A Simple Robust Link Between American Puts and Credit Insurance

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Background: Linkages between equity and debt markets

- Merton (74): “Structural Models”
  Equity is a call on the firm.
  ⇒ American puts on stock are linked to credit [in conjunction with leverage and firm value dynamics] in complicated ways. (Black, Cox (76), ... Hull, Nelken, White (04), ... Cremers, Driessen, Maenhout, 08)

- Merton (76): “Reduced-Form Models”
  When a company defaults, its stock price drops to zero.
  ⇒ American puts are linked to credit through the joint specification of stock price, return volatility, default arrival dynamics. (Carr, Wu (05), Carr, Linetsky (06))

- Empirical evidence: Credit spreads co-vary with stock price, realized return volatility, option implied volatilities, and implied volatility skew (Collin-Dufresne, Goldstein, Martin (01), Cremers, Driessen, Maenhout, Weinbaum (04), Zhang, Zhou, Zhu (06), Cao, Yu, Zhong (07), Berndt, Ostrovnaya (07)...)

Evidence: Linkages between equity and debt markets

- GM: Default risk and stock price
  - Negative stock price
  - CDS spread

- GM: Default risk and long-term implied volatility
  - ATMV
  - CDS spread

- GM: Default risk and long-term implied volatility skew
  - Negative skew
  - CDS spread

Carr & Wu American Puts & Credit Insurance
A new, simple, robust linkage between out-of-the-money American puts on the stock and credit insurance

What’s new?
- We use an American put spread to replicate a standardized credit insurance contract that pays one dollar whenever default occurs.
- The linkage is based on cash flow matching, not stat. co-movements.
- The linkage is direct: It does not operate through firm value, leverage, or assumed co-movements between stock volatility and default arrival.

How simple?
- The American put spread has the same payoff as a pure credit contract.
- No Fourier transform, simulation, or PIDEs, not even HW trees.
- No model parameter estimation/calibration.

How robust?
- The linkage remains valid, regardless of specifications on default arrival, interest rates, and pre- and post-default price dynamics (as long as the stock price stays out of a default corridor).
The Default Corridor

- We assume:
  - The stock price $S$ stays above level $B > 0$ before default.
  - The stock price drops below $A < B$ at default and stays below thereafter.
- $[A, B]$ defines a default corridor that the stock price can never be in.
- Mnemonic: $B$ is below $S$ before default; $A$ is above $S$ after default.

**What generates the default corridor?**

- **Strategic default:** Debt holders have incentives to induce default before the equity value vanishes ($B > 0$). (Anderson, Sundaresan (96), Mella-Barral, Perraudin (97), Fan, Sundaresan (00), Broadie, Chernov, Sundaresan (07), Carey, Gordy (07), Hackbarth, Hennessy, Leland (07).)

- The default procedure generates sudden drops in equity value ($B > A$): legal fees, liquidation costs, loss of continuation option on projects...

- Incomplete information: Announcement of default reveals that the company is worse than investors had expected. Stock price drops.
Suppose that we can trade in two out-of-the-money American puts on the same stock with the same expiry $T$, with strikes lying inside the company’s default corridor: $A \leq K_1 < K_2 \leq B$.

The scaled spread between the two American put options, $u(t, T) \equiv (p_t(K_2, T) - p_t(K_1, T))/(K_2 - K_1)$ is the cost of replicating a standardized default insurance contract that: pays one dollar at default if the company defaults prior to the option expiry $T$ and zero otherwise.

If no default occurs before $T$, $S_t > B$, neither put option will be exercised. The payoff at maturity is zero.

If default occurs at $\tau \leq T$, it is optimal to exercise both puts (as long as $r \geq q$). The payoff is $((K_2 - S_\tau) - (K_1 - S_\tau))/(K_2 - K_1) = 1$.

We label this contract as a unit recovery claim (URC).

Replicating a URC is simple: Just spread two American puts.

Important special case: Under zero equity recovery ($A = 0$), a URC is replicated with one put by setting $K_1 = 0$: $u(t, T) = p_t(K_2, T)/K_2$. 

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Under the Black-Merton-Scholes (BMS) model, the stock price follows a geometric Brownian motion (GBM).

**G1:** Merton (1976): Company can default via a Poisson process ($\lambda$). The stock price follows a GBM before default, and jumps to zero upon default.

- The BMS call pricing formula still holds, by replacing $r$ with $r + \lambda$.
- One can still dynamically replicate a European call using the stock and a defaultable zero-coupon bond with zero recovery upon default.
- The BMS implied volatility displays negative skew against strike.

