Decomposing Long Bond Returns

Liuren Wu
Joint work with Peter Carr

Baruch College

March 9th, 2017
Objective dictates modeling framework

Different modeling frameworks are built for different purposes:

- **Expectation hypothesis (EH):** Predict short rate move with yield curve slope.
  - Yield curve shape combines information from expectation, risk premium, and convexity, but expectation dominates the short term.

- **Dynamic Term Structure Models (DTSM):** Value the whole yield curve based on assumptions on the full risk-neutral dynamics of the short rate.
  - Uses one yardstick to measure everything else for cross-sectional consistency.
  - Deviations from DTSM valuations can be used to construct statistical arbitrage trading on the yield curve.

- **HJM-type models:** Price interest rate options based on the current forward curve and views of forward rate volatility.
  - Highlight the volatility contribution for option valuation while delta-hedge away the yield curve exposure.
Our objective: Analyzing returns on long-dated bonds

- EH uses long rates to predict short rate move, not the other away around.
  - How to predict long rate movements based on the yield curve shape, while accounting for risk premium and convexity?
  - More importantly, how to predict excess returns on long bonds?

- Modeling long rates with DTSM stretches the modeler’s imagination on how short rate should move in the next 30-60 years...
  - Mean reversion calibrated to time series or short end of the yield curve implies much smaller movements than observed from long rates.
  - Long rates are neither (easily) predictable, nor converging to a constant. — They move randomly, and with substantial volatility.
  - Can we say something intelligent & accurate about a 50-year rate without making a 50-year projection?

- (Current) volatility can be much more accurately estimated than mean using historical data, how can one effectively use such information?

The distinct behaviors of long bonds ask for a distinct modeling approach.
A new modeling approach

We propose a new modeling framework that is particularly suited for analyzing long bond returns:

- Links pricing directly to P&L attribution of bond investments.
  - The attribution makes it clear on what to bet/hedge
- Prices each rate based on its own behavior, not that of the short rate.
  - Localization allows one to make less ambitious but more confident statements.
  - The model can say/do something useful about a 50-yr bond investment without making a 50-year projection, especially if you just want to hold the bond for short term.
- Generates predictions on bond returns, even with no prediction on rates.
  - via a decomposition of expectation/risk premium from convexity.
  - The separation can even help us better predict long rates...
Let $B_t$ be the time-$t$ price of a default-free coupon bond (portfolio) with fixed future cash flows $\{C_j\}$ at times $\{t + \tau_j\} \geq t$ for $j = 1, 2, \ldots, N$.

The classic valuation of this coupon bond can be represented as

$$B_t = \sum_j C_j \mathbb{E}_t^P \left[ M_{t,t+\tau_j} \right] = \sum_j C_j \mathbb{E}_t^P \left[ \left( \frac{dQ}{dP} \right) e^{-\int_{t+\tau_j}^{t+\tau_j} r_u du} \right]$$

$$= \sum_j C_j \mathbb{E}_t^Q \left[ e^{-\int_t^T r_u du} \right].$$

- $\mathbb{E}_t [\cdot]$ — expectation under time-$t$ filtration,
- $M_{t,T}$ — the pricing kernel linking value at time $t$ to value at time $T$
- $P$ — the real world probability measure,
- $Q$ — the so-called risk-neutral measure,
- $r_t$ — instantaneous short rate
- $\frac{dQ}{dP}$ defines the measure change from $P$ to $Q$. It is the martingale component of the pricing kernel that defines the pricing of various risks.

The yield-to-maturity of the bond is defined via the following transformation:

$$B_t \equiv \sum_j C_j \exp(-y_t \tau_j).$$
Yield decomposition under the classic setting

of a zero coupon bond — Inverting the coupon bond yield is harder.

