Predictability of Interest Rates and Interest-Rate Portfolios

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The evolution of interest-rate forecasting

- **History**: Expectation hypothesis regressions (since 1970s):
  - The current term structure contains useful information about future interest-rate movement.

- **Recent**: Multifactor dynamic term structure models (DTSM)
  - Affine (Duffie, Kan, 96; Duffie, Pan, Singleton, 2000)
  - Quadratic (Leippold, Wu, 2002; Ahn, Dittmar, Gallant, 2002)

- **Now**: Use DTSM to explain expectation hypothesis regression results.

- **Question**:
  - *Why don’t we directly use DTSM to forecast interest rate movements?*
Let’s try

We estimate several three-factor affine DTSM models using 12 interest-rate series.

- The forecasting performances of these models are no better than the random walk hypothesis!

- This result is not particularly dependent on model design.

- The models fit the term structure well on a given day.

- All three factors are highly persistent and hence difficult to forecast.

- The pricing errors are much more transient (predictable) than the factors or the raw interest rates.
What do we do now?

• We use the DTSM not as a forecasting vehicle, but as a *decomposition* tool.

\[ y_t^\tau = f(X_t, \tau) + e_t^\tau \]

• What the DTSM captures \((f(X_t, \tau))\) is the persistent component, which is difficult to forecast.
• What the model misses (the pricing error \(e_t\)) is the more transient and hence more predictable component.

• We propose to form interest-rate portfolios that
  • neutralize their first-order dependence on the persistent factors.
  • only vary with the transient residual movements.

• Result: The portfolios are strongly predictable, even though the individual interest-rate series are not.
  \[ \Rightarrow \text{What is left out from the factors can also be economically significant.} \]
Three-factor affine DTSMs

- Affine specifications:
  - Risk-neutral factor dynamics:
    \[ dX_t = \kappa^* (\theta^* - X_t) \, dt + \sqrt{S_t} \, dW_t^*, \quad [S_t]_{ii} = \alpha_i + \beta_i^T X_t. \]
  - Short rate function: \( r(X_t) = a_r + b_r^T X_t \)

- Bond pricing: Zero-coupon bond prices:
  \[ P(X_t, \tau) = \exp \left( -a(\tau) - b(\tau)^T X_t \right). \]

- Affine forecasting dynamics:
  \[ \gamma(X_t) = \sqrt{S_t} \lambda_1 + \sqrt{S_t^-} \lambda_2 X_t. \]
  - Does not matter for bond pricing.
  - Specification is up to identification.

- Dai, Singleton (2000): \( A_m(3) \) classification with \( m = 0, 1, 2, 3 \).

- We estimate all four generic specifications.
Data

- Data: 12 interest rate series on U.S. dollar: 1, 2, 3, 6, and 12-month LIBOR; 2, 3, 5, 7, 10, 15, and 30-year swap rates.


- Quoting conventions: actual/360 for LIBOR; 30/360 with semi-annual payment for swaps.

\[
\text{LIBOR}(X_t, \tau) = \frac{100}{\tau} \left( \frac{1}{P(X_t, \tau)} - 1 \right), \quad \text{SWAP}(X_t, \tau) = 200 \times \frac{1 - P(X_t, \tau)}{\sum_{i=1}^{2\tau} P(X_t, i/2)}.
\]

- Average weekly autocorrelation (\(\phi\)) is 0.991: Half-life = \(\ln \phi / 2 \) / \(\ln \phi \approx 78\) weeks (1.5 years)

*Interest rates are highly persistent; forecasting is difficult.*
Estimation: Maximum likelihood with UKF

- State propagation (discretization of the forecasting dynamics):

\[ X_{t+1} = A + \Phi X_t + \sqrt{Q_t} \varepsilon_{t+1}. \]

- Measurement equation:

\[ y_t = \begin{bmatrix} \text{LIBOR}(X_t, i) \\ \text{SWAP}(X_t, j) \end{bmatrix} + e_t, \quad i = 1, 2, 3, 6, 12 \text{ months} \]
\[ j = 2, 3, 5, 7, 10, 15, 30 \text{ years}. \]

- Unscented Kalman Filter (UKF) generates conditional forecasts of the mean and covariance of the state vector and observations.

- Likelihood is built on the forecasting errors:

\[ l_{t+1}(\Theta) = -\frac{1}{2} \log |\overline{A}_{t+1}| - \frac{1}{2} \left( (y_{t+1} - \overline{y}_{t+1})^\top (\overline{A}_{t+1})^{-1} (y_{t+1} - \overline{y}_{t+1}) \right). \]
### Estimated factor dynamics: $A_0(3)$

\[
\begin{align*}
\mathbb{P} : \, dX_t &= -\kappa X_t \, dt + dW \\
\mathbb{P}^* : \, dX_t &= (-b \gamma - \kappa^* X_t) \, dt + dW^*
\end{align*}
\]

<table>
<thead>
<tr>
<th>Forecasting dynamics $\kappa$</th>
<th>Risk-neutral dynamics $\kappa^*$</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
0.002 & 0 & 0 \\
0.02 & \text{--} & \text{--} \\
-0.186 & 0.480 & 0 \\
0.42 & 1.19 & \text{--} \\
-0.749 & -2.628 & 0.586 \\
1.80 & 3.40 & 2.55
\end{bmatrix}
\] |
| \[
\begin{bmatrix}
0.014 & 0 & 0 \\
11.6 & \text{--} & \text{--} \\
0.068 & 0.707 & 0 \\
1.92 & 20.0 & \text{--} \\
-2.418 & -3.544 & 1.110 \\
10.7 & 12.0 & 20.0
\end{bmatrix}
\] |

- The $t$-values are smaller for $\kappa$ than for $\kappa^*$.
- The largest eigenvalue of $\kappa$ is 0.586
  \[\Rightarrow\] Weekly autocorrelation 0.989, half life 62 weeks.
The errors are small. The 3 factors explain over 99%.

