that investors charge the maximum premium allowed by no arbitrage on downside jumps but that they charge only a moderate price on upside jumps. Figlewski and Freund [1994] and Green and Figlewski [1999] analyze the issue from the supply side. They argue that the strategy of writing put options and dynamically hedging with the underlying index involves significant market risk and model risk. Carr and Wu [2002] further show that the dynamic delta-hedging strategy breaks down during large market movements. Thus, the high premiums for out-of-the-money put options are a combined result of 1) concerns for market crashes because of the concentrated exposure to common downside movements and 2) the inherent difficulty in hedging against market crash risk.

THE MATURITY PATTERN OF IMPLIED VOLATILITY SMIRKS

In plotting the implied volatility smirks in Exhibit 2, we define moneyness as the strike as a percentage of the spot level: $100 \times K/S$, where K denotes the strike price and S denotes the spot level. This choice of moneyness is mainly for convenience since the available implied volatility quotes are at fixed moneyness levels according to this definition. We can obtain a more transparent link between the shape of the implied volatility smirk and the conditional non-normality of the risk-neutral return distribution using a standardized measure, d, for moneyness,

$$d \equiv \frac{\ln(K/S)}{\sigma\sqrt{\tau}} \tag{1}$$

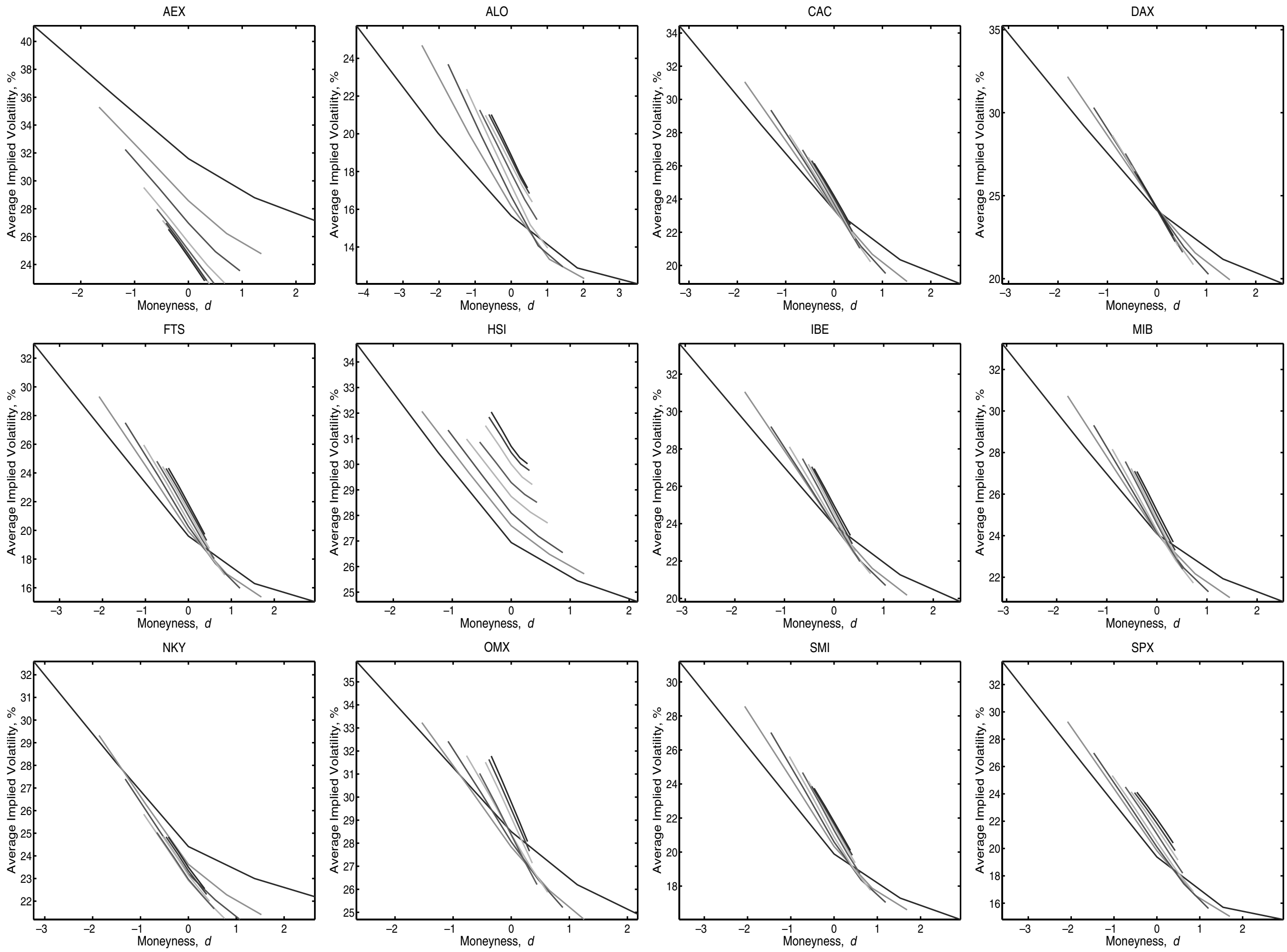
where τ denotes the time to maturity of the option and σ is some measure of the volatility level. This definition has become an industry standard. The $\sqrt{\tau}$ scaling is important in making the scaling of the moneyness comparable across different maturities. The use of some volatility measure σ in the denominator is to make the moneyness measure also comparable across different securities of different volatility levels. The definition in Equation (1) also allows an intuitive interpretation of moneyness as roughly the number of standard deviations that the log strike is away from the log spot price in the Black-Scholes world.

