This paper examines the implications of offering households the choice between traditional fully-amortizing mortgages that require a substantial down payment (CPMs) and mortgages that involve lower initial payments (LIPs) because of a lower down payment requirement and a non-traditional amortization structure. I examine the issue in an equilibrium model in which households make decisions as if they discount hyperbolically rather than exponentially. I find that expected lifetime utility of a newly born household is higher in the economy that allows households access to LIPs as well as CPMs. Allowing households access to LIPs exacerbates rather than mitigates the undersaving problem, however.
1 Introduction

Mortgages that have lower initial payments than traditional fully amortizing mortgages have been at the heart of the recent foreclosure crisis. The mortgage products with the highest default rates have been those that involve no down payment or very small down payments and those that involve no amortization or negative amortization for a short period of time after origination. This has spurred calls for more regulation of the mortgage products available to consumers and the creation of a national Consumer Financial Protection Agency (see, e.g., Bar-Gill and Warren [2008]). While it is almost certain that limiting access to low initial payment mortgages (LIPs) would reduce foreclosures rates, there has been little economic analysis of the costs and benefits of allowing households access to LIPs.

If households make decisions as if they have standard, time-consistent preferences and households fully understand their mortgage contracts, expanding the household’s choice set to include LIPs is welfare-enhancing since households always choose the mortgage that gives them the highest utility. Financial intermediaries accurately price the higher default risk these mortgages entail such that, in the absence of externalities, there is no welfare loss from the higher default rate. However, economists increasingly recognize that household behavior is inconsistent with preferences that assume a constant intertemporal discount rate. Rather, the evidence suggests that households behave as though they discount the immediate future much more heavily than the distant future.\(^1\)

In this paper, I examine the welfare implications of offering households the choice between traditional 30 year mortgages requiring substantial down payments (constant payment mortgages, hereafter CPMs) and LIPs in a general equilibrium model in which households’ preferences are characterized as making decisions as though they were quasi-hyperbolic discounters. Households in my model are sophisticated quasi-hyperbolic discounters such that, as Krusell, Kuruscu, and Smith (2010) show, preferences are a special case of the temptation preferences of Gul and Pesendorfer (2001). I evaluate welfare as though the household discounted geometrically consistent with the Gul and Pesendorfer (2001) framework.

\(^1\)See, for example, Angeletos, Laibson, Repetto, Tobacman, and Weinberg (2001), Laibson, Repetto, and Tobacman (2007), Skiba and Tobacman (2008), Paserman (2008), Fang and Silverman (2009), and DellaVigna (2009).
Features of LIPs that improve welfare in an exponential setting will continue to offer some welfare benefits in a hyperbolic setting. Most importantly, households that expect higher future income can use an LIP to buy a home earlier in their life cycle or a more appropriately sized home when they do choose to buy. Indeed, Gerardi, Rosen, and Willen (2010) empirically show that innovation in the mortgage market, including the introduction of non-standard mortgage products such as LIPs, improves consumers’ ability to align their housing consumption with future income.

Hyperbolic households may, however, be tempted to enter home ownership in situations in which they cannot afford it or to buy a home that is bigger than the one they can afford. Such households may subsequently default on their mortgages or struggle to make payments such that they would be better off in a world in which they did not have the option of an LIP. There may also be benefits from LIPs in a hyperbolic economy that are not present when households discount the future exponentially. In particular, when households suffer from temptation preferences, home ownership offers them a commitment mechanism enabling them to save more adequately for their retirement (Laibson, 1997). LIPs enable more households to become home owners and access this commitment device; this effect may improve welfare.

To study the net effect of allowing households access to LIPs, I calibrate a general equilibrium life cycle model of mortgage choice. The model is an endowment economy with an exogenous stock of housing and riskless interest rate. Mortgage rates are endogenous. The model has elements similar to that of Corbae and Quintin (2010) and Garriga and Schlagenhauf (2009). The key difference between these models and mine is that I characterize the households intertemporal discounting as hyperbolic while households in Corbae and Quintin (2010) and Garriga and Schlagenhauf (2009) discount the future geometrically. Although the focus of their analysis is not on the welfare implications of LIPs, Corbae and Quintin (2009) do compute welfare in a world with and without LIPs. They find welfare is higher in a world with LIPs.