- The zero yield can be decomposed generically into three components:

\[
y_t(T) = \frac{1}{T} E_t^P \left[ \int_t^T r_u du \right] \quad \text{(Expectation)}
\]

\[
+ \frac{1}{T} E_t^P \left[ \left( \frac{dQ}{dP} - 1 \right) \int_t^T r_u du \right] \quad \text{(Risk premium)}
\]

\[
- \frac{1}{T} \ln E_t^Q \left[ \exp \left( - \left( \int_t^T r_u du - E_t^Q \int_t^T r_u du \right) \right) \right] \quad \text{(Convexity)}
\]

1. The 1st term is the investor \textit{expectation} of the average future short rate.
2. The 2nd term measures the \textit{covariance} between \( \frac{dQ}{dP} \) and the average future short rate, capturing the risk premium on the interest rate risk.
3. The 3rd term measures convexity induced by short rate \textit{variation}.

   - If \( \int_t^T r_u du \) is normally distributed with variance \( V \), the convexity term is simply half of the variance \(- \frac{1}{2} V\).
   - More generally, it captures the sum of variance and higher-order cumulants, \(- \sum_{n=2}^{\infty} \frac{\kappa_n}{n!} \), where \( \kappa_n \) denotes the \( n \)th-cumulant of \( \int_t^T r_u du \).

\textit{All three components depend on projections from today \( t \) to the bond expiry \( T \).}
If we focus on the P&L over the near term (the next second/day/year), we can decompose the instantaneous bond return wrt its own yield movement via a Taylor expansion:

$$dB_t = \frac{\partial B_t}{\partial t} dt + \frac{\partial B_t}{\partial y} dy + \frac{1}{2} \frac{\partial^2 B_t}{\partial y^2} (dy)^2 + o(dt),$$

- $o(dt)$ denotes higher-order terms of $dt$ when yield moves diffusively.
- This local, near-term decomposition is local/particular to the bond and is tied to the movement of the yield on this particular bond.
- The decomposition is equally applicable (and equally operational) for coupon bonds (portfolios).
  - The analysis is no longer zero or short rate centric, but centric to the particular investment portfolio under investigation.
Expected return on bond investments

- Given the decomposition

\[ dB_t = \frac{\partial B_t}{\partial t} dt + \frac{\partial B_t}{\partial y} dy + \frac{1}{2} \frac{\partial^2 B_t}{\partial y^2} (dy)^2 + o(dt), \]

- We can write the expected return of the bond investment as

\[
\mathbb{E}^P_t \left[ \frac{dB_t}{B_t dt} \right] = \frac{\partial B_t}{B_t \partial t} + \frac{\partial B_t}{B_t \partial y} \mu_{t,y} + \frac{1}{2} \frac{\partial^2 B_t}{B_t \partial y^2} \sigma_{t,y}^2
\]

\[ = y_t - \mu_{t,y} \tau + \frac{1}{2} \sigma_{t,y}^2 \tau^2 \]

- \( \mu_{t,y} \) — the time-\( t \) level of the drift of the yield.
- \( \sigma_{t,y}^2 \) — the time-\( t \) level of the instantaneous variance.
- \( \tau \) and \( \tau^2 \) — value-weighted maturity (duration) and maturity squared of the coupon bond:

\[ \tau = \sum_j \frac{C_j e^{-y_t \tau_j}}{B_t} \tau_j, \quad \tau^2 = \sum_j \frac{C_j e^{-y_t \tau_j}}{B_t} \tau_j^2. \]

They are simply maturity and maturity squared for zeros.
Decomposing expected return on bond investments

\[
\mathbb{E}_t^P \left[ \frac{dB_t}{B_t dt} \right] = y_t - \mu_{t,y} \tau + \frac{1}{2} \sigma_{t,y}^2 \tau^2
\]

- **Expected bond return** comes from (1) carry, (2) rate expectation, and (3) convexity.
  1. Bonds with a higher yield have higher returns due to carry.
  2. Expected rate hike reduces expected return.
  3. Since bond price and yield exhibit a convex relation, random shaking of yield (without direction) leads to a positive return.
     - A duration neutral portfolio that is long longer-term bonds (convexity) is similar to a delta-neutral long options positions.

