A Simple Robust Link Between American Puts and Credit Insurance

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Joint work with Peter Carr

Ziff Brothers Investments, April 2nd, 2010
The role of credit risk in stock option pricing

- Black, Scholes, Merton (73): Stock price follows a GBM.
  - Stock price can never hit zero.
  - Credit risk does not play a role in stock option pricing.

- Merton (74): Equity is a call on firm value.
  - Stock options are linked to credit [in conjunction with leverage and firm value dynamics] in complicated ways.

- Merton (76): When a company defaults, its stock price drops to zero.
  - Stock options are linked to credit risk through the joint specification of stock price, return volatility, default arrival dynamics. (Carr, Wu (05), Carr, Linetsky (06))

- This paper: Deep-out-of-money (DOOM) American puts are purely credit contracts!
Evidence: Linkages between equity and debt markets

GM: Default risk and stock price

GM: Default risk and long-term implied volatility

GM: Default risk and long-term implied volatility skew

Liuren Wu (Baruch)
A new, simple, robust linkage

between DOOM American puts on the stock and credit insurance on the company.

- **What’s new?**
  - We use an American put spread to replicate a standardized credit insurance contract that pays one dollar whenever default occurs.
  - The linkage is based on cash flow matching, not stat. co-movements.
  - The linkage is direct: It does not operate through firm value, leverage, or assumed co-movements between stock volatility and default arrival.

- **How simple?**
  - The American put spread has the same payoff as a pure credit contract.
  - No Fourier transform, simulation, or PIDEs, not even HW trees.
  - No model parameter estimation/calibration.

- **How robust?**
  - The linkage remains valid, regardless of specifications on default arrival, interest rates, and pre- and post-default price dynamics (**as long as the stock price stays out of a default corridor**).
We assume:

- The stock price $S$ stays above level $B > 0$ before default.
- The stock price drops below $A < B$ at default and stays below thereafter.

$[A, B]$ defines a default corridor that the stock price can never be in.

Mnemonic: $B$ is below $S$ before default; $A$ is above $S$ after default.

What generates the default corridor?

- Strategic default: Debt holders have incentives to induce default before the equity value vanishes ($B > 0$). (Anderson, Sundaresan (96), Mella-Barral, Perraudin (97), Fan, Sundaresan (00), Broadie, Chernov, Sundaresan (07), Carey, Gordy (07), Hackbarth, Hennessy, Leland (07).)

- The default procedure generates sudden drops in equity value ($B > A$): legal fees, liquidation costs, loss of continuation option on projects...

- Incomplete information: Announcement of default reveals that the company is worse than investors had expected. Stock price drops.
Linking equity American puts to credit protection

- Suppose that we can trade in two DOOM American puts on the same stock with the same expiry $T$, with strikes lying inside the company's default corridor: $A \leq K_1 < K_2 \leq B$.

- The scaled spread between the two DOOM puts, $U^p(t, T) \equiv (P_t(K_2, T) - P_t(K_1, T))/(K_2 - K_1)$ is the cost of replicating a standardized default insurance contract that: pays one dollar at default if the company defaults prior to the option expiry $T$ and zero otherwise.

  - If no default occurs before $T$, $S_t > B$, neither put option will be exercised. The payoff at maturity is zero.
  - If default occurs at $\tau \leq T$, it is optimal to exercise both puts (as long as $r \geq q$). The payoff is $((K_2 - S_\tau) - (K_1 - S_\tau))/(K_2 - K_1) = 1$.

- We label this contract as a unit recovery claim (URC).

  - Replicating a URC is simple: Just spread two American puts.
  - Important special case: Under zero equity recovery ($A = 0$), a URC is replicated with one put by setting $K_1 = 0$: $U^p(t, T) = P_t(K_2, T)/K_2$. 

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Analytical pricing of American puts

- For American put options struck within the default corridor, \( K \in [A, B] \), it is optimal to exercise whenever default occurs.

- Assume constant interest rate and default arrival rate,

\[
P_0(K, T) = \mathbb{E}_Q^\mathbb{Q}[e^{-r\tau}[K - R(\tau)]]
\]

\[
= \int_0^T \lambda e^{-\lambda t} e^{-rt}[K - Ae^{-r(T-t)}]dt
\]

\[
= \lambda \left[ K \frac{1-e^{-(r+\lambda)T}}{r+\lambda} - Ae^{-rT} \frac{1-e^{-\lambda T}}{\lambda} \right].
\]

- The American put value depends **purely** on credit risk (\( \lambda \)) and interest rates (\( r \)).