The main finding of the paper is that steady state welfare is higher when households have access to LIPs and CPMs than when they have access only to CPMs. In the benchmark parameterization, the expected lifetime utility of a newly born household is 0.1% higher in
the economy with LIPs. This is largely because LIPs raise the home ownership rate among young households and retirees. It is not because allowing households access to LIPs increases the savings rate. For most age groups, savings are in fact lower, both in absolute terms and relative to what exponential households would save, when households are able to finance home ownership with an LIP. The intuition behind this result is that, when households can only become home owners by making a substantial down payment and owner-occupying is preferred to renting, households begin saving earlier. When a down payment is no longer required, households save less because they can be home owners with little to no saving. Although allowing households access to LIPs raises the expected lifetime utility at birth for all income groups, the welfare gains are highest for households born as the highest income earners. The gain to a household born as a low or a middle income earner is less than half that of a household born as a high income earner.

The welfare calculations in this paper do not incorporate any externalities from a higher foreclosure or home ownership rate. The foreclosure and home ownership rates are substantially higher in the economy with LIPs. In the benchmark economy in which households can borrow using only CPMs, the three year foreclosure rate is 4.5% while, in the economy with both CPMs and LIPs, the three year foreclosure rate is 9.5%. The welfare calculations in the model do not assume that foreclosures entail any negative externalities. While evidence (e.g., Campbell, Giglio, and Pathak, 2010) suggests that foreclosures lower the prices of nearby homes not in foreclosure, it is unclear how to translate such price declines into aggregate welfare and I have not attempted to do so here. The welfare gains would shrink if there are large externalities associated with foreclosures. The home ownership rate rises from 68% to 79% as a result of the introduction of LIPs. If home ownership entails positive externalities, introducing LIPs would result in larger welfare gains than I present here. In interpreting the results, it is also important to bear in mind that I compare welfare only in the two steady states; I do not study the economy as it transitions to the new steady state with LIPs.

This paper is the first that I know of to study the implications of hyperbolic discounting for mortgage choice. In addition to Corbae and Quintin (2010) and Garriga and Schlagenhauf (2010), earlier work by Chambers, Garriga, and Schlagenhauf (2009) and Piskorski and Tschitsy (2010) has studied the choice between a CPM and LIPs. Chambers, Garriga, and
Schlagenhauf (2009) show that, when consumers discount the future exponentially, there is strong age dependence in mortgage choice. I also find strong age dependence in mortgage choice in a hyperbolic setting. Piskorski and Tschitsyi (2010) show that the key features of an option ARM, a very popular kind of LIP, are consistent with the optimal mortgage contract when consumers discount the future exponentially. A much longer literature (see, for example, Campbell and Cocco, 2003 and Koijen, Van Hemert, and Van Nieuwerburgh and the references therein) studies consumers’ choices between adjustable and fixed rate mortgages. I study only fixed rate mortgages in this paper.

The remainder of the paper proceeds as follows: The next section presents the model used to study mortgage choice and the equilibrium in the two economies. Section 3 presents the benchmark parameterization of the model. The results are in Section 4. Section 5 concludes and suggests directions for future research.

2 The Model

I study an overlapping generations endowment economy in which households live for at most $J$ periods of which $J_{RET} < J$ are spent “working”. Each period, the household makes decisions regarding its tenure, assets, and mortgage choice. Similar to Campbell and Cocco (2010) and Corbae and Quintin (2010), there is no option to refinance to keep the model computationally tractable. If the household chooses to rent, it must rent a house of size $h_1$. If a household transitions to own its home, it chooses what size of home to buy, and selects between a CPM and an LIP. The mortgage rate for each mortgage type is computed as the mortgage rate that makes the expected present value of the mortgage equal to the mortgage balance at origination. There are a small number of home sizes, a decision made to reduce the computation required to solve the model.

