Are Interest Rate Derivatives Spanned by the Term Structure of Interest Rates?

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The U.S. dollar LIBOR market includes both interest rates and interest rate options. We ask whether a common finite-dimensional system spans both types of instruments. We find that the options market exhibits factors seemingly *independent* of the underlying yield curve. There are three common factors in LIBOR and swap rates, yet these factors can explain only a little over half of the variation in the swaption implied volatilities. Three additional factors are needed to capture the movement of the implied volatility surface.

Our interest in this issue arises because we see two groups of practitioners in the fixed-income market take sharply different approaches in applying interest rate models. Those trading the yield curve take an approach quite similar to academicians who work on equilibrium or structural models. They choose a two- or three-factor structural model, estimate the model parameters from the time series data, and then on each day choose the level of the state variables (factors) to fit the current term structure. The discrepancies between the fitted yield curve and the market prices are perceived as potential trading opportunities.

Interest rate options traders take the yield curve as a given, with minimal or no smoothing. To fit the observed yield curve perfectly, they often allow some of the model parameters to be time-inhomogeneous. The model is then solved based on no-arbitrage conditions as in Heath, Jarrow, and Morton [1992], assuming some term structure for the volatilities. Even so, they may need to recalibrate the model every day, changing the parameters to match the newly observed implied volatility surface.

The fact that yield curve traders apply a low-dimensional structural model implies that a finite set of state variables are sufficient to capture most of the common variations on the yield curve. The idiosyncratic part is potentially due to transient technical dislocations. Yet the full yield curve fitting practice in the options market implies that option traders do not want to be exposed to the idiosyncratic risks in the interest rate market, potentially because they have their own independent risks to deal with.

Our empirical findings confirm such conjectures. Similar to Litterman and Scheinkman [1991], we identify three common interest rate factors that explain 99.50% of the variation on the yield curve. When applied to swaption implied volatilities, however, these three factors explain only 59.48% of the variation in that market. Simulation analysis further indicates that the low percentage is not an artifact of the principal components analysis technique we use, but rather a true feature of the data.

To adequately explain the variation in the implied volatility surface, we need three additional *volatility factors*. These volatility factors are *independent* of the three interest rate factors, yet they are crucial in explaining the movement of interest rate options. Together with the three interest rate factors, they explain 97.62% of the variation in the implied volatility surface.

Furthermore, we find that the three volatility factors mainly capture the independent variation of the implied volatility surface along the option expiration dimension. Analogous to the roles played by the three interest rate factors on the yield curve, the three volatility factors capture the level, the slope, and the curvature of the option expiration term structure of the implied volatility surface. Variations of the surface along the swap maturity dimension are captured mostly by the interest rate factors, but are relatively insensitive to the volatility factors.

Most germane to this article is research by Litterman and Scheinkman [1991] and Knez, Litterman, and Scheinkman [1994], who perform factor analysis on the U.S. Treasury bond market and the money market. Also related is work by Wadhwa [1999], who conducts a principal components analysis of the implied volatility surface in the swaptions market. We extend this analysis to include both interest rates and interest rate options, and investigate whether the same finite-dimensional system spans both markets.

Motivated by similar findings, Collin-Dufresne and Goldstein [2001] provide mathematical conditions in an affine structure, under which stochastic factors driving the volatility do not influence the yield curve. Singleton and Umantsev [2002] provide an alternative explanation based on risk premiums; there are shocks to the volatility of yields under the risk-neutral measure that do not affect the correlation structure of swap yields under the objective measure.

I. DATA AND ESTIMATION

We first describe our data and the estimation techniques.

Data

The data set is obtained from Lehman Brothers. It consists of 1) LIBOR at maturities of 1, 2, 3, 6, and 12 months, 2) swap rates at maturities of 2, 3, 5, 7, 10, 15, and 30 years, and 3) at-the-money swaption Black [1976] implied volatilities at option maturities of 1, 3, and 6 months, and 1, 2, 3, 4, 5, 7, and 10 years. At each of the ten option maturities, we have seven contracts with different underlying swap maturities: one, two, three, four, five, seven, and ten years. All interest rates and interest rate options are on U.S. dollars. The data are daily closing mid-quotes from October 30, 1995, through July 23, 2001 (1,434 observations).

Swaptions are options to enter a swap contract of a certain tenor at a pre-fixed swap rate (strike). As a standard industry practice, the options contracts (e.g., caps, swaptions) are quoted in terms of Black implied volatilities. Given the quoted volatility, the delivery price is determined according to the current term structure of interest rates and the Black option pricing formula.

Exhibit 1 reports the summary statistics. All series are highly autocorrelated, and particularly so for interest rates. The mean level of interest rates increases with maturity, while the standard deviation exhibits a plateau around one and two years of maturity. The swaption implied volatilities also exhibit a humped-shape mean term structure along the option expiration dimension. The standard deviation of the implied volatility, however, declines monotonically with option maturity.

Principal Components Analysis

First, we perform principal components analysis of the LIBOR and swap rates to identify common factors underlying the yield curve. Second, we regress the swaption implied volatilities on the yield curve factors to investigate how these factors influence the variation in the implied volatility surface. Our analysis shows that the yield curve factors explain only 59.48% of the variation in the implied volatility surface.

We hence proceed to identify additional volatility factors (principal components) from the regression residuals of the swaption implied volatilities. Due to the inherent indeterminacy problem of factor identification, our sequential procedure guarantees that the additional volatility factors are independent of the common factors identified from the yield curve.