- Once controlled for credit risk, the American put value does not depend on the stock price level (zero delta), nor does it depend on the diffusion volatility \( \sigma \) (zero vega).

- American puts within the default corridor are a **linear** function of the strike price.
Linking American puts to unit recovery claims (URC)

- **URC**: pays one dollar at $\tau$ if default occurs at $\tau \leq T$, zero otherwise.

- Under constant $\lambda$ and $r$, its time-$t$ value is,
  \[
  U(t, T) = \mathbb{E}_t^Q [e^{-r\tau} 1(\tau < T)] = \lambda \frac{1-e^{-(r+\lambda)(T-t)}}{r+\lambda}.
  \]

- Compare: The risk-neutral default probability — the forward price of paying one dollar at $T$ if $\tau < T$: $\mathbb{D}(t, T) = \mathbb{E}_t^Q [1(\tau < T)]$.
  \[
  U(t, T) \leq \mathbb{D}(t, T) \leq e^{rT} U(t, T).
  \]

- American put spread: $U^P(t, T) \equiv \frac{P_t(K_2, T) - P_t(K_1, T)}{K_2 - K_1}$, with $A \leq K_1 < K_2 \leq B$

  - If no default occurs, $S_t > B$, neither put option will be exercised. The payoff at maturity is zero. The spread is worth zero at expiry.

  - If default occurs at $\tau \leq T$, it is optimal to exercise both puts. The payoff is $((K_2 - S_\tau) - (K_1 - S_\tau))/(K_2 - K_1) = 1$.

- **The cash flow from the put spread is the same as that from the URC.**
Linking CDS to URC

The most actively traded credit derivative is the credit default swap (CDS):

**A1:** Bond recovery rate ($R^b$) is known. ⇒ The value of the protection leg of the CDS is linked to the value of a URC by $V^{prot}(t, T) = (1 - R^b)U(t, T)$.

**A2:** Deterministic interest rates. ⇒ The value of the premium leg of the CDS can also be linked to the whole term structure of URCs,

$$A(t, T) = k(t, T)\mathbb{E}_t^Q \int_t^T e^{-\int_t^s (r(u)+\lambda(u))du} ds$$

$$= k(t, T) \int_t^T \left[ e^{-\int_t^s r(u)du} - U(t, s) + \int_t^s r(u)e^{-\int_u^s r(v)dv} U(t, u)du \right] ds.$$

**A1+A2:** We can strip URC term structure from a term structure of CDS, without assuming how default occurs.

**A3:** Constant bond recovery, interest rates, and default rates. ⇒ we can infer the value of a URC from the CDS spread ($k(t, T)$):

$$\lambda^c(t, T) = \frac{k(t, T)}{1 - R^b}, \quad U^c(t, T) = \lambda^c(t, T)\frac{1 - e^{-\left(r(t, T) + \lambda^c(t, T)\right)(T-t)}}{r(t, T) + \lambda^c(t, T)}.$$
Empirical implications

- **American put** spreads struck within the default corridor replicate a pure credit contract (URC).

- We can also infer the value of the URC from other traded credit contracts, such as credit default swaps (CDS).

- The URC values calculated from the American puts and the CDS should be similar in magnitude, and move together.

- Current deviations between the two markets should predict future movements.
  - The direction and strength of the predictability reflect the information flow between the two markets.
Company selection and data construction

- Stock options are from OptionMetrics. CDS are from Bloomberg.
  - Sample period: January 2005 to August 2008.
  - Construct a weekly date list on every Wednesday.
- Select companies on each date that have viable DOOM puts:
  - Non-zero bid price and open interest — genuine market interest.
  - Time-to-maturity is greater than 360 days — reduce maturity mismatch with CDS.
  - Strike price is $5 or less; and the absolute delta is 15% or less — approximately in default corridor.
- Choose one put contract with the highest open interest, and compute the unit recovery value assuming zero equity recovery $(B = 0)$:
  \[ U^p(t, T) = \frac{P_t(K, T)}{K}. \]
- Find companies with viable 5-year CDS quotes.
  - 5-year is the most liquid and the most widely available — trade-off between maturity mismatch and data availability.
  - Compute unit recovery at the put option maturity assuming constant bond recovery $(R^b = 40\%)$, interest rate, and default arrival:
  \[ U^c(t, T) = \xi k_t \frac{1-e^{-(r(t, T) + \xi k_t)(T-t)}}{r(t, T) + \xi k_t}, \quad \xi = \frac{1}{1-R^b}. \]
Sample characteristics

- Further filtering:
  - $U^p(t, T) < 1$ — filter out data error.
  - $U^c(t, T) \geq 3\%$ — reduce selection bias.