There is no intentional bequest motive in the model such that all bequests are accidental. This is consistent with the empirical evidence in Hurd (1989); Hurd finds that most bequests are accidental and that the intentional bequest motive is quite small. Not including an intentional bequest motive in the model also simplifies the computation since the policy function in the last period of life is known such that the optimal policy functions for other
periods of life can be solved for recursively. Other life cycle models solved recursively by assuming all bequests are accidental include Huggett (1996), Rios-Rull (1996), and Nakajima (2010).

The housing stock is fixed at $H_K$ and depreciates at rate $\delta$ each period. If a financial intermediary is forced to foreclose on a borrower, it incurs a cost $\chi$ (a percentage of the home value at the time of foreclosure) to rehabilitate the home to the size it was at the time of foreclosure. Households face idiosyncratic income and home price risk. Stochastic home prices are represented by assuming the home will decrease or increase in size with exogenously given probability; the house size follows a Markov chain. The stochastic home price shocks are such that the size of the aggregate housing stock does not change over time.

The timing in the model is as in Corbae and Quintin. At the beginning of each period, the household learns its income for that period and, if it is an owner, whether its home has appreciated or depreciated in value. The household then makes its tenure, housing, mortgage termination, mortgage product, and consumption decisions. If the household chooses to enter into a new mortgage contract, it makes the down payment at the start of the period. At the end of the period, the household receives its income, consumes, and makes rent or mortgage payments. As in Corbae and Quintin’s benchmark specification, mortgages are non-recourse in the sense that the lender cannot seize assets other than the house if the borrower defaults on the mortgage.

2.1 Households

Households that choose to own a home take on a $T$ period mortgage. The household’s state vector is \( \{j, a, H, h, n, h_O, \kappa, y\} \) where \( j \in \{0, ..., J - 1\} \) represents the household’s age, \( a \) represents the household’s assets, \( H \in \{0, 1\} \) is the household’s tenure, \( h \in \{h_1, h_2, h_3\} \) is the house size, \( n \in \{0, ..., T\} \) is the number of periods the household has remaining on in its current mortgage, and \( h_O \in \{h_2, h_3\} \) denotes the house size that the household chose at origination. As in Gervais (2002), Corbae and Quintin (2010), and Nakajima (2010), the smallest home a household can buy is $h_2$ rather than $h_1$. Income, $y$, is exogenously given and follows a Markov process. \( \kappa \in \Omega = \{CPM, LIP\} \) represents the household’s mortgage type.
I interpret hyperbolic discounting a special case of temptation preferences (Krusell, Kuruscu, and Smith, 2010) and thus avoid the multiple selves’ problem of computing welfare. Households therefore make decisions discounting the next period by $\beta \alpha$, $\beta \leq 1$, but their actual welfare is computed using geometric discounting, i.e., $\beta = 1$. For $\beta < 1$, households are sophisticated hyperbolic discounters in that they are aware of their temptation problem.

The household aged $j$ that enters the period with assets $a$, tenure $H$, house size $h$, $n$ periods remaining on its mortgage, a mortgage $\kappa$, and income $y$ thus chooses its tenure, housing, mortgage, and assets to maximize

$$(1) \quad u(c, h', H') + \beta \alpha \pi_j EV (j + 1, a', y', H', h', n + 1, h'_O, \kappa', y')$$

where

$$V(j, a, H, h, n, h_O, \kappa, y) = \max_{\{a', h', \kappa, 1_d\}} \begin{cases} u(c, h', H') + \\ \alpha \pi_j EV (j + 1, a', H', h', n, h'_O, \kappa', y') \end{cases},$$

$$n = (1 - 1_B - 1_D) \max (0, n - 1) + 1_B (T - 1) + 1_D 0,$$

The indicator function $1_B$ takes on a value of one if the household buys a new home in that period, and hence takes on a new mortgage, and 0 otherwise. The indicator $1_D$ takes on a value of 1 if the household chooses to default in that period, 0 otherwise. $\pi_j$ is the probability that a household that has survived to age $j$ survives to age $j + 1$.