To determine the optimal number of factors in the principal components analysis, we construct a modified version of the test statistic in Connor and Korajczyk [1993]. The modification explicitly accounts for the sharp difference in liquidities between the interest rate market and the swaptions market. Specifically, we perform a downward adjustment to the cross-sectional means of the estimated squared residuals for each market by a maximum of their respective squared bid-ask spreads. The adjusted mean squared residuals are set to zero when they are negative. This adjustment ensures that pricing errors smaller than the bid-ask spread are ignored since they do not represent trading opportunities.

We apply an average bid-ask spread of 0.5 basis point for the interest rate data and 0.5% for the implied volatility series. According to this adjusted test statistic and at a 2.5% confidence level, we find that the optimal number

EXHIBIT 1
Summary Statistics

Maturity	Mean	Std	Skewness	Kurtosis	Auto
LIBOR and Swap Rates					
1 m	5.582	0.568	-0.293	1.422	0.993
2 m	5.612	0.579	-0.406	1.575	0.994
3 m	5.643	0.600	-0.444	1.504	0.995
6 m	5.701	0.639	-0.452	1.184	0.995
1 y	5.847	0.691	-0.268	0.393	0.995
2 y	6.015	0.681	-0.089	-0.395	0.995
3 y	6.144	0.650	-0.016	-0.635	0.995
5 y	6.307	0.613	-0.013	-0.771	0.995
7 y	6.420	0.593	-0.057	-0.831	0.995
10 y	6.547	0.572	-0.105	-0.905	0.994
15 y	6.684	0.543	-0.136	-1.010	0.994
30 y	6.750	0.504	-0.176	-1.089	0.995
Swaption Implied Volatility					
1 m	15.720	3.557	0.702	0.222	0.989
3 m	15.884	3.227	0.682	0.283	0.991
6 m	16.081	2.795	0.493	-0.248	0.991
1 y	16.313	2.301	0.246	-0.870	0.992
2 y	16.280	1.831	0.197	-0.679	0.991
3 y	15.942	1.645	0.163	-0.585	0.990
4 y	15.555	1.487	0.138	-0.497	0.989
5 y	15.139	1.349	0.114	-0.458	0.988
7 y	14.007	1.272	0.220	-0.281	0.989
10 y	12.531	1.330	0.253	-0.260	0.989

Average statistics for swaptions at each option maturity. Data are daily closing mid quotes form Lehman Brothers, from October 30, 1995, to July 23, 2001 (1,434 observations).

of common factors identified from the yield curve is three, and that the optimal number of additional volatility factors identified from the implied volatility regression residuals is also three.

Principal components analysis is traditionally performed on (excess) returns of an asset. We perform principal components analysis directly on the observed LIBOR and swap rates and implied volatilities. There are a number of reasons for this. First, to compute returns on the swap and swaption contracts, we need to infer a zero curve or a forward rate curve from the finite number of observations on the term structure, as in Litterman and Scheinkman [1991]. Interpolation and smoothing are hence needed. Yet, Björk and Christensen [1999] and Filipovic [1999] show that the commonly used smoothing functions, such as the Nelson-Siegel family and cubic splines, are not consistent with any diffusion dynamics. Hence, any such interpolation schemes may induce spurious effects into the factor analysis.

Second, the movement of the implied volatility surface is relatively separate from the interest rate movement. Thus, principal components analysis on implied volatilities instead of option prices or returns facilitates our identification of independent factors in the options market.

Third, when interest rates are cointegrated, analysis of excess returns may lead to singular problems (see Engle and Granger [1987]). When factors are significantly more persistent than the errors, principal components analysis of the levels provides an efficient way to identify the more persistent factors. Our posterior analysis of the regression residuals (not reported) confirms that the residuals are much less persistent than the explained components.

II. RESULTS AND DISCUSSION

We find both common and independent factors.

Common Factors in Interest Rates

Exhibit 2 reports the cumulative variance explained for each interest rate series by the three common factors.

EXHIBIT 2
Common Factors Explaining Variation in U.S. Dollar LIBOR and Swap Rates

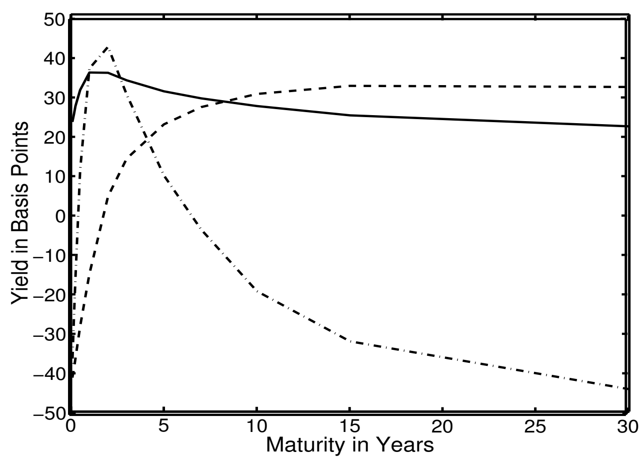
Factors	1	2	3
1 m	58.94	95.70	98.09
2 m	66.31	98.20	99.33
3 m	72.94	98.66	99.06
6 m	85.26	99.06	99.21
1 y	94.85	98.26	99.74
2 y	97.57	97.83	99.85
3 y	95.66	98.72	99.85
5 y	90.51	99.64	99.76
7 y	85.85	99.78	99.81
10 y	80.33	99.28	99.91
15 y	74.07	97.99	99.89
30 y	67.78	95.06	99.21
Aggregate	82.57	98.28	99.50

The first factor explains 82.57% of the aggregate variation in interest rates. It explains as much as 97.57% of the variation in the two-year swap rates, but only 58.94% of the variation in the one-month rate and 67.78% of the variation in the 30-year swap rate. Incorporating the second factor significantly improves the percentage of variation explained at both the short and the long ends of the yield curve. The third factor further improves the variance explained at the short and long maturities. Altogether, the three factors explain 99.50% of the variation in the interest rate market.