**G2:** Rubinstein (1983): The stock price follows a displaced diffusion:

$$S_t = Be^{rt} + (S_0 - B)e^{rt+\sigma W_t-\frac{1}{2}\sigma^2 t}, \quad B \in (-\infty, S_0), \quad S_t \in (Be^{rt}, \infty).$$

- Stock price never falls below $B$.
- The BMS implied volatility displays positive skew when $B > 0$.

Each generalization captures a certain aspect of the stock price dynamics.
A tale of two generalizations

The Black-scholes implied volatility skews from the two generalizations:

- Both models can generate skews, neither model can generate a smile.
  - Merton: $IV(d_2) \approx \sigma + \frac{N(d_2)}{N'(d_2)} \sqrt{T} \lambda$, $IV'(d_2)|_{d_2=0} \approx \sqrt{T}\lambda$, at small $\lambda$.
  - Rubinstein: $IV(K) \approx \frac{\int_S^K \frac{1}{x} \, dx}{\int_S^K \frac{1}{(x-B)\sigma} \, dx} = \frac{\ln(K/S)}{\ln(K-B)/(S-B)\sigma}$, at small $T$.

Model parameters: $r = 5\%$, $q = 0$, $\sigma = 40\%$, $S_0 = 100$, $t = 1$. $\lambda = 1\%$ for Merton and $B = 60$ for Rubinstein.
Implied volatility smiles and skews in reality

Typical Implied Volatility Smiles/Skews on GM: 03–Jan–2006

\[ \frac{\ln(K/S)}{\sigma \sqrt{\tau}} \]

Implied Volatility

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Defaultable Displaced Diffusion

Idea: Combine Merton’s jump-to-default with Rubinstein’s positive displacement.

- **DDD Description:**
  - Company can default via a Poisson process ($\lambda$).
  - Prior to default, the stock price follows Rubinstein’s displaced diffusion with positive displacement ($B > 0$).
  - Upon default at $t \leq T$, the stock price drops to $R(t) = Ae^{-r(T-t)}$.

- **DDD Representation:**

  $$S_t = e^{rt} \{ R(0) + J_t [B - R(0) + (S_0 - B)G_t] \}$$

  with $G_t = e^{\sigma W_t - \frac{1}{2} \sigma^2 t}$ a GMBgl and $J_t = 1(N_t = 0)e^{\lambda t}$ a compensated Poisson process.

  - Prior to default, $J_t = e^{\lambda t}$, $G_t > 0$, the stock price $S_t$ exceeds $B(t) \equiv R(0)e^{rt} + [B - R(0)]e^{(r+\lambda)t} \geq B$.
  - After default, $S_t = Ae^{-r(T-t)} \leq A$.
  - The region $[A, B]$ forms the default corridor, which the stock price cannot enter prior to option expiry.

- European option pricing remains analytical.
Combine two skews to generate a smile

Merton (1976) Jump to Default

Rubinstein (1983) Positively Displaced Diffusion

Defaultable Displaced Diffusion

\[ r = 5\%, q = 0, \sigma = 40\%, S_0 = 100, \lambda = 1\%, B = 60, A = 10, t = 1. \]
For American put options struck within the default corridor, \( K \in [A, B] \), it is optimal to exercise whenever default occurs.

\[
P_0(K, T) = \mathbb{E}^Q \left[ e^{-r\tau} [K - R(\tau)] \right] \\
= \int_0^T \lambda e^{-\lambda t} e^{-rt} [K - Ae^{-r(T-t)}] dt \\
= \lambda \left[ K \frac{1 - e^{-(r+\lambda)T}}{r+\lambda} - Ae^{-rT} \frac{1 - e^{-\lambda T}}{\lambda} \right].
\]

1. The American put value depends \textit{purely} on credit risk (\( \lambda \)) and interest rates (\( r \)).
2. Once controlled for credit risk, the American put value does not depend on the stock price level (zero delta), nor does it depend on the diffusion volatility \( \sigma \) (zero vega).
3. American puts within the default corridor are a \textit{linear} function of the strike price.
Linking American puts to unit recovery claims (URC)

- **URC**: pays one dollar at $\tau$ if default occurs at $\tau \leq T$, zero otherwise.