For a household that starts the period as a renter ($H = 0$), the constraint on (1) is

$$(2) \quad c + a' = y + (1 + r) (a - H' 1_{CPM} \nu q h') - H' (p_0 (\kappa) + \delta h') - (1 - H') R h_1$$

where $q$ is the price per unit of housing, $p_n (\kappa)$ is the payment due on a mortgage of type $\kappa$ in period $n$, $\delta$ is the depreciation rate, and $R$ is the rental rate. The indicator function $1_{CPM}$ takes on a value of 1 if the household uses a CPM to finance home ownership and 0 otherwise such that (2) captures the fact that the household need only make a down payment if it both becomes an owner ($H' = 1$) and chooses a CPM.

If the household starts the period as an owner ($H = 1$), it decides whether to default on
its mortgage and whether to sell its home. If the household decides to default, \( H' = 0 \). Let \( 1_B \) be an indicator that takes on a value of one if the household chooses to buy a new home. The constraints on (1) if \( H = 1 \) are thus

\[
\begin{align*}
c + a' &= y + 1_B [q (1 + \sigma) h - b_n (\kappa)] + (1 + r) (a - H' \nu 1_B 1_{CPM} q h') \\
&\quad - H' [(1 - 1_B) p_n (\kappa) + 1_B p_0 (\kappa') + \delta h'] - (1 - H') R h_1, \\
(3) \\
H' &\equiv 0 \text{ if } 1_D = 1, \\
(4) \\
\kappa' &= \begin{cases} 
\{CPM, LIP\} & \text{if } 1_B = 1 \\
\kappa & \text{if } 1_B = 0 \\
\emptyset & \text{if } H' = 0 
\end{cases}, \text{ and} \\
(5) \\
h'O &\equiv h_O \text{ if } 1_B = 0.
\end{align*}
\]

where \( \sigma \) represents the transactions cost of selling a home.

The interpretation of (3) is that if the household chooses to default on its mortgage, it must rent for that period. Equations (4) and (5) represent the fact that the household cannot refinance. Equation (4) says that the household can only enter into a new mortgage contract when it buys a new home and \( \kappa \) is null if the household chooses to rent. Equation (4) is mechanical: it says merely that the household’s state variable for the home size at origination does not change if the household does not buy a new home.

2.1.1 The Benefits of Home Ownership

In this framework, there are two benefits of owning a home relative to renting. First a premium for owning relative to renting is built into the felicity function through its dependence on tenure chosen in that period, \( H' \). In this respect, I follow Hu (2005), Chatterjee and Eyigungor (2009), and Corbae and Quintin (2010). Households can only rent a house of size \( h_1 \); if a household wants to consume housing services associated with a house of size \( h_2 \) or \( h_3 \), it must be a home owner. I follow Corbae and Quintin (2010) in this respect. These assumptions are important to generate home ownership rates similar to what we observe in the data. The assumptions are important for understanding the results regarding welfare.
2.2 Financial Intermediary

As in Corbae and Quintin (2010), the financial intermediary is an infinitely lived company that accepts household savings and makes mortgage loans. It earns the exogenously given rate \( r \) on savings. It also holds a stock of housing capital which it can rent out at rate \( R \) per unit or sell to households as owner-occupied housing. It incurs the maintenance cost \( \delta \) on its housing stock and a cost \( \chi q_h \) of rehabilitating housing units it acquires through foreclosure. In equilibrium, it must make zero profits. Since the value of a home must be equal to the present value of future rents, in equilibrium each unit of housing rents at rate \( R = rq + \delta \) where \( q \) is the price per unit of housing.

2.3 Mortgage Choice

A mortgage contract, denoted by \( \kappa \), may be a traditional constant-payment, fixed-rate mortgage (CPM) or an LIP mortgage. A CPM entails a down payment of \( \nu \) percent of the value of the home, payments designed to fully amortize the mortgage over \( T \) periods, and an interest rate \( r_{CPM} \). If the household takes out an LIP mortgage, it makes no down payment and pays only the interest on the mortgage during the first \( n_{LIP} \) periods. The principal on an LIP is amortized over the remaining \( T - n_{LIP} \) periods. An LIP mortgage is characterized by an interest rate \( r_{LIP} \). The equilibrium concept in this paper lies is the same as that in Garriga and Schlagenhauf (2009) and Athreya (2002): the equilibrium is a pooling equilibrium where the financial intermediary offers the same interest rate to all borrowers in a particular product category. In Corbae and Quintin (2010), the mortgage interest rate is specific to a single household’s asset, income, and housing combination such that it represents financial intermediaries assessing the risk of individual households.