Exhibit 3 depicts the loadings of the three factors on interest rates. The plots are similar to Figure 2 in Litterman and Scheinkman [1991]. The three factors are the *level* factor (solid line), the *slope* factor (dashed line), and the *curvature* factor (dash-dotted line).

The loading of the first factor is relatively flat across interest rate maturities. The second factor drops the short rate by about 40 basis points but increases the long rate by about 30 basis points. The first two factors combined generate an upward-sloping mean yield curve. The curvature factor contributes positively to the middle of the yield curve and negatively to both ends of the curve. It thus enhances the curvature of the yield curve.

EXHIBIT 3
Factor Loadings on Interest Rates



Lines depict the factor loadings (in basis points) on interest rates of different maturities. Solid line: level factor; dashed line: slope factor; and dash-dotted line: curvature factor.

Independent Factors in Swaptions Market

Once we have extracted the common factors from the interest rate market, we regress the swaption implied volatilities on the yield curve factors and investigate how much variation these yield curve factors explain in the swaptions market. Panel A of Exhibit 4 reports the explained percentage variance of the implied volatility by the three yield curve factors. On average, the three yield curve factors explain only 59.48% of the variance in the swaption implied volatility. Additional volatility factors are needed.

In Panel B of Exhibit 4, we report the percentage of variance explained by incorporating one additional volatility factor. Compared to the case with only three interest rate factors (Panel A), the explained variance is increased by approximately 44% to 85.73%. Most of the improvement is at short option expirations across all swap maturities. To illustrate the significance of the improvement, we shade all series that are explained by more than 90%.

Incorporating two additional factors, as indicated in Panel C, explains 96.11% of the total variation. With the second volatility factor, most implied volatilities at moderate option and swap maturities can be explained by more than 90%. The darker shading in the middle part of Panel C highlights the region of implied volatilities that need two volatility factors for their variation to be explained by more than 90%. The right-hand bottom corner of the panel is unshaded, as the two additional volatility factors are still not enough to improve the explained percentage variation to over 90%. Note the ten-year option on ten-year swaps is explained by merely 56.79%.

Incorporating a third factor increases the aggregate explained variance to 97.62%, as shown in Panel D. More important, almost all the implied volatility series are explained by more than 90% (except for three series on ten-year swaps). The darker shading at the right-hand bottom corner of Panel D indicates that the third volatility factor mainly contributes to the long-maturity options on long-maturity swaps.

Factor Loadings on the Implied Volatility Surface

Exhibit 5 plots the loading of the three common factors on the implied volatility surface. Overall, the interest rate level factor (first panel) contributes negatively to the implied volatility surface. The impact is greatest at -1.26% on one-month options, but slightest at -0.18% on ten-year options, both on one-year swaps.

The impact of the interest rate slope factor (middle panel) is mostly positive. The greatest impact is again on one-month options on one-year swaps at 1.70%, but its loadings on long-term options on long-maturity swaps are close to zero.

Of the three interest rate factors, the greatest impact on implied volatility comes from the curvature (third) factor, as depicted in the last panel of Exhibit 5. While the impacts of the level and slope factors are both less than 2%, the impact of the curvature factor can be as high as 10%. A positive shock to the curvature factor generates a large negative shock to the implied volatility level,

EXHIBIT 4
Variance Decomposition on Implied Volatilities—Cumulative Percentage Variation Explained

Maturity	1	2	3	4	5	7	10	Average
A. Three Common Factors in the Yield Curve								
0.1	63.18	62.88	60.82	59.90	59.59	56.54	52.35	59.32
0.3	66.23	66.95	64.36	63.42	63.17	59.78	54.59	62.64
0.5	71.29	69.11	67.35	66.52	66.77	63.77	58.19	66.14
1	67.00	66.97	66.58	65.85	66.09	64.39	59.80	65.24
2	58.27	60.73	61.30	61.00	60.30	55.94	49.16	58.10
3	52.89	56.21	56.02	55.54	53.74	49.00	42.36	52.25
4	47.98	51.83	50.56	48.54	46.66	42.97	39.22	46.82
5	42.93	46.84	45.13	43.29	42.75	39.06	38.20	42.60
7	46.89	48.94	46.44	44.15	41.27	37.72	37.04	43.21
10	36.57	36.49	33.14	29.88	25.78	23.77	22.57	29.74
Average	55.32	56.69	55.17	53.81	52.61	49.30	45.35	**59.48**
B. One Additional Factor In Derivatives								
0.1	89.59	93.54	94.13	94.03	93.43	93.14	90.91	92.68
0.3	91.27	96.26	96.62	96.88	96.58	95.77	93.09	95.21
0.5	95.75	97.88	98.58	98.92	98.90	98.24	95.02	97.61
1	93.62	94.18	93.63	92.75	91.91	89.85	86.01	91.71
2	85.68	85.53	83.62	82.16	78.08	72.60	66.90	79.22
3	78.48	78.12	76.13	72.59	68.30	61.95	55.52	70.16
4	69.68	70.64	66.38	61.94	58.33	53.02	48.66	61.24
5	61.13	61.33	57.86	54.25	51.63	46.52	45.19	53.99
7	55.77	53.86	49.97	46.20	42.26	38.28	37.59	46.27
10	38.83	36.90	33.17	29.96	26.31	24.59	23.26	30.43
Average	75.98	76.82	75.01	72.97	70.57	67.40	64.22	**85.73**