- Under constant $\lambda$ and $r$, its time-$t$ value is,
  \[ U(t, T) = \mathbb{E}^Q_t [e^{-r\tau} 1(\tau < T)] = \lambda \frac{1-e^{-(r+\lambda)(T-t)}}{r+\lambda}. \]

- Compare: The risk-neutral default probability — the forward price of paying one dollar at $T$ if $\tau < T$: \[ D(t, T) = \mathbb{E}^Q_t [1(\tau < T)]. \]

  \[ U(t, T) \leq D(t, T) \leq e^{rT} U(t, T). \]

- **American put spread**: \[ U^o(t, T) \equiv \frac{P_t(K_2, T) - P_t(K_1, T)}{K_2 - K_1}, \text{ with } A \leq K_1 < K_2 \leq B \]

  - If no default occurs, $S_t > B$, neither put option will be exercised. The payoff at maturity is zero. The spread is worth zero at expiry.

  - If default occurs at $\tau \leq T$, it is optimal to exercise both puts. The payoff is \( ((K_2 - S_\tau) - (K_1 - S_\tau))/(K_2 - K_1) = 1. \)

- *The cash flow from the put spread is the same as that from the URC.*
Robustness of the linkage: DDD generalizations

- As long as the default corridor exists, the linkage holds true,
- regardless of specifications on
  - Random interest rates.
  - Random stock recovery.
  - Stochastic default arrival.
  - Stochastic volatility and jumps.
- The stochastic differential equation that governs the stock price process prior to default can be generalized to,

\[
\begin{align*}
    dS_t &= (r_t + \lambda_t)(S_t - R_t)dt + r_t R_t dt + [S_t^- - R_t^- - J_t^- e^{\int_0^t r_s ds} (B - R_t)]dM_t^G, \\
t &\in [0, \tau \wedge T].
\end{align*}
\]
Linking CDS to URC

The most actively traded credit derivative is the credit default swap (CDS) contract:

A1: Bond recovery rate ($R^b$) is known. $\Rightarrow$ The value of the protection leg of the CDS is linked to the value of a URC by $V^\text{prot}(t, T) = (1 - R^b)U(t, T)$.

A2: Deterministic interest rates. $\Rightarrow$ The value of the premium leg of the CDS can also be linked to the whole term structure of URCs,

$$A(t, T) = k(t, T)E_t^Q \int_t^T e^{-\int_t^s (r(u) + \lambda(u))du} ds$$
$$= k(t, T) \int_t^T \left[ e^{-\int_t^s r(u)du} - U(t, s) + \int_t^s r(u)e^{-\int_u^s r(v)dv} U(t, u)du \right] ds.$$

A1+A2: We can strip URC term structure from a term structure of CDS, without assuming how default occurs.

A3: Constant bond recovery, deterministic interest rates and default rates. $\Rightarrow$ we can infer the value of a URC from the CDS spread ($k(t, T)$):

$$\lambda^c(t, T) = k(t, T)/(1 - R^b), \quad U^c(t, T) = \lambda^c(t, T) \frac{1 - e^{-(r(t, T) + \lambda^c(t, T))(T-t)}}{r(t, T) + \lambda^c(t, T)}.$$
Empirical implications

- **Known**: Credit spreads co-move with implied volatility and volatility skew. This evidence is consistent with our model (and many other models).

- What are the unique empirical implications of our theoretical result?
  - **American put** spreads struck within the default corridor replicate a pure credit contract (URC).
  - We can also infer the value of the URC from other traded credit contracts, such as credit default swaps (CDS) and recovery swaps.
  - The URC values calculated from the American puts and the CDS should be similar in magnitude, and move together.
  - When they differ significantly, we should be able to design arbitrage trading strategies.
    - Remember: the American put spreads replicate the cash flow of the standardized insurance contract.
  - Their deviation should not depend on the other usual suspects such as stock price, stock return volatility (RV, IV), which are determinants of American puts in most other models.
Collect data on American stock put options from OptionMetrics and CDS spreads from various sources, on a list of companies.