**Implications**

- If one has no view on direction, form duration-neutral portfolios.
- Long/short convexity based on view on volatility estimates
- Adjust carry trades for convexity.
Example: Arbitrage a parallel-shifting flat yield curve

\[ \mathbb{E}_t^P \left[ \frac{dB_t}{B_t dt} \right] = y_t - \mu_{t,y} \tau + \frac{1}{2} \sigma_{t,y}^2 \tau^2 \]

- Imagine a situation where
  - zero-coupon yields at long maturities (e.g., 10, 15, 30) are flat and move in parallel: \( y_t(\tau) = y_t \).
  - The yields move by substantial amounts, \( \sigma_t^2(\tau) = \sigma_t^2 \gg 0 \).

- We can form a self-financing, riskless portfolio that makes money:
  - Make the portfolio dollar neutral — Since the yield level is the same. Dollar-neutral leads to zero carry and hence self-financing.
  - Make the portfolio duration neutral — Since they move in parallel driving by the same risk source, duration-neutral cancels out the risk and hence makes the portfolio riskless.
  - Make the portfolio long convexity — positive expected profits.

- Example: Long $300 10-yr and $100 30-yr zeros, short $400 15-yr zero.
  - Dollar neutral: \( 300 + 100 - 400 = 0 \)
  - Duration neutral: \( \frac{3}{4} 10 + \frac{1}{4} 30 - 15 = 0 \).
  - Long convexity: \( \frac{3}{4} 10^2 + \frac{1}{4} 30^2 - 15^2 = 300 - 225 = 75 \).
Example: The payoff of a fly

\[ dB_t = \frac{\partial B_t}{\partial t} dt + \frac{\partial B_t}{\partial y} dy + \frac{1}{2} \frac{\partial^2 B_t}{\partial y^2} (dy)^2 + o(dt) \]

\[ = y_t B_t dt - \tau B_t dy + \frac{1}{2} \tau^2 B_t (dy)^2 + o(dt) \]

\[ dFly_t = 0 + 0 + \frac{1}{2} \left[ \sum_j (w_j \tau_j^2) \right] (dy)^2 + o(dt) \]
No dynamic arbitrage pricing on bond investments

Given the P&L decomposition on the bond investment,

\[ dB_t = \frac{\partial B_t}{\partial t} dt + \frac{\partial B_t}{\partial y} dy + \frac{1}{2} \frac{\partial^2 B_t}{\partial y^2} (dy)^2 + o(dt), \]

Take expectation under \( \mathbb{Q} \), and set the instantaneous expected return to \( r_t \) by *no dynamic arbitrage* (NDA):

\[
\mathbb{E}^\mathbb{Q}_t \left[ \frac{dB_t}{B_t dt} \right] = r_t = \frac{\partial B_t}{B_t \partial t} + \frac{\partial B_t}{B_t \partial y} \mu^\mathbb{Q}_{t,y} + \frac{1}{2} \frac{\partial^2 B_t}{B_t \partial y^2} \sigma^2_{t,y}
\]

NDA leads to a simple pricing relation for the long bond yield:

\[ y_t = r_t + \mu^\mathbb{Q}_{t,y} \tau - \frac{1}{2} \sigma^2_{t,y} \tau^2. \]

The fair value of the yield spread \( (y_t) \) on the bond investment is determined by its *current* risk-neutral drift \( (\mu^\mathbb{Q}_{t,y}) \) and volatility \( (\sigma_{t,y}) \) estimates.
Bond pricing based on *local, near-term dynamics*

\[
y_t = r_t + \mu_{t,y} \tau - \frac{1}{2} \sigma_{t,y}^2 \tau^2
\]  

(1)

- **Local**: The fair valuation of the bond investment in (1) does not depend on short-rate dynamics, but only depend on the *dynamics of its own* — the dynamics of the yield in consideration.
  - The dynamics of the short rate, or any other rates, do not enter the pricing relation.