The average persistence of the pricing errors (0.69, half life 3 weeks) is much smaller than that of the interest rates (0.991, 1.5 years).
4-week ahead forecasting

Three strategies:
(1) random walk (RW); (2) AR(1) regression (OLS); (3) DTSM.

Explained Variation = 100 × [1 − var(Err)/var(ΔR)]

<table>
<thead>
<tr>
<th>Maturity</th>
<th>RW</th>
<th>OLS</th>
<th>DTSM</th>
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<tbody>
<tr>
<td>6 m</td>
<td>0.00</td>
<td>0.53</td>
<td>-31.71</td>
</tr>
<tr>
<td>2 y</td>
<td>0.00</td>
<td>0.02</td>
<td>-7.87</td>
</tr>
<tr>
<td>3 y</td>
<td>0.00</td>
<td>0.13</td>
<td>-0.88</td>
</tr>
<tr>
<td>5 y</td>
<td>0.00</td>
<td>0.44</td>
<td>0.81</td>
</tr>
<tr>
<td>10 y</td>
<td>0.00</td>
<td>1.07</td>
<td>-3.87</td>
</tr>
<tr>
<td>30 y</td>
<td>0.00</td>
<td>1.53</td>
<td>-36.64</td>
</tr>
</tbody>
</table>

- OLS is not that much better than RW, due to high persistence (max 1.5%).

- **DTSM is the worst!** DTSM can be used to fit the term structure (99%), but not forecast interest rates.
Use DTSM as a decomposition tool

- We linearly decompose the LIBOR/swap rates \((y)\) as

\[ y^i_t \approx H^\top_i X_t + e^i_t, \quad H_i = \left. \frac{\partial y^i_t}{\partial X_t} \right|_{X_t=0} \]

- We form a portfolio \((m = [m_1, m_2, m_3, m_4]^\top)\) of 4 LIBOR/swap rates so that

\[ p_t = \sum_{i=1}^{4} m_i y^i_t \approx \sum_{i=1}^{4} m_i H^\top_i X_t + \sum_{i=1}^{4} m_i e^i_t = \sum_{i=1}^{4} m_i e^i_t. \]

- We choose the portfolio weights to hedge away its dependence on the three factors: \(Hm = 0\). 

\[\]
Example: A 4-rate portfolio (2-5-10-30)

Portfolio weights: \( m = [0.0277, -0.4276, 1.0000, -0.6388] \).
Long 10-yr swap, use 2, 5, and 30-yr swaps to hedge.

Hedged 10-yr swap
\( \phi \) (half life): 0.816 (one month) vs. 0.987 (one year).
\[ \Delta R_{t+1} = -0.0849 - 0.2754 R_t + e_{t+1}, \quad R^2 = 0.14, \]
\[ (0.0096) \quad (0.0306) \]
\( R^2 = 1.07\% \) for the unhedged 10-year swap rate.
• 12 rates can generate 495 4-instrument portfolios.
• Robust: Improved predictability for all portfolios (against unhedged single rates)
No guaranteed success for spread (2-rate) and butterfly (3-rate) portfolios.

Predictability improves dramatically after the 3rd factor.
A simple buy and hold investment strategy on interest-rate portfolios

- Form 4-instrument swap portfolios ($m$). Regard each swap contract as a par bond.

- Long the portfolio (receive the fixed coupon payments) if the portfolio swap rate is higher than the model value. Short otherwise:

\[ w_t = c \left[ m^\top (y_t - SWAP(X_t)) \right] \]

- Hold each investment for 4 weeks and liquidate.

- Remark: The (over-simplified) strategy is for illustration only; it is not an optimized strategy.
Profitability of investing in four-instrument swap portfolios
The sources of the profitability

- Risk and return characteristics
  - The investment returns are *not* related to traditional stock and bond market factors (the usual suspects): $R_m$, $HML$, $SMB$, $UMD$, Credit spread, interest rate volatility,...
  - But are positively related to some swap market liquidity measures.

- Interpretation
  - The first 3 factors relate to systematic economic movements: Inflation rate, output gap, monetary policy, ...
  - What is left is mainly due to short-term liquidity shocks.
  - By providing liquidity to the market, one can earn economically significant returns.
Robustness check

- **Other models** ($A_m(3)$ with $m = 1, 2, 3$):
  - Better model choice generates higher predictability for the portfolios.
  - Portfolios from all models are more predictable than single interest rates.

- **Out-of-sample**: Remains strong.

- **Other currencies**: Similar conclusions.

- **Other markets**: ...
After thoughts: The role of no-arbitrage models

- No-arbitrage models provide relative valuation across assets, and hence can best used for cross-sectional comparison.
  - If the price of a stock is $100, assuming no interest rate, dividend, or other carrying costs/benefits, no-arbitrage theory dictates that the forward price of the stock is $100.
  - Is $100 the fair forward price? Will the price go up or down? How big is the risk premium on the stock? Is it time varying?
- No-arbitrage theory does not tell us how to predict the factors, but it does tell us how each instrument is related to the factor risk (factor loading).
- ⇒ It is the most useful for hedging:
  - Hedge away the risk, exploit the opportunity.