- Sample period: January 2005 to June 2007
- Company selection criteria:
  - OptionMetrics have non-zero bid quotes on one or more put options struck more than one standard deviation below the current spot price and with maturities over 180 days.
  - Reliable CDS quotes are available at 1-, 2-, 3-year maturities.
  - The average CDS spreads at 1-year maturity is over 30bps.
## List of selected companies

<table>
<thead>
<tr>
<th>Equity Ticker</th>
<th>Cusip Number</th>
<th>Company Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR</td>
<td>00176510</td>
<td>American Airline</td>
</tr>
<tr>
<td>CTB</td>
<td>21683110</td>
<td>Cooper Tire &amp; Ribber</td>
</tr>
<tr>
<td>DDS</td>
<td>25406710</td>
<td>Dillard’s Inc.</td>
</tr>
<tr>
<td>EK</td>
<td>27746110</td>
<td>Eastman Kodak Co</td>
</tr>
<tr>
<td>F</td>
<td>34537086</td>
<td>Ford Motor Co</td>
</tr>
<tr>
<td>GM</td>
<td>37044210</td>
<td>General Motors Corp</td>
</tr>
<tr>
<td>GT</td>
<td>38255010</td>
<td>Goodyear Tire &amp; Rubber Co</td>
</tr>
<tr>
<td>KBH</td>
<td>48666K10</td>
<td>KB Home</td>
</tr>
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</table>
Construct URC from American stock put options

**In theory:** \( U^o(t, T) = \frac{P_t(K_2, T) - P_t(K_1, T)}{K_2 - K_1}, \quad A \leq K_1 < K_2 < B. \)**

**In practice:**
- **How to identify the default corridor \([A, B]\)?**
  - If we have put quotes for a continuum of strikes, we can identify the default corridor based on the option price behaviors across strikes.
    - The put option prices are a convex function of the strike outside the corridor, but are a linear function of the strike within the corridor.
  - With only discrete strikes available, we consider 3 choices:
    - C1 Set \( K_1 = 0 \) and \( K_2 \) to the lowest strike with non-zero bid.
    - C2 Set \((K_1, K_2)\) to the two lowest strikes with non-zero bids.
    - C3 Estimate the convexity of the put mid quotes across adjacent strikes, choose the strike at which the convexity is the lowest as \( K_2 < S_t \), and the adjacent lower strike as \( K_1 \).
  - These different choices generate largely similar results. We report results using C1 based on its simplicity and the lower transaction cost of using one instead of two options.
    - One can also try to optimize the choice via ex post analysis — C1 represents a simple, conservative choice.

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Take CDS quotes available at fixed maturities (1, 2, 3 years). Linear interpolation to obtain CDS spread at the longest option maturity.

From the interpolated CDS spread $k(t, T)$, infer the default arrival rate based on constant interest rate and default rate assumptions and an assumed 40% bond recovery: $\lambda^c(t, T) = k(t, T)/(1 - R^b)$.

One can use default recovery swap to fix the recovery.

Compute the unit recovery claim value according to,

$$U^c(t, T) = \lambda^c(t, T) \frac{1 - e^{-(r(t, T) + \lambda^c(t, T))(T-t)}}{r(t, T) + \lambda^c(t, T)}.$$
Summary statistics of unit recovery claims

<table>
<thead>
<tr>
<th>Ticker</th>
<th>$U^c$</th>
<th></th>
<th></th>
<th>$U^o$</th>
<th></th>
<th></th>
<th>Cross-market Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std</td>
<td>Auto</td>
<td>Mean</td>
<td>Std</td>
<td>Auto</td>
<td></td>
</tr>
<tr>
<td>AMR</td>
<td>0.265</td>
<td>0.167</td>
<td>0.995</td>
<td>0.116</td>
<td>0.073</td>
<td>0.982</td>
<td>0.933</td>
</tr>
<tr>
<td>CTB</td>
<td>0.052</td>
<td>0.042</td>
<td>0.993</td>
<td>0.087</td>
<td>0.039</td>
<td>0.964</td>
<td>0.369</td>
</tr>
<tr>
<td>DDS</td>
<td>0.032</td>
<td>0.018</td>
<td>0.990</td>
<td>0.063</td>
<td>0.026</td>
<td>0.981</td>
<td>0.709</td>
</tr>
<tr>
<td>EK</td>
<td>0.037</td>
<td>0.019</td>
<td>0.989</td>
<td>0.043</td>
<td>0.019</td>
<td>0.969</td>
<td>0.869</td>
</tr>
<tr>
<td>F</td>
<td>0.136</td>
<td>0.066</td>
<td>0.989</td>
<td>0.103</td>
<td>0.044</td>
<td>0.967</td>
<td>0.806</td>
</tr>
<tr>
<td>GM</td>
<td>0.165</td>
<td>0.106</td>
<td>0.994</td>
<td>0.085</td>
<td>0.060</td>
<td>0.968</td>
<td>0.941</td>
</tr>
<tr>
<td>GT</td>
<td>0.073</td>
<td>0.034</td>
<td>0.986</td>
<td>0.075</td>
<td>0.034</td>
<td>0.970</td>
<td>0.869</td>
</tr>
<tr>
<td>KBH</td>
<td>0.026</td>
<td>0.010</td>
<td>0.984</td>
<td>0.048</td>
<td>0.038</td>
<td>0.983</td>
<td>0.774</td>
</tr>
<tr>
<td>Average</td>
<td>0.098</td>
<td>0.058</td>
<td>0.990</td>
<td>0.077</td>
<td>0.042</td>
<td>0.973</td>
<td>0.784</td>
</tr>
</tbody>
</table>