The main reason I choose the pooling equilibrium concept is computational. Because the model here is a life cycle model and households can transition between renting and owning at any point in time, unlike the infinite horizon model in Corbae and Quintin (2010), it is computationally very challenging to construct equilibrium interest rates in this setting. Introducing interest rates specific to each individual is unlikely to qualitatively change the predictions of the model. Indeed, despite having different equilibrium concepts, household
leverage has similar implications in Corbae and Quintin (2010) and Garriga and Schlagenhauf (2009).

2.4 House Prices

As in Corbae and Quintin (2010), stochastic house prices are captured by households facing an exogenously given probability that their house changes in size and, hence, value. In particular, home owners that currently a home of size $h_2$ face a probability $\lambda$ that the home will increase to size $h_3$ and a probability $\lambda$ that the home will decrease to size $h_1$. Home owners that currently own a home of size $h_3$ face a probability $\lambda$ that the house will depreciate to size $h_2$. A home owner that owns a home of size $h_1$ faces a probability $\lambda$ that the home will increase to a home of size $h_2$. Rental units, all of which are of size $h_1$, do not change in size.

2.5 Steady State Equilibrium and Computation

In equilibrium, lenders make zero pro… ts. This implies that the contract rates $r_{CPM}$ and $r_{LIP}$ are the rates that equate the expected present value of the mortgage to the loan balance at origination. The opportunity cost of the lender’s funds is the riskless interest rate, $r$; it costs lenders $\phi$ to service the mortgage rate. Lenders thus compute the present value of the mortgage rate by discounting the expected cash ‡ ows by $r + \phi$. The equilibrium price of a unit of housing in terms of units of the consumption good is the one that clears the market for homes, i.e.,

$$\sum_{1_H=0} h_1 + \sum_{1_H=1, h=h_1} h_1 + \sum_{1_H=1, h=h_2} h_2 + \sum_{1_H=1, h=h_3} h_3 = H_K$$

To solve for the equilibrium, I simulate the model over 20,000 households for 1,000 periods for different combinations of interest rates. I drop the first 100 periods as burn-in iterations. For each set of interest rates, I then compute the average present value of a mortgage contract $\kappa$. An equilibrium is a set of interest rates $\{r_{CPM}, r_{LIP}\}$ such that the average present value of a mortgage contract $\kappa$ is equal to the size of the mortgage at origination. With a large
enough number of households and periods, the average present value of the mortgage contract will also be the expected present value of the mortgage contract.

I know of no theory that guarantees either existence or uniqueness of the equilibrium in this framework. While Krusell and Smith (2008) state that the household’s problem has a unique equilibrium in a finite horizon setting such as the current model, that guarantees only that, conditional on a particular interest rate, the household’s problem has a unique solution; there is nothing that guarantees that with many agents the equilibrium interest rate will be unique. As a result, I use grid search to find the equilibria in the model. While this process is time consuming, it is greatly accelerated by the use of parallel processing. In particular, I use the openMP framework to find all equilibria. The interest rate grid starts at \( r + \phi \) and increases to 25\% in increments of 25 basis points.

3 Parameterization

Table 1 summarizes the parameterization of the model. Several of the parameters are fixed based on empirical estimates. The remaining parameters are used to ensure that the model matches certain moments in the data. I choose these parameters to match historical home-ownership rates, average mortgage rates, average foreclosure rates, and loan-to-income ratios at origination. I target a home-ownership rate of 67\%, a three-year real mortgage rate of 16\%, a three-year foreclosure rate of 4.5\%, and a loan-to-income ratio at origination of 91\%.