- **Near-term**: The pricing of the yield does not even depend on its own full dynamics, but only depends on the current level of the drift and volatility.
  - The drift \( \mu_{t,y} \) and volatility \( \sigma_{t,y} \) can each follow some stochastic process, and/or depend on other rates/economic state variables ...
  - None of these enter into the pricing relation.

- **Estimates, no questions asked**: One can bring in forecasts/estimates on volatility, risk premium/rate prediction, and examine their implications on the yield (curve).
  - The estimates can come from any model assumptions, any algorithms, allowing maximum flexible cross-field collaboration.
Different frameworks serve different purposes

**Classic DTSM**
- Full short rate dynamics prices bond of all maturities.
- Good to maintain cross-sectional consistency across the whole curve.
- Hard to reconcile long rates with *actual* short rate dynamics.
- Better suited to construct smooth curves with cross-sectional consistency.

**New framework**
- Each yield is priced according to its own near-term dynamics.
- Good to specify the most relevant dynamics for the pricing.
- Hard to maintain cross-sectional consistency across all bonds.
- Better suited to analyze specific bond (portfolios) and connect to their real, current behaviors.
Decomposing long-bond returns with no rate predictions

- It is difficult to predict long rate movements. So we start by assuming random walk on floating (constant maturity) long rates:

\[ dy_t(\tau) = \sigma_t(\tau) dW_t^\mathbb{P}. \]

- A risk-neutral drift is induced by market pricing of bond risk \((-dW_t)\):

\[ dy_t(\tau) = \lambda_t \sigma_t dt + \sigma_t(\tau) dW_t^\mathbb{Q}. \]

- Market price of interest rate tends to be negative, leading to positive market price of bond risk \((\lambda_t)\) on average.

- The risk-neutral drift of the \textit{fixed-expiry} rates is further adjusted by the local shape of the yield curve ("sliding"):

\[ \mu_t^\mathbb{Q} = \lambda_t \sigma_t - y_t'(\tau). \]

- The pricing is based on the yield dynamics of a fixed contract, but it is easier to model/estimate floating rate dynamics (e.g., 10-yr rate).
Plugging the no-prediction assumption into the pricing relation leads to

\[ \frac{\partial [y_t \tau]}{\partial \tau} = r + \lambda_t \sigma_t(\tau) \tau - \frac{1}{2} \sigma_t^2(\tau) \tau^2. \]

For zeros, \( \frac{\partial [y_t \tau]}{\partial \tau} = f(\tau) \) is the instantaneous forward rate.

Define instantaneous volatility weighted duration and convexity as

\[ d_t = \sigma_t(\tau) \tau, \quad c_t = \sigma_t^2(\tau) \tau^2. \]

Integrate

\[ y_t = r + \lambda_t D_t - \frac{1}{2} C_t, \]

with \( D \) and \( C \) denoting the integrated duration and convexity

\[ D_t \equiv \left[ \frac{1}{\tau} \int_0^\tau d_t(s)ds \right], \quad C_t \equiv \left[ \frac{1}{\tau} \int_0^\tau c_t(s)ds \right] \]

What matters is not just sensitivity \((\tau)\), but also volatility.

In absence of prediction, risk premium drives the long rate up, convexity drives the long rate down.
We can extract bond risk premium from long yield and yield volatility:

\[ \lambda_t = \frac{y_t - r_t + \frac{1}{2} C_t}{D_t} \]

- Long rates \((y_t)\) and financing cost \((r_t)\) are directly observed.
- Variance term structure \(\sigma_t(\tau)\) can be estimated using recent history — making use of information from the reasonably accurate variance estimators.

We can then examine whether the ex-ante risk premium predicts ex post bond excess return, without ever the need to fit a predictive regression.
Empirical analysis: Data

- Data: US and UK swap rates 1995.1.3-2016.5.11, 5378 business days
  - Based on 6-month LIBOR Maturity, 2,3,4,5,7,10,15,20,30
  - Extended maturity since
    - US: 2004/11/12 for 40 & 50 years
    - UK: 1999/1/19 for 20 & 30 years, 2003/08/08 for 40 & 50 years

- Stripped Treasury zero rates for robustness check
Swap rate variance term structure estimators

Estimate variance $\sigma_t^2$ on each floating swap rate series with a 1y rolling window.