With one year's worth of exchanged-traded S&P 500 index options data, Carr and Wu [2003] find that when they plot the implied volatility against the measure of moneyness defined in Equation (1), the implied volatility smirk shape does not flatten out but instead steepens as maturity increases up to the observable horizon of two years. Since the slope of the smirk reflects the asymmetry (skewness) of the risk-neutral distribution for the index return, the fact that the smirk does not flatten out implies that the asymmetry of the conditional risk-neutral distribution persists as the conditioning horizon increases.

Exhibit 3 plots the implied volatility smirks at different maturities for all 12 equity indexes under the standard moneyness measure d. To highlight the maturity pattern of the smirk, we plot the implied volatility smirks at different maturities within the same panel for each equity index. The 12 panels correspond to the 12 equity indexes. Since the implied volatility quotes are at fixed strikes as percentages of the underlying spot level, the range of the standardized moneyness d shrinks as the option maturity increases. Thus, we can easily distinguish the different maturity of each line. The longest line in each panel represents the smirk of the options at the shortest maturity (1 month), and the shortest line represents the smirk at the longest maturity (5 years).

Exhibit 3 reveals a strikingly similar maturity pattern across all 12 equity indexes. When plotted against the moneyness measure d, the implied volatility smirk does not flatten out but actually steepens as maturity increases. These findings are consistent with those in Carr and Wu [2003] for S&P 500 index options, but our evidence extends their results in two important dimensions. First, the maturity pattern not only holds for the United States but also extends to all major equity indexes around the world. Second, the times to maturity of their exchanged-listed index options data are within 2 years, but the time to maturity for the over-the-counter data used in this study extends to 5 years.

EXHIBIT 3
Maturity Pattern of Implied Volatility Smirks



Lines denote the sample averages of the implied volatility quotes, plotted against a standard measure of moneyness $d = \ln(K/S)/(\sigma\sqrt{\tau})$ where K, S, and τ denote the strike price, the spot index level, and the time to maturity in years, respectively. The term σ represents a mean volatility level for each equity index, proxied by the sample average of the implied volatility quotes underlying each equity index. For each equity index, we plot the implied volatility smirks at the 8 different maturities in the same panel. The maturities for each line are 1 month, 3 months, 6 months, 1 year, 2 years, 3 years, 4 years, and 5 years. The length of the line shrinks with increasing maturity, with the longest line representing the shortest maturity (1 month). The 12 panels correspond to the 12 equity indexes.

From an equilibrium perspective, the highly skewed implied volatility smirk at a certain maturity is the combined result of a high demand for put options owing to crash-o-phobia and a weak supply owing to the inherent difficulties in dynamically hedging risks associated with market crashes. The steepening and persistence of the implied volatility smirks over longer maturities further indicate that 1) concerns for market crashes are strong not only in the short term but also in the long run and 2) put-option sellers do not view the risk exposures from dynamically hedging their positions as more benign because of long-term averaging effects. Indeed, the risk exposures can increase over long horizons because the time variation in the volatility level causes the performance of simple delta hedging for long-dated options to further deteriorate.