- **Final selection**: 121 companies over 186 weeks.
  - The number of companies at each week ranges from 10 to 61, averages at 28.
  - 5,276 option contracts: Strikes are $5$ (3,635), $4$ (9), $2.5$ (1,622). Maturities are from 360 to 955 days.
Comparative analysis: One contract, two estimates

The cross-correlation between the two sets of estimates: 70%.

⇒ The two sets of estimates are similar in magnitude and move together, but also show large deviations.
Regressing one estimate against the other

<table>
<thead>
<tr>
<th>Relation</th>
<th>Intercept</th>
<th>Slope</th>
<th>$R^2$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U^p = \alpha_{pc} + \beta_{pc} U^c$</td>
<td>0.019 ( 0.003 )</td>
<td>0.727 ( 0.026 )</td>
</tr>
<tr>
<td></td>
<td>$U^c = \alpha_{cp} + \beta_{cp} U^p$</td>
<td>0.051 ( 0.001 )</td>
<td>0.681 ( 0.015 )</td>
</tr>
</tbody>
</table>

Panel A. Ordinary Least Square

<table>
<thead>
<tr>
<th>Relation</th>
<th>Intercept</th>
<th>Slope</th>
<th>$R^2$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U^p = \alpha_{pc} + \beta_{pc} U^c$</td>
<td>-0.022 ( 0.004 )</td>
<td>1.047 ( 0.036 )</td>
</tr>
<tr>
<td></td>
<td>$U^c = \alpha_{cp} + \beta_{cp} U^p$</td>
<td>0.021 ( 0.003 )</td>
<td>0.955 ( 0.033 )</td>
</tr>
</tbody>
</table>

Panel B. Total Least Square

Null: $\alpha = 0, \beta = 1.$

$\Rightarrow$ Both estimates contain errors.

$\Rightarrow$ On average, the two estimates move in the same direction and in similar magnitudes.
Cross-market deviation

- The cross-market deviation $D = U^p - U^c$ has an unconditional sample average of $-1.56\%$, a median of $-2\%$, and a standard deviation of $7.66\%$.
- The average cross-market deviation varies over time:

\[ U^p - U^c \]

⇒ Puts are historically cheaper, but are no longer so since mid 2007.
### What drives the cross-market deviation?

<table>
<thead>
<tr>
<th>( y )</th>
<th>( X )</th>
<th>( U^p - U^c )</th>
<th>( \ln U^p - \ln U^c )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \beta )</td>
<td>( t\text{-stats} )</td>
</tr>
<tr>
<td>( (U^p + U^c)/2 )</td>
<td>-0.082</td>
<td>(-6.9)</td>
<td>18.1</td>
</tr>
<tr>
<td>(</td>
<td>\text{Delta}</td>
<td>)</td>
<td>0.540</td>
</tr>
<tr>
<td>(\ln(K/S))</td>
<td>0.041</td>
<td>(20.6)</td>
<td>23.5</td>
</tr>
<tr>
<td>(IV_p)</td>
<td>0.018</td>
<td>(4.1)</td>
<td>17.7</td>
</tr>
<tr>
<td>(ATMV)</td>
<td>0.076</td>
<td>(13.6)</td>
<td>20.2</td>
</tr>
<tr>
<td>(RVT^{30})</td>
<td>0.038</td>
<td>(14.6)</td>
<td>21.0</td>
</tr>
<tr>
<td>(RVT^{360})</td>
<td>0.054</td>
<td>(10.8)</td>
<td>19.5</td>
</tr>
<tr>
<td>(TD/BE)</td>
<td>-0.000</td>
<td>(-3.3)</td>
<td>18.6</td>
</tr>
<tr>
<td>(TD/MC)</td>
<td>0.000</td>
<td>(4.5)</td>
<td>18.2</td>
</tr>
<tr>
<td>(DF)</td>
<td>0.080</td>
<td>(9.1)</td>
<td>18.8</td>
</tr>
<tr>
<td>(OI)</td>
<td>-0.000</td>
<td>(-9.0)</td>
<td>18.7</td>
</tr>
</tbody>
</table>
What drives the cross-market deviation?