EXHIBIT 4 (continued)
Variance Decomposition on Implied Volatilities

Maturity	1	2	3	4	5	7	10	Average
			C. Two Additional Factors In Derivatives					
0.1	96.06	98.32	98.27	97.82	97.26	96.75	94.60	97.01
0.3	96.69	99.10	99.22	99.00	98.64	97.60	94.87	97.87
0.5	97.30	98.31	98.82	99.00	98.93	98.25	95.02	97.95
1	94.57	96.30	97.13	97.97	98.52	98.25	95.48	96.89
2	93.44	96.27	97.74	98.54	98.47	96.82	91.19	96.07
3	93.59	97.36	98.49	98.68	98.16	95.40	87.91	95.65
4	93.67	98.02	98.61	98.44	97.44	93.98	87.24	95.34
5	93.07	98.29	98.44	97.89	96.80	92.22	86.31	94.72
7	96.90	97.14	95.65	94.01	91.13	87.48	84.53	92.41
10	85.66	78.85	75.02	71.76	66.15	62.31	56.79	70.93
Average	94.09	95.80	95.74	95.31	94.15	91.91	87.39	**96.11**
			D. Three Additional Factors In Derivatives					
0.1	96.09	98.95	99.30	99.20	98.68	97.81	95.33	97.91
0.3	96.98	99.27	99.47	99.28	98.86	97.67	94.87	98.06
0.5	97.63	98.33	98.86	99.02	98.94	98.27	95.25	98.04
1	95.45	96.96	97.60	98.37	98.83	98.66	95.99	97.41
2	96.16	97.70	98.78	99.22	98.90	97.24	91.94	97.13
3	97.25	99.01	99.49	99.32	98.53	95.80	88.73	96.88
4	97.73	99.23	99.16	98.68	97.52	94.06	87.53	96.27
5	96.96	98.88	98.62	97.90	96.87	92.30	86.31	95.41
7	96.91	98.44	98.61	98.50	98.02	96.17	93.45	97.16
10	92.55	94.55	95.17	95.64	95.78	95.52	91.71	94.42
Average	96.37	98.13	98.51	98.51	98.09	96.35	92.11	**97.62**

Bold numbers represent aggregate percentages. Shading highlights series explained by more than 90%. Darker shading implies that more factors are needed.

especially at the short option and swap maturities.

Exhibit 6 graphs the loading of the additional volatility factors on the implied volatility surface. The loading of the first volatility factor (first panel) is about 0.2% at short option maturities, but is close to zero at long option maturities. Its impact along the swap maturity dimension is relatively flat.

The second volatility factor (middle panel) contributes positively to the implied volatility surface at short option maturities, but negatively at moderate to long option maturities. As a result, the second volatility factor generates a downward slope in the term structure of the implied volatility surface along the option maturity dimension. The term spread generated from this factor is about 0.2% to 0.35%, depending on the underlying swap maturities.

The third volatility factor (last panel) contributes positively at the two ends of the option maturities, but negatively in the middle. The greatest absolute impact is on very long option maturities (ten-year) at around 0.3%.

The distinct actions of the three volatility factors are mostly along the option maturity dimension; their impact along the swap maturity dimension is relatively flat. Analogous to the impacts of the three interest rate factors on the yield curve, the three volatility factors can also be regarded as capturing the level, the slope, and the curvature of the implied volatility term structure along the option maturity dimension.

Mimicking Portfolios

An advantage of principal components analysis is that one can directly extract the factors as a linear combination of the observables. Portfolios that mimic a particular factor allow us to further investigate the properties of the unobservable factors and the impact of these factors on other fixed-income securities.

Exhibit 7 reports correlations between interest rates and the mimicking portfolios of the six factors: the three interest rate factors and the three additional volatility factors. The interest rate level factor is positively correlated with all interest rates, with an almost perfect correlation (0.99) with the two-year swap rate. Hence, as evidenced before, the level factor can be well proxyed by the two-year swap rate.

The mimicking portfolio for the slope factor is negatively correlated with short rates but positively correlated with long rates, and thus generates non-parallel movement in the yield curve. While the eigenvector provides a direct way of forming the mimicking portfolio, we can approximate the second factor with a simpler and more intuitive portfolio. In particular, if we form a spread portfolio by going long one unit of the 30-year swap rate and shorting one unit of the one-month LIBOR, the spread portfolio has an almost perfect correlation (0.98) with the slope factor, thus confirming that the second factor indeed captures the slope of the curve.

The mimicking portfolio for the curvature factor is positively correlated with moderate-maturity swaps but negatively correlated with both short- and long-term interest rates. We construct a portfolio that goes long 1.6 units of the two-year swap rate and shorts one unit of the one-month LIBOR and 1.5 units of the 30-year swap rate. Such a three-instrument portfolio is commonly referred to as a *butterfly*. It captures the curvature of the yield curve.

In the last row of Exhibit 7, we report the correlation of this butterfly portfolio with the six factors. Not surprisingly, its correlation with the third factor (the curvature factor) is nearly perfect (0.99).