From a modeling perspective, the maturity pattern implies that the conditional risk-neutral distribution for the index returns remains heavily non-normal (skewed) as the conditioning horizon increases up to five years. To accommodate the evidence of short-term smirks, researchers have often incorporated a jump component into the asset return process.[1] However, return non-normality generated by most of these jump models disappears rapidly as the conditioning horizon increases by virtue of the classic central limit theorem (Backus, Foresi, and Wu [1997]). To generate return non-normality at longer time horizons, researchers have resorted to introducing a persistent stochastic volatility process.[2] However, to maintain the same (or even higher) return non-normality for horizons as long as five years, the stochastic volatility process must become near non-stationary. A non-stationary stochastic volatility process generates features along other dimensions that do not match the observed implied volatility behavior. For example, Pan [2002] shows that under a non-stationary stochastic volatility process, the implied volatility level grows explosively with increasing maturity, a feature not observed in the data. Hence, the maturity pattern of the implied volatility smirk poses challenges for option pricing modeling.

Another interesting observation from both Exhibits 2 and 3 is that the implied volatility smirks across all maturities and for all 12 equity indexes are decidedly one-sided, implying that the risk-neutral distribution of the index return has a very fat left tail but that the right tail of return distribution is thin. Such evidence is consistent with the one-sided α-stable specification in Carr and Wu [2003] but not with the traditional compound Poisson jump specification in Merton [1976]. Most recently, Bakshi, Carr, and Wu [2005] identify stochastic discount factors in international economies from currency returns and currency options. They can identify a downside jump component but not an upside jump component in the stochastic discount factors, suggesting that even though the aggregate uncertainty of an economy can receive both positive and negative shocks, only downside jumps are priced.

THE TIME SERIES DYNAMICS OF IMPLIED VOLATILITIES

In this section, we document the time series dynamics of the implied volatility quotes and develop insights into the dynamic specification of the stochastic volatility process under both the risk-neutral and the statistical measures.

At each date and maturity and for each equity index, we observe implied volatility quotes at five strike levels that form a smirk pattern. We summarize the information in each implied volatility smirk by its level, slope, and curvature. To do this, we fit a second-order polynomial function to the volatility smirk against moneyness:

$$\mathrm{IV}(d;\, t,\, \tau) = c_0 + c_1 d + c_2 d^2 + e \tag{2}$$

where $\mathrm{IV}(d; t, \tau)$ denotes the implied volatility quote at moneyness d, time t, and time to maturity τ; d denotes the standard moneyness measure defined in Equation (1), with the at-the-money implied volatility at each date and maturity to proxy σ in the definition; and e denotes the fitting error.

Repeating the second-order polynomial fit in Equation (2) for each maturity at each date for each index in the sample, we obtain a time series of coefficients (c_0, c_1, c_2) that capture the level, the slope, and the curvature of the implied volatility smirk, respectively. The intercept c_0 represents the fitted value for the at-the-money ($d = 0$) implied volatility. It is therefore a proxy for the volatility level at that maturity and date. The coefficient c_1 captures the slope of the smirk at $d = 0$ and reflects the asymmetry of the conditional risk-neutral distribution on the index return. The coefficient c_2 measures the global curvature of the smirk and is related to the kurtosis of the risk-neutral return distribution.