- Similar mean magnitudes for the unit recovery values from the two markets.
- CDS spreads ($U^c$) are more persistent than American puts ($U^o$).
- High cross-market correlations.
Relating cross-market deviations to URC levels

\[ U_t^o - U_t^c = D_i - 0.5033 \left( \frac{U_t^o + U_t^c}{2} \right) + 0.0466 \ln K_t + e_t \]

\( D_i \) — company dummy. \( R^2 = 85.24\% \).

- The put-implied URC values \( (U^o) \) are higher for low-URC firms.
  - Our strike choice (for non-zero bid) may over-estimate the URC value.
  - Choosing a higher strike increases this bias.

- The CDS-implied URC values are higher for high-URC firms.
  - If equity recovery \( R(\tau) > 0 \), \( P_t(K, T)/K \) pays \( (K - R(\tau))/K \) at default, less than the $1 payoff from a URC.
    \[ \Rightarrow U^o \text{ underestimates the URC value.} \]
  - The bond recovery assumption \( (R^b = 40\%) \) can bias \( U^c \).
    - If the actual recovery is lower, our assumption would over-estimate the default probability, and hence the URC value.
Contemporaneous regressions on the two URC series

\[ U_t^o = a + bU_t^\xi + \epsilon_t, \]

<table>
<thead>
<tr>
<th>Ticker</th>
<th>( a )</th>
<th>( b )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR</td>
<td>0.008</td>
<td>(1.42)</td>
<td>0.406</td>
</tr>
<tr>
<td>CTB</td>
<td>0.069</td>
<td>(4.53)</td>
<td>0.343</td>
</tr>
<tr>
<td>DDS</td>
<td>0.029</td>
<td>(3.55)</td>
<td>1.049</td>
</tr>
<tr>
<td>EK</td>
<td>0.012</td>
<td>(3.77)</td>
<td>0.847</td>
</tr>
<tr>
<td>F</td>
<td>0.030</td>
<td>(2.44)</td>
<td>0.531</td>
</tr>
<tr>
<td>GM</td>
<td>-0.004</td>
<td>(-0.64)</td>
<td>0.539</td>
</tr>
<tr>
<td>GT</td>
<td>0.010</td>
<td>(1.88)</td>
<td>0.890</td>
</tr>
<tr>
<td>KBH</td>
<td>-0.025</td>
<td>(-2.03)</td>
<td>2.858</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>0.016</td>
<td>(1.87)</td>
<td>0.933</td>
</tr>
</tbody>
</table>

- In parentheses are \( t \)-statistics against the null: \( a = 0 \) and \( b = 1 \).
- Positive slope (co-movements), high \( R^2 \).
- \( U_t \) captures the unexplained component of the American put \( U_t^o = P_t/K \).
  - Non-CDS driven variation in American puts.
Explain non-CDS driven daily variation in American puts

Stock price: $\Delta U_t = a + b\Delta S_t + e_t$,

<table>
<thead>
<tr>
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<th>$a$</th>
<th>$b$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR</td>
<td>0.000</td>
<td>( 0.33 )</td>
<td>-0.003</td>
</tr>
<tr>
<td>CTB</td>
<td>-0.000</td>
<td>( -0.44 )</td>
<td>-0.008</td>
</tr>
<tr>
<td>DDS</td>
<td>0.000</td>
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<td>-0.004</td>
</tr>
<tr>
<td>EK</td>
<td>-0.000</td>
<td>( -0.25 )</td>
<td>-0.003</td>
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<tr>
<td>F</td>
<td>-0.000</td>
<td>( -0.22 )</td>
<td>-0.008</td>
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<tr>
<td>GM</td>
<td>-0.000</td>
<td>( -0.14 )</td>
<td>-0.003</td>
</tr>
<tr>
<td>GT</td>
<td>0.000</td>
<td>( 0.88 )</td>
<td>-0.003</td>
</tr>
<tr>
<td>KBH</td>
<td>-0.000</td>
<td>( -0.14 )</td>
<td>-0.000</td>
</tr>
</tbody>
</table>