- The deviation does not depend on the unit recovery claim value in a clear way.

  *Delta effect* \((\Delta, S)\): The more in the money, the higher the put value.
  - If CDS captures all the credit risk information, delta-dependence shows the violation of the default corridor (zero-delta) assumption.
  - Low stock price may reveal more credit risk than revealed from CDS.

  *Volatility effect* \((IV, RV)\): The higher the volatility, the higher the put value.
  - If CDS captures all the credit risk information, volatility-dependence shows the violation of the default corridor (zero-vega) assumption.
  - High realized/implied volatility may reveal more credit risk than revealed from CDS.

- High put open interest often reveals that put is under-valued?
Cross-market deviation and future market movements

- **H1**: Markets are fully efficient and cross-market deviation reflects purely violations of model/implementation assumptions.
  ⇒ The deviation can be fully explained by contemporary variables.
  ⇒ It has no bearing for the future.

- **H2**: Cross-market deviation reflects temporary market dislocations and asymmetries in information flow.
  ⇒ Current deviation predicts future market movements.

  - **H2a**: The deviation reflects credit risk information from the CDS market not yet in the put market → A positive deviation predicts negative movements in the put value.
  - **H2b**: The deviation reflects credit risk information from the put market not yet in the CDS market → A positive deviation predicts positive movements in CDS.
Hypothesis testing

- **H1**: $D_t = a + bX_t + D_t$ — How much the deviation can be explained by contemporary variables.
  - For each company and at each Wednesday, perform the regression using daily data over the past 30 business days.
  - The median R-squared ($R^2$) from the over 5000 regressions reflects the explanatory power of this regressor.

- **H2a**: $\Delta U^p_{t+\Delta_t} = \alpha^p + \beta^p D_t + e_{t+\Delta_t}$ — Whether the deviation predicts future put movements.
  - $\beta^p$ should be negative under $H2a$.

- **H2b**: $\Delta U^c_{t+\Delta_t} = \alpha^c + \beta^c D_t + e_{t+\Delta_t}$ — Whether the deviation predicts future CDS movements.
  - $\beta^c$ should be positive under $H2b$. 
## Cross-market information flow

<table>
<thead>
<tr>
<th>Horizon</th>
<th>$R^2$</th>
<th>$\beta^p$</th>
<th>$R^p_2$</th>
<th>$\beta^c$</th>
<th>$R^c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t = 7$ Days</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
<td>-0.122</td>
<td>( -9.4 )</td>
<td>1.7</td>
<td>0.138</td>
</tr>
<tr>
<td>$U^p$</td>
<td>79.6</td>
<td>-0.110</td>
<td>( -5.0 )</td>
<td>0.5</td>
<td>0.327</td>
</tr>
<tr>
<td>$U^c$</td>
<td>24.4</td>
<td>-0.188</td>
<td>( -9.5 )</td>
<td>1.7</td>
<td>0.034</td>
</tr>
<tr>
<td>$(U^p + U^c)/2$</td>
<td>54.0</td>
<td>-0.209</td>
<td>( -8.5 )</td>
<td>1.4</td>
<td>0.043</td>
</tr>
<tr>
<td>$</td>
<td>Delt</td>
<td>a$</td>
<td>51.1</td>
<td>-0.203</td>
<td>( -11.6 )</td>
</tr>
<tr>
<td>$ln(K/S)$</td>
<td>27.1</td>
<td>-0.190</td>
<td>( -10.8 )</td>
<td>2.2</td>
<td>0.207</td>
</tr>
<tr>
<td>$IV^p$</td>
<td>54.0</td>
<td>-0.037</td>
<td>( -2.1 )</td>
<td>0.1</td>
<td>0.247</td>
</tr>
<tr>
<td>$ATMV$</td>
<td>23.7</td>
<td>-0.116</td>
<td>( -7.0 )</td>
<td>0.9</td>
<td>0.180</td>
</tr>
<tr>
<td>$RV^{30}$</td>
<td>16.9</td>
<td>-0.137</td>
<td>( -8.0 )</td>
<td>1.3</td>
<td>0.141</td>
</tr>
<tr>
<td>$RV^{360}$</td>
<td>19.5</td>
<td>-0.156</td>
<td>( -8.6 )</td>
<td>1.5</td>
<td>0.189</td>
</tr>
<tr>
<td>$TD/BE$</td>
<td>15.6</td>
<td>-0.162</td>
<td>( -3.7 )</td>
<td>1.2</td>
<td>0.261</td>
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<tr>
<td>$TD/MC$</td>
<td>25.6</td>
<td>-0.183</td>
<td>( -10.2 )</td>
<td>2.1</td>
<td>0.215</td>
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<tr>
<td>$DF$</td>
<td>22.0</td>
<td>-0.124</td>
<td>( -7.4 )</td>
<td>1.1</td>
<td>0.171</td>
</tr>
<tr>
<td>$OI$</td>
<td>18.5</td>
<td>-0.176</td>
<td>( -8.0 )</td>
<td>1.8</td>
<td>0.166</td>
</tr>
</tbody>
</table>