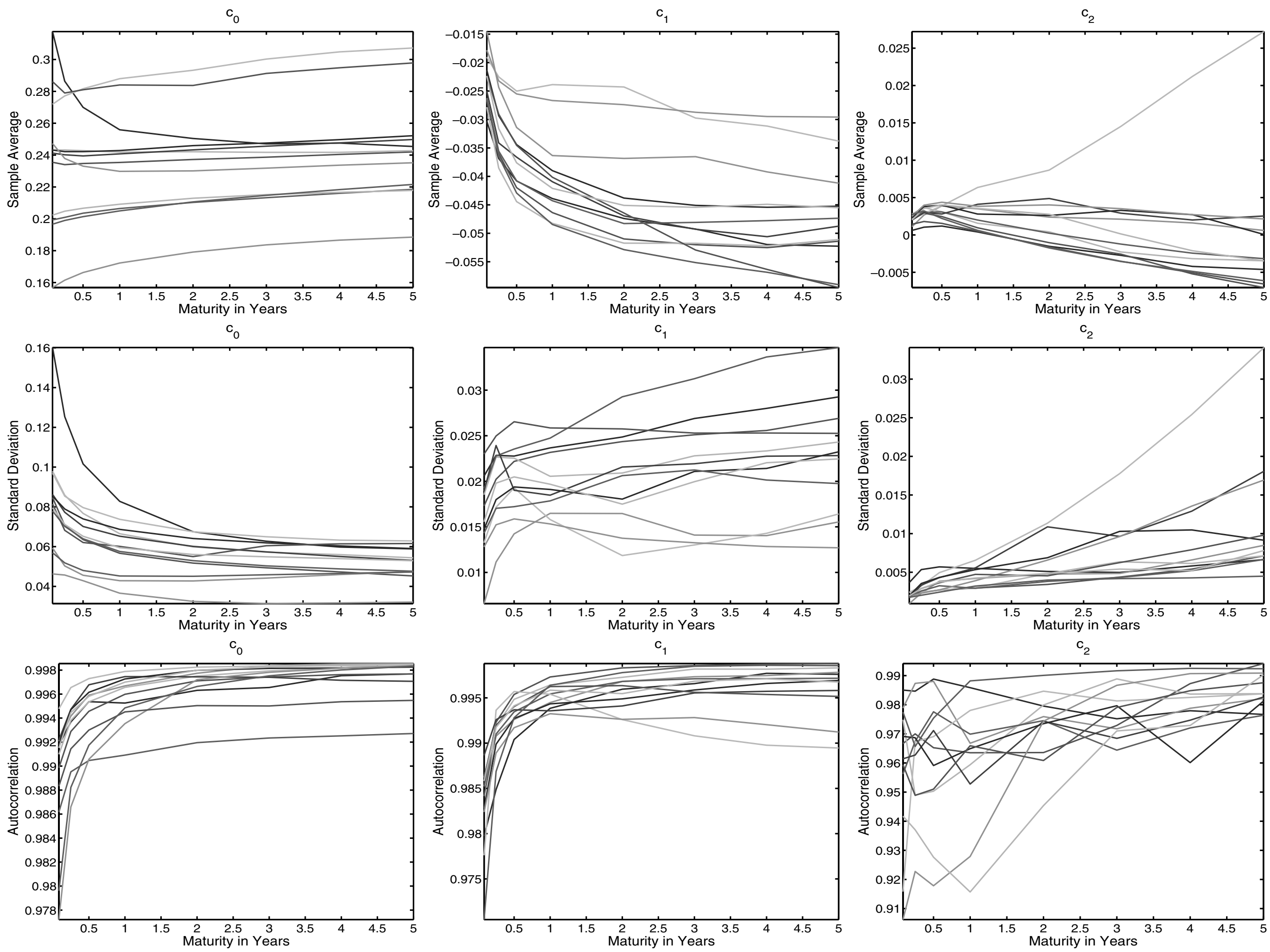
Exhibit 4 plots three summary statistics of each coefficient against maturity: the sample mean, the standard deviation, and the daily autocorrelation of the daily estimates of the coefficient. The panels in the left column summarize the time series properties on the volatility level, proxied by c_0, at different option maturities. Each panel in this column represents one statistic and each line in a panel represents one equity index. The mean term structure of the volatility level c_0 can be either upward sloping or downward sloping, but overall it is relatively flat.

For risk management on derivative positions, it is important to assess the volatility risk, or the volatility of volatility. In Exhibit 4, we measure the term structure of the volatility risk in terms of the standard deviation of the volatility level c_0 (first column, second row). For all equity indexes, the standard deviation estimates decline steadily with increasing maturity. The relatively flat mean term structure and this downward-sloping term structure of standard deviation supprt the stationarity of the volatility process.