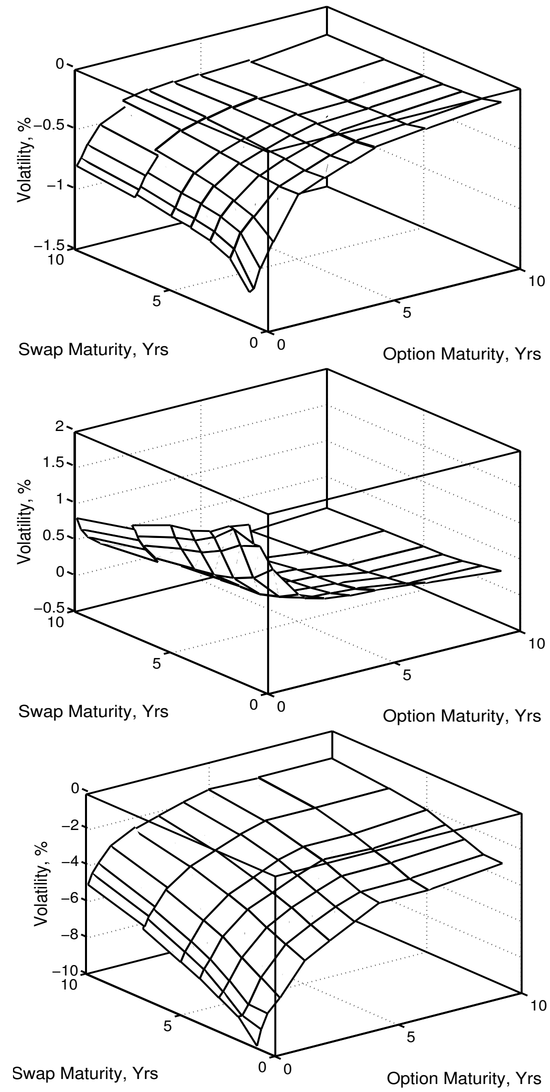
In the language of fixed-income risk management, the three common factors are referred to as the *duration*, *curve*, and *butterfly* risk. Duration hedging essentially focuses on the risk of the level factor where interest rates are assumed to move in parallel. Yet, the yield curve can flatten or steepen over time, thus generating slope changes. Finally, the variation in the curvature of the yield curve can be captured by that of a butterfly portfolio.

The last three columns of Exhibit 7 report the correlations of each interest rate series with the three volatility factors. The correlations are all very close to zero, implying that the volatility factors are not only independent of the three interest rate factors by construction, but also uncorrelated with the underlying yield curve altogether.

Exhibit 8 reports the correlations of each implied volatility series with the mimicking portfolios of the six factors. The correlations between the interest rate level factor and the implied volatilities are all negative, ranging from -0.24 to -0.60, with an average of -0.48. These negative correlations may point to misspecifications of the Black model. In the Black model, the diffusion of the swap rate is assumed to be proportional to the swap rate level. Thus, the negative correlation between the Black volatilities and the interest rate level factor may imply that the proportionality assumption is overstating the dependence of the diffusion on the interest rate level.

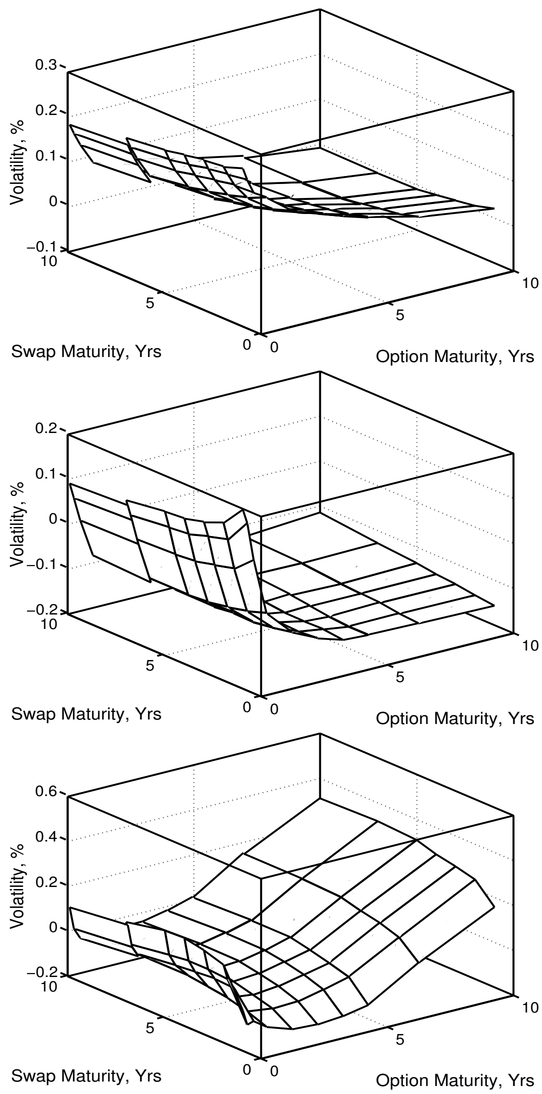
Again, we use shading to highlight the relative con-

EXHIBIT 5
Loadings of Interest Rate Factors on Implied Volatility Surface



Lines depict loadings (in percentage of implied volatility) of the three interest rate common factors on the implied volatility surface of swaptions. Top Panel: impact of the level factor; middle panel: impact of the slope factor; bottom panel: curvature factor.

EXHIBIT 6
Loadings of Volatility Factors on Implied Volatility Surface



Lines depict loadings (in percentage of implied volatility) of the additional volatility factors on the implied volatility of swaptions. Top panel: impact of the first volatility factor; middle panel: impact of the second volatility factor; bottom panel: third volatility factor.

tribution of each factor. In Exhibit 8, we shade all the series whose correlation with the specific factor is higher than 0.5 in absolute value. The shading of the top right-hand triangle in Panel A illustrates that the interest rate level factor is more correlated with options on long swap maturities.

The correlations of the implied volatilities with the interest rate slope factor (Panel B) are all positive, ranging from zero at long swap and option maturities to 0.31 at short swap and option maturities. The correlations are mostly low and all below 0.5. Nevertheless, in contrast to the level factor, the slope factor is more correlated with implied volatilities on short swap maturities.

Similar to the interest rate level factor, the interest rate curvature factor (Panel C) is also highly correlated with the implied volatilities. Unlike the level factor, however, the curvature factor is more correlated with options on short-maturity swaps. Litterman and Scheinkman [1991] claim that the curvature factor is closely related to the volatility level of the interest rates. Our finding confirms such a claim, and further indicates that the curvature factor is more correlated with the volatility level of *short*-term interest rates. On the other hand, the implied volatilities of options on long-term swaps are more correlated with the interest rate level factor.