There is a negative delta component in the American puts:

- Violation of the default corridor.
- $S_t$ contains credit risk information absent from the current CDS quotes.
Realized volatility: $\Delta U_t = a + b\Delta RV_t + e_t$,

<table>
<thead>
<tr>
<th>Ticker</th>
<th>$a$</th>
<th>$b$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR</td>
<td>0.000</td>
<td>(0.13)</td>
<td>0.181</td>
</tr>
<tr>
<td>CTB</td>
<td>-0.000</td>
<td>(-0.71)</td>
<td>0.129</td>
</tr>
<tr>
<td>DDS</td>
<td>-0.000</td>
<td>(-0.16)</td>
<td>-0.249</td>
</tr>
<tr>
<td>EK</td>
<td>0.000</td>
<td>(0.17)</td>
<td>0.105</td>
</tr>
<tr>
<td>F</td>
<td>0.000</td>
<td>(0.30)</td>
<td>-0.139</td>
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<tr>
<td>GM</td>
<td>0.000</td>
<td>(0.04)</td>
<td>0.058</td>
</tr>
<tr>
<td>GT</td>
<td>0.000</td>
<td>(0.15)</td>
<td>-0.049</td>
</tr>
<tr>
<td>KBH</td>
<td>-0.000</td>
<td>(-0.09)</td>
<td>-0.015</td>
</tr>
</tbody>
</table>

Nothing much here. No vega component in the American puts consistent with our default corridor theory.
Explain non-CDS driven daily variation in American puts

At-the-money implied volatility: $\Delta \mathcal{U}_t = a + b \Delta ATMV_t + e_t$,

<table>
<thead>
<tr>
<th>Ticker</th>
<th>$a$</th>
<th>$b$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR</td>
<td>0.000</td>
<td>0.213</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(0.45)</td>
<td>(3.23)</td>
<td></td>
</tr>
<tr>
<td>CTB</td>
<td>-0.000</td>
<td>0.362</td>
<td>0.137</td>
</tr>
<tr>
<td></td>
<td>(-0.77)</td>
<td>(2.81)</td>
<td></td>
</tr>
<tr>
<td>DDS</td>
<td>0.000</td>
<td>0.384</td>
<td>0.188</td>
</tr>
<tr>
<td></td>
<td>(0.84)</td>
<td>(4.61)</td>
<td></td>
</tr>
<tr>
<td>EK</td>
<td>0.000</td>
<td>0.227</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(3.70)</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>-0.000</td>
<td>0.222</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>(-0.05)</td>
<td>(3.53)</td>
<td></td>
</tr>
<tr>
<td>GM</td>
<td>-0.000</td>
<td>0.241</td>
<td>0.066</td>
</tr>
<tr>
<td></td>
<td>(-0.13)</td>
<td>(2.78)</td>
<td></td>
</tr>
<tr>
<td>GT</td>
<td>0.000</td>
<td>0.409</td>
<td>0.203</td>
</tr>
<tr>
<td></td>
<td>(0.78)</td>
<td>(5.25)</td>
<td></td>
</tr>
<tr>
<td>KBH</td>
<td>-0.000</td>
<td>0.262</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(-0.05)</td>
<td>(2.09)</td>
<td></td>
</tr>
</tbody>
</table>

Slopes are all positive and significant. There is a common options market movement that is not priced in the current CDS quotes. This component is either

- a market risk factor (violation of the corridor assumption), or
- a credit risk component not priced in the CDS quotes.
Predicting implications: Hypotheses

- **H1**: $\mathcal{U}_t$ is purely due to transient noise in the options market. 
  $\Rightarrow$ A positive $\mathcal{U}_t$ predicts future **decline** in the American put value.

- **H2**: $\mathcal{U}_t$ reflects credit risk information from the options market that has not shown up yet in the current CDS quotes. 
  $\Rightarrow$ A positive $\mathcal{U}_t$ predicts future **increase** in the CDS spread.

Engle-Granger error-correction regressions:

$$
\Delta U^o_{t+\Delta t} = \alpha^o + \beta^o \mathcal{U}_t + e_{t+\Delta t}, \quad \Delta U^c_{t+\Delta t} = \alpha^c + \beta^c \mathcal{U}_t + e_{t+\Delta t}
$$

with $\mathcal{U}_t = U^o_t - a - b U^c_t$.