3.1 Demographics

A period in the model corresponds to 3 years. The household is born at age 25, such that \( j = 0 \) corresponds to a chronological age of 25. The household lives until at most 85 years of age corresponding to \( J = 20 \). The age at which the household retires, \( J^{RET} \), is 13 such that the household retires at a chronological age of 64. I take the survival probabilities, \( \{\pi_j\}_{j=0}^{J-1} \), from Arias et al. (2008).
Table 1: Benchmark Parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>3-Yr Discount Factor</td>
<td>0.857</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Short Term Discount Factor</td>
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</tr>
<tr>
<td>$\nu$</td>
<td>Down Payment Share for CPMs</td>
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<tr>
<td>$r$</td>
<td>3-Yr Real Risk-Free Rate</td>
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<tr>
<td>$\psi$</td>
<td>Consumption Share</td>
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</tr>
<tr>
<td>$\theta$</td>
<td>Owner-occupied Premium</td>
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</tr>
<tr>
<td>$h_1$</td>
<td>Small House Size</td>
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</tr>
<tr>
<td>$h_2$</td>
<td>Mid-Size House Size</td>
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</tr>
<tr>
<td>$h_3$</td>
<td>Large House Size</td>
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</tr>
<tr>
<td>$\lambda$</td>
<td>House price shock probability</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>Servicing Cost</td>
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</tr>
<tr>
<td>$\chi$</td>
<td>Foreclosure Discount</td>
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<tr>
<td>$\delta$</td>
<td>Housing depreciation</td>
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</tr>
<tr>
<td>$\rho$</td>
<td>Selling costs</td>
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</tr>
<tr>
<td>$T$</td>
<td>Mortgage Contract Term</td>
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<tr>
<td>$J$</td>
<td>Maximum Life Span</td>
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<tr>
<td>$J_{ret}$</td>
<td>Retirement Age</td>
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</tr>
<tr>
<td>$n_{LIP}$</td>
<td>Interest Only Period for LIPs</td>
<td>1</td>
</tr>
</tbody>
</table>

### 3.2 Income

I assume that the income process during working years follows an AR(1) process with a quadratic polynomial in age. That is, the process for income is

\[ y_t = \rho y_{t-1} + \gamma_1 age_t + \gamma_2 age_t^2 + \varepsilon_t \]

where $\varepsilon_t$ has variance $\sigma^2_{\varepsilon}$. I estimate the parameters of (6) using triennial PSID data on earnings from 1967 to 1992. I estimate the model using all heads of households between the ages of 25 and 64 that have positive labor income in the year prior to the survey, that have only high school degrees, and that are not part of the Survey of Economic Opportunities sample. The measure of income is all labor income. I convert income for all years into 1983$ prior to estimation using the CPI (all items). This estimation procedure yields $\hat{\rho} = 0.76$, and $\hat{\sigma}^2_{\varepsilon} = 8817$. I approximate (6) with a three state Markov chain using the approach of Tauchen and Hussey (1991). After retirement, labor income is set to 60% of income in the last working
year following Cocco, Gomes, and Maenhout (2005) and Yao and Zhang (2005).

3.3 Preferences

The key parameter in the model is the short-term discount rate, $\beta$. I set this to 0.7 in the benchmark specification based on the estimates of Laibson, Repetto, and Tobacman (2007). In the same estimation, Laibson, Repetto, and Tobacman (2007) estimate the long-term annual discount rate to 0.95; I thus set $\alpha$ to 0.95$^3$.

The felicity function is

$$u(c, h, H) = \psi \ln c + (1 - \psi) \ln h + \theta 1_{h > h_1}$$

following Corbae and Quintin (2010). I set $\psi$ to 0.76 implying that renters spend 24% of their consumption expenditure on housing based on the estimates Davis and Ortalo-Magné (2009). There are no good estimates for $\theta$ such that I use $\theta$ to calibrate the model to match certain characteristics of the data.