Panel D of Exhibit 8 reports the correlations of the first volatility factor with the implied volatility surface. The correlations vary greatly across option maturities, but are relatively flat across swap maturities. The factor has high positive correlation with implied volatilities of short-term options, but close to zero correlation with long-term option implied volatilities. Such a factor complements the interest rate level and curvature factors in further explaining the level of the volatilities. We hence label it the *volatility level factor*.

Panel E of Exhibit 8 reports the correlation between the second volatility factor and the implied volatility surface. The correlations are positive with short-term options but negative with long-term options. The correlations across the swap dimension are again relatively flat. Hence, the factor contributes to the slope of the implied volatility term structure along the option expiration dimension.

To illustrate this idea, we form a series of spread portfolios by going long one unit of the one-month volatility and shorting four units of the five-year volatility for each underlying swap rate. In the last row of Panel E, we report the correlations of the second volatility factor with these seven spread portfolios. While not perfect, the correlations of the second volatility factor with these spread portfolios are very high, confirming the contribution of this factor to the slope of the option term structure.

We thus label the second volatility factor the *volatility spread factor*. Furthermore, while the volatility level factor is more correlated with short-term options, the slope factor is more correlated with moderate- to long-term options.

The correlations of the third volatility factor with the implied volatilities are summarized in Panel F of Exhibit 8. Overall, the correlations are very low except at very long option and swap maturities. The correlations

EXHIBIT 7
Correlations of Factor Mimicking Portfolios with LIBOR and Swap Rates

Factors	1	2	3	4	5	6
1 m	0.77	-0.61	-0.15	-0.02	0.01	0.01
2 m	0.81	-0.56	-0.11	-0.01	0.01	-0.00
3 m	0.85	-0.51	-0.06	0.01	-0.00	-0.02
6 m	0.92	-0.37	0.04	0.02	-0.01	0.00
1 y	0.97	-0.18	0.12	0.00	-0.01	0.01
2 y	**0.99**	0.05	0.14	-0.00	-0.00	0.00
3 y	0.98	0.17	0.11	-0.01	0.00	-0.00
5 y	0.95	0.30	0.04	-0.01	0.01	-0.00
7 y	0.93	0.37	-0.02	-0.01	0.01	-0.00
10 y	0.90	0.44	-0.08	-0.00	0.00	-0.00
15 y	0.86	0.49	-0.14	0.01	-0.01	-0.01
30 y	0.82	0.52	-0.20	0.01	-0.02	0.01
Spread	-0.03	**0.98**	-0.02	0.03	-0.02	-0.00
Butterfly	0.00	-0.00	**0.99**	-0.01	0.02	-0.04

Bold numbers represent the highest correlation between each mimicking factor and the interest rate portfolios.

are positive at very short and very long option maturities, but negative in the middle, forming a butterfly structure. We exploit this structure and construct a series of butterfly portfolios corresponding to each underlying swap rate by going long one unit of the one-month volatility and two units of ten-year volatility and shorting two units of the seven-year volatility.

The last row of Panel F reports the correlation between the third factor and these portfolios. The correlation is around 0.9 for all swap maturities. We hence label the third volatility factor the *volatility butterfly factor*.

The spread and butterfly portfolios are formed only for illustrative purposes. They serve as motivations for our christening of the three volatility factors. Thus, we make no attempt to optimize the exact formulation. Furthermore, we construct a series of spread and butterfly portfolios corresponding to each underlying swap maturity to illustrate the idea that the volatility factors are mainly capturing the term structure along the option maturity dimension, but are relatively insensitive to the choice of the underlying swap maturity. In practice, obviously, we do not need to form the spread and butterfly portfolios within the same swap maturity, nor do we need to form so many portfolios of each type. One portfolio for each factor is enough.

In short, the three volatility factors capture the level, the slope, and the curvature of the implied volatility term structure along the option maturity dimension. Their contributions along the swap maturity dimension, however, are relatively flat, implying that these factors are mainly capturing the time aggregation and conditioning of the volatility process, which determines the term structure of volatility along the option maturity dimension. The volatility term structure along the swap maturity dimension, on the other hand, is mainly captured by the interest rate factors.

III. SIMULATION ANALYSIS

The principal components analysis assumes an approximate linear and static structure between the common factors and the observable series on interest rates and implied volatilities. LIBOR and swap rates, however, are non-linear functions of one another. More important, the relations between the Black implied volatilities and the LIBOR and swap rates are not only non-linear, but also dynamic, since swaptions price the conditional dynamics of the underlying swaps. Because of this, a natural concern is that the independent movements of interest rates and implied volatilities may be an artifact of the principal components analysis technique.

To investigate the power of the principal components analysis, we perform a simulation analysis. First, we estimate a three-factor Gaussian affine model on LIBOR and swap rates using a quasi-maximum likelihood method in conjunction with an extended Kalman [1960] filter. Second, we use the extracted factors to produce a simulated series of LIBOR and swap rates, as well as at-the-money cap prices and Black implied volatilities. Finally, we perform principal components analysis on the simulated interest rates and implied volatilities.

By design, the simulated series of LIBOR, swap rates, and cap implied volatilities are all governed by the same three dynamic factors, although in non-linear functional forms. Thus, if principal components analysis finds independent movement for implied volatilities, this will serve as evidence against the use of the technique.

The results in Exhibit 9 indicate that three principal components from the simulated interest rate series explain 100% of the total variance of the simulated interest rate series. This result confirms the analysis of Singleton and Umantsev [2003] that LIBOR and swap rates are approximately linear in the state variables within the affine framework.