- Under H1, $\beta^o < 0$
- Under H2, $\beta^c > 0$. 
Predicting options and CDS movements over 1-day horizon

\[ \Delta U^o_{t+\Delta t} = \alpha^o + \beta^o U_t + e_{t+\Delta t}, \quad \Delta U^c_{t+\Delta t} = \alpha^c + \beta^c U_t + e_{t+\Delta t}, \]

\( \Delta t = 1 \) day.

<table>
<thead>
<tr>
<th>Ticker</th>
<th>( \beta^o )</th>
<th>( R^2 )</th>
<th>( \beta^c )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR</td>
<td>-0.070 ( -2.77 )</td>
<td>0.033</td>
<td>0.037 ( 1.94 )</td>
<td>0.012</td>
</tr>
<tr>
<td>CTB</td>
<td>-0.015 ( -1.44 )</td>
<td>0.005</td>
<td>-0.004 ( -0.42 )</td>
<td>0.001</td>
</tr>
<tr>
<td>DDS</td>
<td>-0.021 ( -2.64 )</td>
<td>0.012</td>
<td>-0.001 ( -0.38 )</td>
<td>0.000</td>
</tr>
<tr>
<td>EK</td>
<td>-0.093 ( -4.30 )</td>
<td>0.052</td>
<td>0.014 ( 2.13 )</td>
<td>0.005</td>
</tr>
<tr>
<td>F</td>
<td>-0.079 ( -2.90 )</td>
<td>0.050</td>
<td>-0.028 ( -2.72 )</td>
<td>0.015</td>
</tr>
<tr>
<td>GM</td>
<td>-0.127 ( -3.47 )</td>
<td>0.054</td>
<td>0.051 ( 1.70 )</td>
<td>0.019</td>
</tr>
<tr>
<td>GT</td>
<td>-0.065 ( -3.65 )</td>
<td>0.033</td>
<td>0.003 ( 0.22 )</td>
<td>0.000</td>
</tr>
<tr>
<td>KBH</td>
<td>-0.007 ( -1.22 )</td>
<td>0.002</td>
<td>0.007 ( 2.06 )</td>
<td>0.013</td>
</tr>
<tr>
<td>Average</td>
<td>-0.060 ( -2.80 )</td>
<td>0.030</td>
<td>0.010 ( 0.57 )</td>
<td>0.008</td>
</tr>
</tbody>
</table>

- H1: \( \beta^o \) is significantly negative for 6 of the 8 companies. \( R^2 \) averages at 3%.
- H2: \( \beta^c \) is significantly positive for 4 of 8 companies. \( R^2 \) averages at 0.8%.
Predicting options and CDS movements over 7-day horizon

\[ \Delta U_{t+\Delta t}^o = \alpha^o + \beta^o U_t + e_{t+\Delta t}, \quad \Delta U_{t+\Delta t}^c = \alpha^c + \beta^c U_t + e_{t+\Delta t}, \]
\[ \Delta t = 7 \text{ days}. \]

<table>
<thead>
<tr>
<th>Ticker</th>
<th>$\beta^o$</th>
<th>$R^2$</th>
<th>$\beta^c$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR</td>
<td>-0.174</td>
<td>0.078</td>
<td>0.166</td>
<td>0.036</td>
</tr>
<tr>
<td>CTB</td>
<td>-0.056</td>
<td>0.015</td>
<td>-0.004</td>
<td>0.000</td>
</tr>
<tr>
<td>DDS</td>
<td>-0.087</td>
<td>0.051</td>
<td>-0.013</td>
<td>0.003</td>
</tr>
<tr>
<td>EK</td>
<td>-0.296</td>
<td>0.170</td>
<td>0.021</td>
<td>0.001</td>
</tr>
<tr>
<td>F</td>
<td>-0.204</td>
<td>0.113</td>
<td>-0.144</td>
<td>0.039</td>
</tr>
<tr>
<td>GM</td>
<td>-0.171</td>
<td>0.043</td>
<td>0.213</td>
<td>0.035</td>
</tr>
<tr>
<td>GT</td>
<td>-0.188</td>
<td>0.079</td>
<td>-0.015</td>
<td>0.001</td>
</tr>
<tr>
<td>KBH</td>
<td>-0.002</td>
<td>0.000</td>
<td>0.053</td>
<td>0.075</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>-0.147</strong></td>
<td><strong>0.069</strong></td>
<td><strong>0.035</strong></td>
<td><strong>0.024</strong></td>
</tr>
</tbody>
</table>