- Long rates vary as much as, if not more, than short rates.
The market price of bond risk extracted from different rates are similar in magnitude and move together:
  - Over the common sample, the cross-correlation estimates among the different $\lambda_t$ series average 99.67% for US, and 98.76% for UK.
  - The evidence supports a one-factor structure for the bond risk premium, as in Cochrane & Piazessi.
  - In the US, market price of risk approached zero in late 1998, 2000, and 2007, but tended to be high during recessions.
  - In the UK, the market price became quite negative during 1998 and 2007-2008.
Correlation between ex ante risk premium ($\gamma_t \sigma_t^m$) and ex post excess returns on each par bond, with the average denoting the correlation between the average risk premium and the average bond excess return over the common sample period.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizon: 6-month</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.31</td>
<td>0.28</td>
<td>0.26</td>
<td>0.24</td>
<td>0.27</td>
<td>0.26</td>
<td>0.29</td>
</tr>
<tr>
<td>UK</td>
<td>0.18</td>
<td>0.22</td>
<td>0.22</td>
<td>0.18</td>
<td>0.19</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td><strong>Horizon: One year</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.36</td>
<td>0.36</td>
<td>0.34</td>
<td>0.31</td>
<td>0.31</td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td>UK</td>
<td>0.36</td>
<td>0.42</td>
<td>0.40</td>
<td>0.32</td>
<td>0.33</td>
<td>0.37</td>
<td>0.39</td>
</tr>
</tbody>
</table>

- Paradoxically, the assumption of no prediction on long-dated swap rates lead to significant prediction on bond excess returns.
- The predictors (risk premium) are generated based purely on a variance estimator and the current slope of the yield curve, without estimating predictive regressions.
Predicting stock returns with market price of bond risk

Correlation between ex ante market price of bond risk and ex post excess returns on S&P 500 index and FTSE 100 index, respectively

<table>
<thead>
<tr>
<th>Maturity</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizon: 6-month</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPX</td>
<td>0.10</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.20</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td>FTSE</td>
<td>0.22</td>
<td>0.26</td>
<td>0.27</td>
<td>0.34</td>
<td>0.35</td>
<td>0.36</td>
<td>0.34</td>
</tr>
<tr>
<td>Horizon: One year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPX</td>
<td>0.17</td>
<td>0.18</td>
<td>0.17</td>
<td>0.17</td>
<td>0.29</td>
<td>0.28</td>
<td>0.30</td>
</tr>
<tr>
<td>FTSE</td>
<td>0.28</td>
<td>0.31</td>
<td>0.33</td>
<td>0.39</td>
<td>0.39</td>
<td>0.41</td>
<td>0.39</td>
</tr>
</tbody>
</table>

- The market price of bond risk reflects general market sentiment on risk attitude, which also seems to show up in the stock market.
- The market price of risk inferred from longer term rates generates stronger predictability, potentially due to better fitting of the no-prediction assumption.
Formulating rate expectation

If we can predict rate movement, we can further enhance bond return prediction:

- EH uses the yield curve slope to predict short-rate movements.
  - We can at least perform convexity adjustment to generate a more informative slope:
    \[ AS_t = y_t^L - r_t + \frac{1}{2} C_t^L \]
- EH does not say anything about long-rate prediction, we propose to predict long-rate movements based on anticipated central bank action, which we capture using the yield curve slope at the short end.
  \[ CB_t = y_t^2 - y_t^1 \]
- Anticipated monetary tightening reins in future inflation, ultimately bringing down long-dated rates (Rotemberg & Woodford (1997)).
- Rate prediction at each maturity reflects the combined effects of the two:
  \[ \mu_t(\tau) = \frac{1 - e^{-\kappa\tau}}{\kappa\tau} (AS_t + CB_t) - CB_t, \quad \kappa = 1 \]