The last row of the first column measures the first-order (daily) autocorrelation of the volatility level c_0. The

EXHIBIT 4

Time Series Properties of the Level, Slope, and Curvature of the Implied Volatility Smirk



Lines denote the sample mean (first row), the standard deviation (second row), and the daily autocorrelation (last row) of the daily coefficients estimates $[c_0, c_1, c_2]$ from a second polynomial fit of the implied volatility smirk at each maturity and date: IV $(d; t, \tau) = c_0 + c^1 d + c_2 d^2$. The statistics are computed over the 10-year sample for each index from May 31, 1995, to May 31, 2005.

estimates are high, indicating that the implied volatility process is highly persistent. Furthermore, the first-order autocorrelation estimates for the volatility level increase with increasing maturity. Had the returns variance followed a one-factor affine process such as the square-root process of Heston [1993a], we would have expected to recover the same autocorrelation estimates for volatilities of different maturities. The upward-sloping term structure of autocorrelation estimates suggests non-linear dynamics or the presence of multiple volatility factors, with the less persistent factors(s) dominating the short end of the term structure and the more persistent factor(s) dominating the long end.

The panels in the middle column of Exhibit 4 summarize the time series properties of the smirk slope coefficient (c_1). For all 12 equity indexes, the mean term structure of the smirk slope c_1 is downward sloping. Thus, the slope of the implied volatility smirk becomes more negative as maturity increases, a maturity pattern already documented in the previous section. Both the standard deviation and autocorrelation estimates for the smirk slope c_1 increase with maturity.

The panels in the last column of Exhibit 4 summarize the properties of the curvature (c_2) of the implied volatility smirk. The mean values of the curvature estimates are close to zero. The small mean estimates are consistent with our visual observation that the implied volatility smirks look like straight lines.

The fact that the persistence increases with maturity for both the level and the slope of the implied volatility smirk indicates that there are potentially multiple factors governing the dynamics of volatility. It is interesting to see how these different factors interact with one another to govern the evolution of the conditional return distribution both along the maturity dimension and over time. To study how the volatility level and the conditional asymmetry of the index return distribution interact with each other, we measure the cross correlation between the volatility level proxy c_0 and the slope proxy c_1, both in levels and in daily differences. The estimates are plotted in Exhibit 5 across different maturities, with each line representing one equity index. The correlation estimates on both levels and daily changes are predominantly negative. Since the slope of the implied volatility smirk is negative, a negative correlation between the volatility level and the smirk slope indicates that the volatility smirk steepens and becomes more negative when the volatility level is high.

Economically, the evidence in Exhibit 5 indicates that the excess demand for the out-of-the-money put options relative to the out-of-the-money call options increases with increasing volatility. When fear of crashes increases, the market charges a higher premium on all options, but more so for out-of-the-money put options than for out-of-the-money call options.

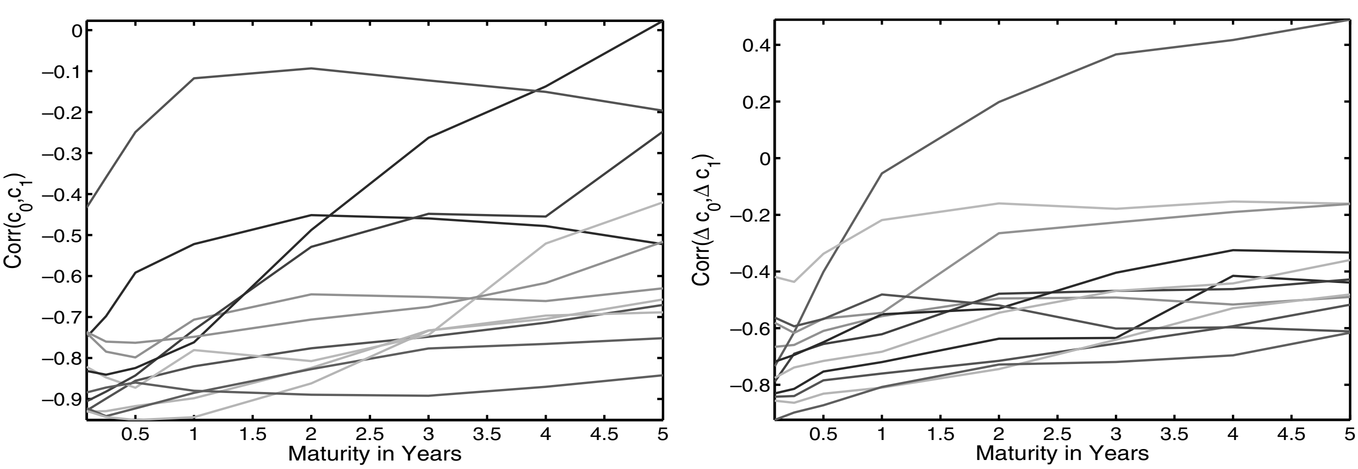
COMOVEMENTS OF WORLDWIDE IMPLIED VOLATILITIES