More important, we find that, despite the non-linear and dynamic relations, the three interest rate principal components explain 97.90% of the variation in the simulated series of cap implied volatilities. The test statistic on the optimal number of factors also indicates that no additional volatility factors are needed. In contrast, the first three principal components extracted from the interest rates data explain only 62.70% of the movement in the real data of the cap Black implied volatilities, very similar to those found for swaptions.

Therefore, we conclude that the independent volatility movement we identify is not an artifact of the principal components analysis, but rather a true feature of the data.

IV. CONCLUDING REMARKS

Inspired by the sharply different behaviors of interest rate traders and option players, we try to clarify whether the same finite-dimensional dynamic system spans both the yield curve and the implied volatility surface of interest rate options. The answer is negative. The options market exhibits factors independent of the underlying yield curve. This finding has important implications for pricing and hedging in the fixed-income market, particularly so for securities in both markets, such as callable and puttable bonds and mortgage-backed securities.

At the head of our research agenda is to investigate the structural nature of these independent volatility factors and to develop a consistent term structure model that captures the behaviors of both the interest rate market and the interest rate options market.

EXHIBIT 8
Correlations of Mimicking Portfolios with Implied Volatility Surface

Maturity	1	2	3	4	5	7	10	Average
A. Factor One: Interest Rate Level								
0.1	-0.53	-0.51	-0.51	-0.52	-0.53	-0.54	-0.55	-0.53
0.3	-0.57	-0.51	-0.51	-0.53	-0.55	-0.57	-0.58	-0.55
0.5	-0.58	-0.50	-0.51	-0.53	-0.56	-0.57	-0.59	-0.55
1.0	-0.54	-0.52	-0.53	-0.55	-0.56	-0.58	-0.60	-0.55
2.0	-0.44	-0.49	-0.51	-0.53	-0.54	-0.55	-0.55	-0.52
3.0	-0.41	-0.46	-0.48	-0.50	-0.51	-0.53	-0.53	-0.49
4.0	-0.38	-0.43	-0.45	-0.47	-0.48	-0.51	-0.54	-0.47
5.0	-0.34	-0.39	-0.42	-0.44	-0.46	-0.51	-0.55	-0.45
7.0	-0.32	-0.38	-0.40	-0.43	-0.45	-0.49	-0.53	-0.43
10.0	-0.24	-0.29	-0.32	-0.34	-0.35	-0.36	-0.38	-0.32
Average	-0.44	-0.45	-0.46	-0.48	-0.50	-0.52	-0.54	**-0.48**
B. Factor Two: Interest Rate Slope								
0.1	0.31	0.33	0.31	0.30	0.30	0.26	0.23	0.29
0.3	0.29	0.34	0.31	0.29	0.28	0.25	0.21	0.28
0.5	0.32	0.32	0.30	0.28	0.27	0.24	0.20	0.28
1.0	0.27	0.24	0.22	0.21	0.21	0.20	0.19	0.22
2.0	0.21	0.17	0.15	0.16	0.15	0.14	0.13	0.16
3.0	0.18	0.15	0.15	0.14	0.14	0.12	0.08	0.14
4.0	0.18	0.16	0.15	0.14	0.14	0.11	0.05	0.13
5.0	0.20	0.16	0.15	0.15	0.14	0.09	0.01	0.13
7.0	0.18	0.11	0.10	0.08	0.07	0.04	-0.00	0.08
10.0	0.13	0.10	0.08	0.06	0.06	0.06	0.04	0.08
Average	0.23	0.21	0.19	0.18	0.18	0.15	0.11	**0.18**

EXHIBIT 8 (continued)
Correlations of Mimicking Portfolios with Implied Volatility Surface

Maturity	1	2	3	4	5	7	10	Average
C. Factor Three: Interest Rate Curvature								
0.1	-0.50	-0.51	-0.51	-0.49	-0.48	-0.45	-0.41	-0.48
0.3	-0.50	-0.54	-0.53	-0.51	-0.50	-0.47	-0.41	-0.50
0.5	-0.53	-0.58	-0.57	-0.55	-0.53	-0.50	-0.44	-0.53
1.0	-0.56	-0.58	-0.58	-0.56	-0.55	-0.52	-0.46	-0.54
2.0	-0.58	-0.58	-0.57	-0.55	-0.53	-0.48	-0.41	-0.53
3.0	-0.57	-0.57	-0.55	-0.53	-0.51	-0.44	-0.37	-0.51
4.0	-0.55	-0.55	-0.53	-0.50	-0.46	-0.39	-0.32	-0.47
5.0	-0.52	-0.54	-0.50	-0.47	-0.44	-0.36	-0.28	-0.44
7.0	-0.58	-0.58	-0.54	-0.50	-0.45	-0.37	-0.30	-0.47
10.0	-0.54	-0.52	-0.47	-0.42	-0.37	-0.32	-0.28	-0.42
Average	-0.54	-0.56	-0.54	-0.51	-0.48	-0.43	-0.37	**-0.49**
D. Derivative Factor One								
0.1	0.51	0.55	0.58	0.58	0.58	0.60	0.62	0.58
0.3	0.50	0.54	0.57	0.58	0.58	0.60	0.62	0.57
0.5	0.49	0.54	0.56	0.57	0.57	0.59	0.61	0.56
1.0	0.52	0.52	0.52	0.52	0.51	0.50	0.51	0.51
2.0	0.52	0.50	0.47	0.46	0.42	0.41	0.42	0.46
3.0	0.51	0.47	0.45	0.41	0.38	0.36	0.36	0.42
4.0	0.47	0.43	0.40	0.37	0.34	0.32	0.31	0.38
5.0	0.43	0.38	0.36	0.33	0.30	0.27	0.26	0.33
7.0	0.30	0.22	0.19	0.14	0.10	0.07	0.07	0.16
10.0	0.15	0.06	0.02	-0.03	-0.07	-0.09	-0.08	-0.01
Average	0.44	0.42	0.41	0.39	0.37	0.36	0.37	**0.40**