### Two way information flow
## Cross-market information flow

<table>
<thead>
<tr>
<th>Horizon</th>
<th>$R^2$</th>
<th>$\beta^p$</th>
<th>$R^2_p$</th>
<th>$\beta^c$</th>
<th>$R^2_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
<td>-0.194</td>
<td>0.9</td>
<td>0.274</td>
<td>14.6</td>
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<tr>
<td>$U_p$</td>
<td>79.6</td>
<td>-0.252</td>
<td>0.6</td>
<td>0.565</td>
<td>18.2</td>
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<tr>
<td>$U_c$</td>
<td>24.4</td>
<td>-0.212</td>
<td>0.5</td>
<td>0.075</td>
<td>2.6</td>
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<tr>
<td>$(U_p + U_c)/2$</td>
<td>54.0</td>
<td>-0.380</td>
<td>1.0</td>
<td>0.097</td>
<td>2.7</td>
</tr>
<tr>
<td>$</td>
<td>\Delta t</td>
<td>$</td>
<td>51.1</td>
<td>-0.320</td>
<td>1.3</td>
</tr>
<tr>
<td>$\ln(K/S)$</td>
<td>27.1</td>
<td>-0.312</td>
<td>1.3</td>
<td>0.353</td>
<td>13.8</td>
</tr>
<tr>
<td>$IV_p$</td>
<td>54.0</td>
<td>-0.047</td>
<td>0.0</td>
<td>0.433</td>
<td>17.2</td>
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<td>$ATMV$</td>
<td>23.7</td>
<td>-0.147</td>
<td>0.3</td>
<td>0.352</td>
<td>14.8</td>
</tr>
<tr>
<td>$RV^{30}$</td>
<td>16.9</td>
<td>-0.102</td>
<td>0.2</td>
<td>0.248</td>
<td>10.0</td>
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<td>$RV^{360}$</td>
<td>19.5</td>
<td>-0.080</td>
<td>0.1</td>
<td>0.327</td>
<td>12.4</td>
</tr>
<tr>
<td>$TD/BE$</td>
<td>15.6</td>
<td>-0.039</td>
<td>0.0</td>
<td>0.535</td>
<td>9.1</td>
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<tr>
<td>$TD/MC$</td>
<td>25.6</td>
<td>-0.271</td>
<td>1.0</td>
<td>0.379</td>
<td>14.5</td>
</tr>
<tr>
<td>$DF$</td>
<td>22.0</td>
<td>-0.169</td>
<td>0.4</td>
<td>0.311</td>
<td>12.6</td>
</tr>
<tr>
<td>$OI$</td>
<td>18.5</td>
<td>-0.132</td>
<td>0.2</td>
<td>0.341</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Predictability on CDS becomes stronger at longer horizon.
Concluding remarks

- We identify a simple robust theoretical linkage between out-of-the-money American put options on a company’s stock and the company’s credit risk.
  - **Simple:** A simple spread between two American put options replicates a *pure* credit insurance contract.
  - **Robust:** The replication is valid as long as there exists a default corridor, irrespective of pre-default and post-default stock price dynamics, interest rate movements, or credit risk fluctuations.

- The theoretical linkage has strong *empirical support*:
  - The values of the credit contract inferred from American put options and CDS spreads are similar in magnitude and move together.
  - Their deviations predict future market movements, reflecting two-way cross-market information flow.

- The assumed default corridor also has theoretical support.
  - The corridor can be readily and reasonably accommodated by both reduced-form and structural models.