3.4 Housing Costs

Based on the estimates of Campbell, Giglio, and Pathak (2010), I set $\chi$ to 0.25. I choose $\lambda$, the probability of an idiosyncratic house price shock, the house sizes, $h_1$, $h_2$, and $h_3$, and the mortgage servicing cost, $\phi$, to calibrate the model to match the key moments in the data. I set $T$, the mortgage term, to 10 such that mortgages have 30 year terms. For CPMs, households must make a 20% down payment such that $\nu = 0.2$. LIPs have one period of interest only payment such that $n_{LIP} = 1$ and the mortgage corresponds to something similar to a 3/27, a very popular subprime mortgage product. The risk-free rate, $r$, is 12%. Selling costs, $\rho$, are 8% of the value of the home as in Cocco (2004). I use $\delta$, the per period depreciation rate on housing, to calibrate the model to match particular moments in the data.
4 Equilibrium

4.1 Equilibrium with only CPMs

Columns 2 and 3 of Table 2 present equilibrium statistics regarding the model when the household has access only to CPMs. There is a unique equilibrium with positive home ownership. By construction, the home ownership rate, the mortgage rate, the foreclosure rate, and the average loan-income ratio are close to those in the data (column 1) when households exhibit hyperbolic discounting. Perhaps surprisingly, the home ownership rate when households discount the future exponentially ($\beta = 1$) is higher than when households discount the future hyperbolically; the home ownership rate in the hyperbolic economy is 68% while it is 79% in the exponential economy.

As Figure 1 illustrates, the aggregate home ownership rate is lower in the economy with hyperbolic discounting primarily because hyperbolic discounters often transition out of home ownership at fairly young ages (such as between ages 2 and 3) and become renters earlier into their retirement. The home ownership rate among households aged 19 periods (82 years chronologically) is only 50% in the hyperbolic economy while it remains at above 75% in the exponential economy owing to greater asset holdings by older households in that economy. The home ownership rate among households aged 0 to 2 periods is quite similar across the two economies.

In the hyperbolic economy, more home owners transition out of home ownership into renting by a foreclosure as evidenced by the foreclosure rate in the hyperbolic economy. The foreclosure rate in the hyperbolic economy is 4.5% while it is 2.8% in the exponential economy. The higher foreclosure rate in the hyperbolic economy in turn implies a slightly higher mortgage interest rate in that economy: the equilibrium mortgage interest rate in the exponential economy is 15.25% while it is 16% in the hyperbolic economy.

Home ownership is concentrated among higher income earners in both the hyperbolic and the exponential economies. In the hyperbolic economy, the home ownership rate among the lowest earning households is only 26% while it is 79% for middle income earners and 90% for high income earners. In the exponential economy, the home ownership rate among low earners is 53%, more than double that in the hyperbolic economy. Middle income
### Table 2: Steady State Equilibria

<table>
<thead>
<tr>
<th>Moment Data</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Only CPMs Available</td>
<td>Hyperbolic Discounting $\beta=0.7$</td>
<td>Exponential Discounting $\beta=1.0$</td>
<td>Hyperbolic Discounting $\beta=0.7$</td>
</tr>
<tr>
<td>3-Yr CPM Mortgage Rate</td>
<td>16.00%</td>
<td>16.00%</td>
<td>15.25%</td>
<td>15.25%</td>
<td>15.00%</td>
</tr>
<tr>
<td>3-Yr LIP Mortgage Rate</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>23.50%</td>
</tr>
<tr>
<td>Average Mortgage Rate</td>
<td>16.0%</td>
<td>16.0%</td>
<td>15.25%</td>
<td>15.25%</td>
<td>18.28%</td>
</tr>
<tr>
<td>Home Ownership Rate</td>
<td>67.0%</td>
<td>67.9%</td>
<td>79.3%</td>
<td>74.1%</td>
<td>84.6%</td>
</tr>
<tr>
<td>3-Yr Foreclosure Rate</td>
<td>4.50%</td>
<td>4.48%</td>
<td>2.80%</td>
<td>9.45%</td>
<td>4.82%</td>
</tr>
<tr>
<td>Average Loan-Income Ratio at Origination</td>
<td>0.91</td>
<td>0.90</td>
<td>1.26</td>
<td>0.87</td>
<td>0.96</td>
</tr>
<tr>
<td>Average Rent-Income Ratio</td>
<td>0.33</td>
<td>0.32</td>
<td>0.40</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>Expected Lifetime Utility of Age 0 Agent</td>
<td>68.13</td>
<td>68.26</td>
<td>68.19</td>
<td>68.33</td>
<td></td>
</tr>
<tr>
<td>Increase in Welfare with LIPs</td>
<td>0.10%</td>
<td>0.11%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 1: Home Ownership Rates by Age in the Hyperbolic and Exponential Economies (CPMs Only)