This is just a starting point...
Predicting future swap rate changes

<table>
<thead>
<tr>
<th>Maturity</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizon: 6-month</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.19</td>
<td>0.23</td>
<td>0.32</td>
<td>0.26</td>
<td>0.24</td>
<td>0.23</td>
<td>0.23</td>
<td>0.22</td>
<td>0.21</td>
<td>0.31</td>
<td>0.31</td>
<td>0.25</td>
</tr>
<tr>
<td>UK</td>
<td>0.20</td>
<td>0.21</td>
<td>0.24</td>
<td>0.23</td>
<td>0.21</td>
<td>0.18</td>
<td>0.18</td>
<td>0.16</td>
<td>0.10</td>
<td>0.08</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td><strong>Horizon: One year</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.33</td>
<td>0.36</td>
<td>0.39</td>
<td>0.28</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.25</td>
<td>0.32</td>
<td>0.32</td>
<td>0.30</td>
</tr>
<tr>
<td>UK</td>
<td>0.34</td>
<td>0.34</td>
<td>0.38</td>
<td>0.36</td>
<td>0.33</td>
<td>0.30</td>
<td>0.31</td>
<td>0.28</td>
<td>0.20</td>
<td>0.19</td>
<td>0.28</td>
<td>0.30</td>
</tr>
</tbody>
</table>

- Our formulation generates strong prediction on swap rate changes at both short and long maturities.
Enhancing bond return predictions with rate prediction

<table>
<thead>
<tr>
<th>Maturity</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizon: 6-month</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.20</td>
<td>0.41</td>
<td>0.41</td>
<td>0.38</td>
<td>0.36</td>
<td>0.34</td>
<td>0.32</td>
<td>0.30</td>
<td>0.29</td>
<td>0.35</td>
<td>0.35</td>
<td>0.37</td>
</tr>
<tr>
<td>UK</td>
<td>0.13</td>
<td>0.28</td>
<td>0.33</td>
<td>0.32</td>
<td>0.31</td>
<td>0.29</td>
<td>0.30</td>
<td>0.27</td>
<td>0.22</td>
<td>0.23</td>
<td>0.25</td>
<td>0.28</td>
</tr>
<tr>
<td>Horizon: One year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.32</td>
<td>0.41</td>
<td>0.40</td>
<td>0.40</td>
<td>0.42</td>
<td>0.41</td>
<td>0.40</td>
<td>0.38</td>
<td>0.35</td>
<td>0.39</td>
<td>0.38</td>
<td>0.42</td>
</tr>
<tr>
<td>UK</td>
<td>0.23</td>
<td>0.36</td>
<td>0.42</td>
<td>0.43</td>
<td>0.44</td>
<td>0.44</td>
<td>0.46</td>
<td>0.43</td>
<td>0.37</td>
<td>0.35</td>
<td>0.41</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Adding rate prediction ($\mu(\tau) - y'(\tau)$) to risk premium further enhances the bond excess return prediction.
Predictive correlation in graphic representation

Universal rate and excess return predictive power across short and long maturities
Similar results from stripped Treasury spot rates over a longer period (1986-
Concluding remarks

- We propose a new modeling framework that is particularly suited for analyzing returns on a bond or bond portfolio.
- The framework does not try to model the full dynamics of an instantaneous short rate, but focus squarely on the behavior of the bond yield in question.
- It does not even ask for the full dynamics specification of this bond yield, but only needs estimates of its current expectation, risk premium, and volatility.
  - It can readily accommodate results from other models and algorithms.
- The model framework decomposes each yield into three components: expectation, risk premium, and volatility.
  - One can estimate the volatility from historical time series, or infer it from the curvature of the yield curve, or interest rate options.
  - Separating risk premium from expectation can be a very challenging, but very fruitful endeavor.
- We show that we can predict bond excess returns, without running predictive regressions, even by assuming no prediction on interest rates.
- Our first take on rate prediction generates strongly positive results, and further enhances bond return prediction.