The evidence so far points to a common feature among the world equity index options markets. The premiums for out-of-the-money put options are significantly and persistently higher than the premiums for the corresponding out-of-the-money call options. To understand whether this common feature is driven by different or common sources of risk, we study the comovements of the implied volatility surfaces across the 12 countries. For this purpose, we perform principal component analysis on the daily changes in the volatility levels (c_0) and separately on the volatility slopes (c_1). We have also performed the principal component analysis on the curvature estimates (c_2), but the results are noisy because of the small curvature estimates. To save space, we do not report the results on curvatures.

The principal component analysis is based on the decomposition of the covariance matrix of the sample series. The normalized eigenvalues of the covariance matrix can be interpreted as the percentage variation

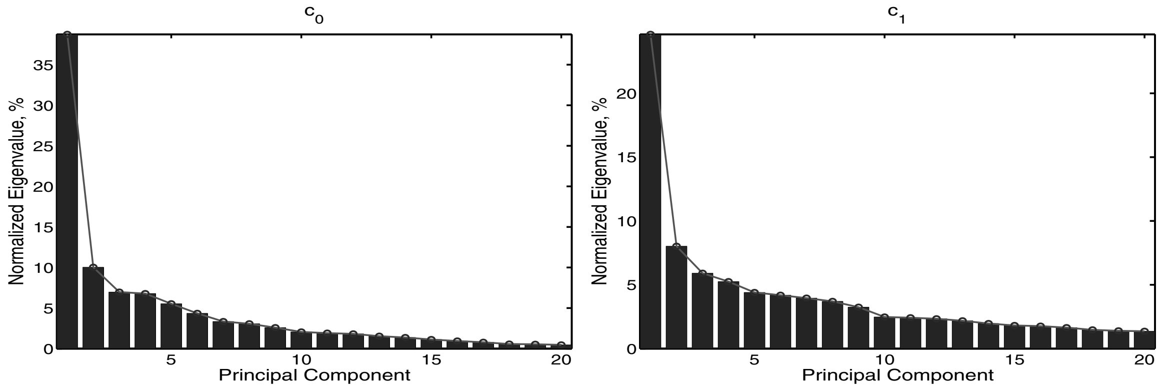
EXHIBIT 5

Cross Correlations between Volatility Level and Smirk Slope



Lines denote the cross-correlation estimates between the volatility level proxy (c_0) and the volatility smirk slope proxy (c_1). The left panel measures the correlation based on daily estimates, the right panel measures the correlation based on daily changes of the estimates.

EXHIBIT 6
Percentage Variance Explained by Each Principal Component



Principal Component	c_0	c_1
1	38.74	24.62
2	10.03	8.02
3	6.97	5.91
4	6.79	5.25
5	5.53	4.40
6	4.35	4.18
7	3.34	3.96
8	3.07	3.70
9	2.59	3.25
10	2.06	2.48