- **H1:** $\beta^o$ is significantly negative for 6 of the 8 companies. $R^2$ averages at 6.9%.
- **H2:** $\beta^c$ is significantly positive for 3 of 8 companies. $R^2$ averages at 2.4%.
Predicting options and CDS movements over 30-day horizon

\[ \Delta U_{t+\Delta t}^o = \alpha^o + \beta^o U_t + e_{t+\Delta t}, \quad \Delta U_{t+\Delta t}^c = \alpha^c + \beta^c U_t + e_{t+\Delta t}, \]
\[ \Delta t = 30 \text{ days}. \]

<table>
<thead>
<tr>
<th>Ticker</th>
<th>( \beta^o )</th>
<th>( R^2 )</th>
<th>( \beta^c )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR</td>
<td>-0.594 ( -3.69 )</td>
<td>0.240</td>
<td>-0.333 ( -1.30 )</td>
<td>0.031</td>
</tr>
<tr>
<td>CTB</td>
<td>-0.143 ( -1.06 )</td>
<td>0.028</td>
<td>-0.010 ( -0.09 )</td>
<td>0.000</td>
</tr>
<tr>
<td>DDS</td>
<td>-0.288 ( -3.13 )</td>
<td>0.161</td>
<td>-0.067 ( -0.84 )</td>
<td>0.017</td>
</tr>
<tr>
<td>EK</td>
<td>-0.963 ( -9.42 )</td>
<td>0.415</td>
<td>-0.177 ( -1.26 )</td>
<td>0.020</td>
</tr>
<tr>
<td>F</td>
<td>-0.583 ( -3.34 )</td>
<td>0.234</td>
<td>-0.487 ( -2.62 )</td>
<td>0.095</td>
</tr>
<tr>
<td>GM</td>
<td>-0.241 ( -1.08 )</td>
<td>0.023</td>
<td>0.724 ( 1.73 )</td>
<td>0.064</td>
</tr>
<tr>
<td>GT</td>
<td>-0.598 ( -2.29 )</td>
<td>0.186</td>
<td>-0.133 ( -0.65 )</td>
<td>0.011</td>
</tr>
<tr>
<td>KBH</td>
<td>0.046 ( 0.55 )</td>
<td>0.008</td>
<td>0.175 ( 3.77 )</td>
<td>0.171</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>-0.421 ( -2.93 )</td>
<td>0.162</td>
<td>-0.038 ( -0.16 )</td>
<td>0.051</td>
</tr>
</tbody>
</table>

- **H1:** \( \beta^o \) is significantly negative for 5 of the 8 companies. \( R^2 \) averages at 16.2%.
- **H2:** \( \beta^c \) is significantly positive for 2 of 8 companies. \( R^2 \) averages at 5.1%.
Predicting implications: Summary

Error-correction regressions:

\[ \Delta U^o_{t+\Delta t} = \alpha^o + \beta^o U_t + e_{t+\Delta t}, \quad \Delta U^c_{t+\Delta t} = \alpha^c + \beta^c U_t + e_{t+\Delta t} \]

- R-squares from the first regression are higher than that from the second regression.
- There are more significantly negative \( \beta^o \) estimates than significantly positive \( \beta^c \) estimates.
- Implications:
  - The credit risk information mainly flows from the CDS market to the American put options market.
  - For a few companies, the credit risk information also flows the other way around.
We identify a simple robust theoretical linkage between out-of-the-money American put options on a company’s stock and the company’s credit risk.

- **Simple:** A simple spread between two American put options replicates a *pure* credit insurance contract.

- **Robust:** The replication is valid as long as there exists a default corridor, irrespective of pre-default and post-default stock price dynamics, interest rate movements, or credit risk fluctuations.

The theoretical linkage has strong *empirical support*:

- The values of the credit contract inferred from American put options and CDS spreads have strong, positive correlations.
- Their deviations predict future movements on American put options.
An example: Linking American puts on GM stock ...

Date: June 23, 2008. Expiry: January 2010

At $K = 5$ (high open interest), put mid value is $1.22$

⇒ URC $= 1.22/5 = 24.2\%$.

Default probability is slightly higher due to rates.
Default probability at 9/20/2009 is 19.87%.
Default probability at 9/20/2010 is 40.16%.
⇒ Default probability at 1/15/2010 (Linear interpolation) ≈ 26.4%.
Red — CDS \((a + bU^c_t)\), Blue — American put ask, Green — American put bid.