Maturity	1	2	3	4	5	7	10	Average
E. Derivative Factor Two								
0.1	0.25	0.22	0.20	0.19	0.20	0.19	0.19	0.21
0.3	0.23	0.17	0.16	0.15	0.14	0.14	0.13	0.16
0.5	0.12	0.07	0.05	0.03	0.02	0.01	0.01	0.04
1.0	-0.10	-0.15	-0.19	-0.23	-0.26	-0.29	-0.31	-0.22
2.0	-0.28	-0.33	-0.38	-0.40	-0.45	-0.49	-0.49	-0.40
3.0	-0.39	-0.44	-0.47	-0.51	-0.55	-0.58	-0.57	-0.50
4.0	-0.49	-0.52	-0.57	-0.60	-0.63	-0.64	-0.62	-0.58
5.0	-0.57	-0.61	-0.64	-0.66	-0.67	-0.68	-0.64	-0.64
7.0	-0.64	-0.66	-0.68	-0.69	-0.70	-0.70	-0.69	-0.68
10.0	-0.68	-0.65	-0.65	-0.65	-0.63	-0.61	-0.58	-0.64
Spread	**0.91**	**0.94**	**0.95**	**0.95**	**0.95**	**0.92**	**0.87**	**0.93**
F. Derivative Factor Three								
0.1	-0.02	0.08	0.10	0.12	0.12	0.10	0.09	0.08
0.3	-0.05	0.04	0.05	0.05	0.05	0.03	0.00	0.02
0.5	-0.06	0.01	0.02	0.01	0.01	-0.01	-0.05	-0.01
1.0	-0.09	-0.08	-0.07	-0.06	-0.06	-0.06	-0.07	-0.07
2.0	-0.17	-0.12	-0.10	-0.08	-0.07	-0.06	-0.09	-0.10
3.0	-0.19	-0.13	-0.10	-0.08	-0.06	-0.06	-0.09	-0.10
4.0	-0.20	-0.11	-0.07	-0.05	-0.03	-0.03	-0.05	-0.08
5.0	-0.20	-0.08	-0.04	-0.01	0.03	0.03	0.01	-0.04
7.0	-0.01	0.11	0.17	0.21	0.26	0.29	0.30	0.19
10.0	0.26	0.40	0.45	0.49	0.54	0.58	0.59	0.47
Butterfly	**0.64**	**0.88**	**0.92**	**0.93**	**0.93**	**0.92**	**0.88**	**0.87**

Bold numbers represent grand averages. Shading highlights correlations that are greater than 0.5 in absolute magnitude.

EXHIBIT 9
Principal Components Analysis of Simulated and Real Data

	Interest Rate Factors			Additional Implied Volatility Factors			
Factors	1	2	3	0	1	2	3
Simulated Series	83.20	98.90	100.00	97.90	99.48	99.95	100.00
Real Data	82.57	98.28	99.50	62.70	94.13	99.24	99.87

ENDNOTE

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REFERENCES

Björk, Tomas, and Bent Jesper Christensen. "Interest Rate Dynamics and Consistent Forward Rate Curves." *Mathematical Finance*, 9 (1999), pp. 323-348.

Black, Fischer. "The Pricing of Commodaity Contracts." *Journal of Financial Economics*, 3 (1976), pp. 167-179.

Collin-Dufresne, Pierre, and Robert S. Goldstein. "Do Bonds Span the Fixed Income Markets? Theory and Evidence for Unspanned Stochastic Volatility." Working paper, Carnegie Mellon University, 2001.

Connor, Gregory, and Robert A. Korajczyk. "A Test for the Number of Factors in an Approximate Factor Model." *Journal of Finance,* 48 (1993), pp. 1263-1291.

Engle, Robert F., and C.W.J. Granger. "Co-integration and Error Correction: Representation, Estimation, and Testing." *Econometrica,* 55 (1987), pp. 251-276

Filipovic, Damir. "A Note on the Nelson-Siegel Family." *Mathematical Finance,* 9 (1999), pp. 349-359.

Heath, David, Robert Jarrow, and Andrew Morton. "Bond Pricing and the Term Structure of Interest Rates: A New Technology for Contingent Claims Valuation." *Econometrica,* 60 (1992), pp. 77-105.

Kalman, Rudolph Emil. "A New Approach to Linear Filtering and Prediction Problems." *Transactions of the ASME—Journal of Basic Engineering*, 82 (1960), pp. 25-45.

Knez, Peter J., Robert Litterman, and Jose Scheinkman. "Explorations into Factors Explaining Money Market Returns." *Journal of Finance,* 49 (1994), pp. 1861-1882.

Litterman, Robert, and Jose Scheinkman. "Common Factors Affecting Bond Returns." *The Journal of Fixed Income,* 1 (1991), pp. 54-61.

Singleton, Kenneth, and Len Umantsev. "The Price of Volatility Risk Implicit in Swaption Prices." Working paper, Stanford University, 2002.

——. "Pricing Coupon-Bond Options and Swaptions in Affine Term Structure, Models." Forthcoming, *Mathematical Finance*, 2003.

Wadhwa, Pavan. "An Empirical Analysis of the Common Factors Governing U.S. Dollar-LIBOR Implied Volatility Movements." *The Journal of Fixed Income,* 9 (1999), pp. 61-68.