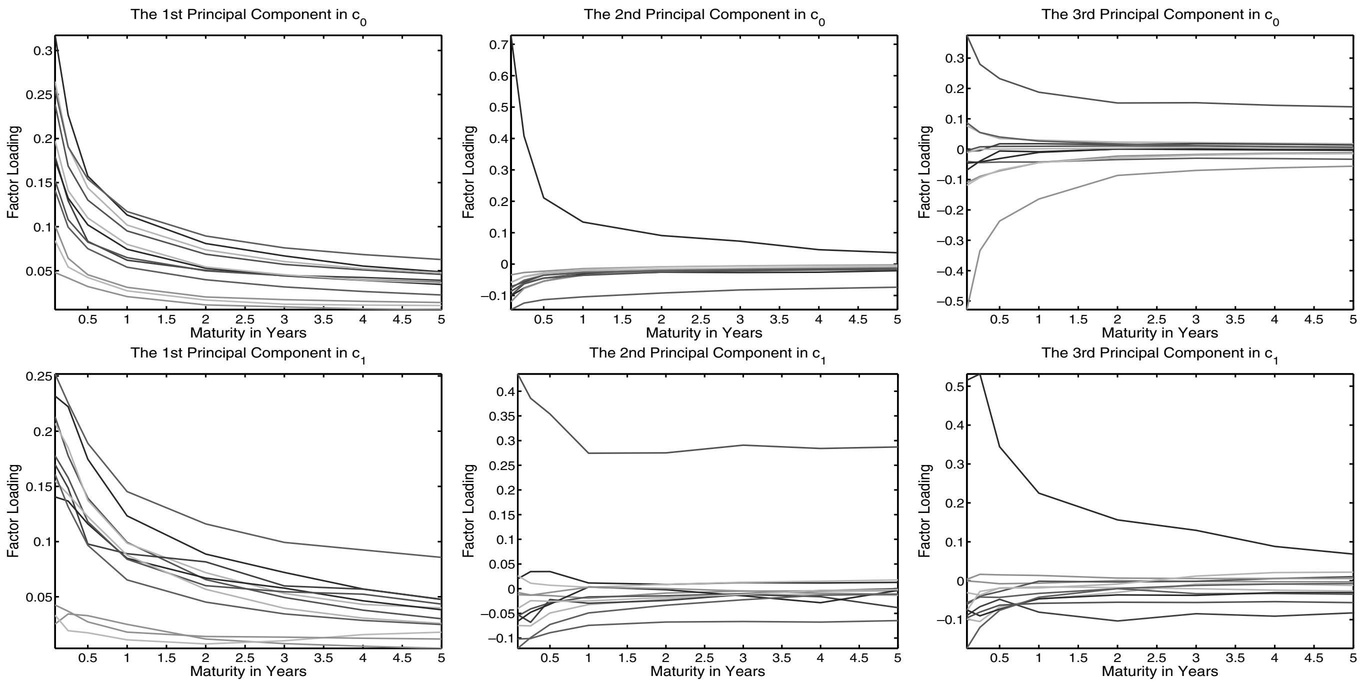
The bar charts and the table report the percentage explained variation of each of the first 10 (20 in the bar charts) principal components on the implied volatility level proxy c_0 and the implied volatility slope proxy c_1. We compute the percentage variance based on the eigenvalues of the covariance matrix of daily changes on the c_0 and c_1 series.

explained by each principal component. The normalized eigenvector corresponding to each principal component represents the loading of this principal component to each of the data series.

In Exhibit 6, we use bar charts to show the percentages of aggregate variation explained by each of the first 20 principal components for the volatility level (left panel) and the smirk slope (right panel). We link the bars with a line to highlight the speed of decay for the percentages. We report the estimates for the first 10 principal components in the table. For the volatility level c_0, the first principal component accounts for 38.74% of the variation in daily changes of the 480 volatility series across 12 indexes. The contribution of the second principal component is much lower at 10.03%. The bar charts show an obvious slope change after the first principal component. The literature has not arrived at a consensus criterion in determining the optimal number of common factors in principal component analysis. Although some researchers have proposed statistical tests (e.g., Connor and Korajzcyk [1993]) a common practice is to plot the eigenvalues as in Exhibit 6 and visually inspect for slope changes. Principal components with eigenvalues falling on different slopes of the plot represent different levels of commonality. Thus, based on the bar chart for c_0, we conclude that the first principal component of the volatility level represents a different level of commonality from the other principal components.

For the volatility slope c_1, the first principal component explains about a quarter of the variation, and the

Exhibit 7
Factor Loading



Lines represent the value of the eigenvector of the covariance matrix corresponding to the first three principal components (from left to right). We perform principal component analysis on the implied volatility level estimates c_0 (the first row) and the implied volatility slope estimates c_1 (the second row) separately. Each line represents one equity index.

second principal component explains 8% of the variation. In this case, the slope change in the bar charts is less obvious, and the decay is much slower across all the principal components. Hence, although there may also exist a common component in the smirk slope variation, a larger proportion of the variation is idiosyncratic.

Exhibit 7 plots the factor loading (eigenvector) for the first three principal components on different indexes and at different maturities. The first row denotes the factor loadings on the volatility level c_0 and the second row denotes the loading on the slope c_1. In each row, the first column denotes the loading of the first principal component from the respective data series, the second column denotes the loading of the second factor, and the third row the loading of the third factor. Each line denotes the loading on each index across different maturities.

For the volatility level c_0, the loadings of the first principal component are all positive and exhibit similar maturity patterns across all 12 equity indexes. The loading on the first principal component declines as maturity increases, an indication that short-dated option implied volatilities are more volatile than long-dated option implied volatilities.

To check whether a similar commonality exists in actual (as opposed to implied) volatilities, we also estimate a GARCH(1,1) model using the time series of returns of each equity index and obtain a series of volatility estimates for each index. Then, we perform a principal component analysis on the estimated 12 volatility series. The analysis shows that 46.58% of the variation in the 12 GARCH volatilities can be explained by one principal component. Although the numbers are not directly comparable to those from the implied volatility series, both pieces of evidence suggest the existence of a global volatility factor.

The loading of the second principal component in the volatility level c_0 is quite different. This second component has a large loading on only one index series, but its loadings on other indexes are close to zero. Therefore, this second principal component looks more like a country-specific component. The third factor has significant loadings on only two indexes. The loadings of the subsequent five principal components exhibit similar patterns: They are significant on either one or two index series but close to zero for all other indexes. Therefore, we conclude that principal components 2-8 are either country specific or shared by only a small number of countries. Only the first principal component of the volatility level represents a global common factor.

For the smirk slope, the first principal component has positive loadings on 9 of the 12 indexes. Indexes in the Asia-Pacific region (Australia [ALO], Hong Kong [HSI], and Japan [NKY]) display smaller loading estimates. Thus, the first principal component on the smirk slope seems to represent a global common factor excluding the Asia-Pacific region. The loading patterns of the second and third principal components obviously represent country-specific factors.

Overall, the principal component analysis shows that the world shares one common global component in market risk, as captured by the comovement in index return volatilities. The variations on the slope of the volatility smirks also share some common movements but contain a larger proportion of idiosyncratic variation. For international asset pricing, our evidence suggests a global return factor that contains a downside jump component and generates stochastic volatility.

CONCLUSION

Worldwide, options on equity indexes show strikingly similar behaviors. Along the moneyness dimension, implied volatilities of options on all major equity indexes show a heavily skewed pattern, implying that out-of-the-money put options are more expensive than the corresponding out-of-the-money call options and that the implied risk-neutral distributions for these index returns are heavily skewed to the left. This highly asymmetric risk-neutral return distribution is in sharp contrast to the much more symmetric statistical distribution estimated from the time series of returns. The discrepancy between the two probability distributions suggests that the market charges a high premium on downside index movements but asks little for upside index movements. We attribute this asymmetry to a combination of the high demand for out-of-the-money put options as a result of global concerns for market crashes and their limited supply because of the inherent difficulties in hedging the risks associated with market crashes.

Along the maturity dimension, the implied volatility smirk does not flatten out with increasing maturity but becomes steeper. Economically, this feature implies that the concern for market crash remains acute for long-term investments and that the risk exposure from delta-hedged short put option positions is not mitigated in the long run. Finally, principal component analysis indicates that the movements of equity index volatilities around the world share one global component. Thus, for international asset pricing, a reasonable model should incorporate one global return factor that contains a downside jump component and generates stochastic volatility.

ENDNOTES

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[1]See e.g., Merton [1976]; Bates [1991]; Madan and Milne [1991]; Heston [1993b]; Barndorff-Nielsen [1998]; Eberlein, Keller, and Prause [1998]; Madan, Carr, and Chang [1998]; Carr, Geman, Madan, and Yor [2002].

[2]See e.g., Hull and White [1987]; Heston [1993a]; Bates [1996; 2000]; Bakshi, Cao, and Chen [1997]; Duffie, Pan, and Singleton [2000]; Anderson, Benzoni, and Lund [2002]; Pan [2002]; Eraker [2004]; Eraker, Johannes, and Polson [2003]; Huang and Wu (2004).

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