- **Hyperbolic**
- **Exponential**

**Note:** Home ownership rates are at start of period.
Figure 2: Average Net Worth to Income Ratios by Age in the Hyperbolic and Exponential Economies

Note: Net Worth to Income at Start of Period.
earners in the exponential economy have a home ownership rate of 87% and high income earners have a home ownership rate of 93%. The low home ownership rate among low income households is due to both the minimum size of owner occupied housing and the persistence of the income processes. The large difference in home ownership rates for low income households between the exponential and hyperbolic economies suggests that the down payment constraint is a major impediment for low income households; when households discount the future hyperbolically, low income households have a great deal of difficulty acquiring the down payment due to the undersaving problem.

Households in the hyperbolic economy save less as Figure 2, which shows the ratio of net worth to income by age, illustrates. Although the net worth to income ratio is always lower in the hyperbolic economy than in the exponential economy, the undersaving is particularly pronounced for older households. The sharp spike in the figure at the retirement age is because the denominator is income at that age which drops off steeply at retirement. The gap between net worth in the hyperbolic and exponential economies grows at retirement because households in the hyperbolic economy exit home ownership much earlier than households in the exponential economy. Without the required payments that home ownership requires, payments that increase home equity, hyperbolic households quickly deplete their assets. Most exponential households stay home owners until the very last period of life and only then liquidate their housing holdings.

Nevertheless, the expected lifetime utility of an age 0 household is not substantially lower in the hyperbolic economy than in the exponential economy. Welfare is only 0.2% lower in the hyperbolic economy than in the exponential economy. The small difference in lifetime expected utility of a newly born households masks differences in welfare across age groups. As Figure 3 shows, the average expected lifetime utility of retirees in the hyperbolic economy is 3 to 5% lower than that of retirees in the exponential economy. This is largely because it is in retirement that households in the hyperbolic economy face the consequences of their undersaving. Retirees in the hyperbolic economy would be better off if they were prevented from selling their home; it is giving into the temptation of selling their home that causes the reduction in average expected lifetime utility for middle-aged households and retirees.
Figure 3: Ratio by Age of Lifetime Expected Utility in Hyperbolic Economy to Lifetime Expected Utility in Exponential Economy (CPMs Only)

Note: Expected Lifetime Utility at Start of Period
4.2 Equilibrium with CPMs and LIPs

Column 4 of Table 2 presents the equilibrium solution to the model when the household has access to the LIP mortgage which, in the benchmark calibration, is a mortgage without any down payment and for which the payments are interest only for the first 3 years. When households discount the future hyperbolically, the home ownership rate rises by 11 percentage points, to 79.3%, from the equilibrium in which the household only has access to traditional CPMs. As Figure 4 illustrates, the home ownership rate is higher in the economy with LIPs largely because households enter into home ownership earlier. In the economy with only CPMs, no household has acquired the down payment required to become a home owner until age 2. Once the household is able to enter home ownership without a down payment, 73% of households are home owners by the start of period 1. The home ownership rate in the economy with CPMs and LIPs remains substantially above that in the economy with only CPMs until household reach about 9 periods of age. The home ownership rate for middle aged and elderly households is similar in the two economies.

In the hyperbolic economy, the introduction of LIPs does not raise the home ownership rate of all income groups, however. The home ownership rate of low income households in fact falls from 26% to 22% as a result of the introduction of LIPs. This is likely due to low income households choosing foreclosure more frequently after the introduction of LIPs. The home ownership rate of middle income earners rises from 79% to 91% as a result of introducing LIPs while that of high income households rises from 90% to 97%.

The equilibrium interest rate on CPMs falls by 100 basis points to 15.25% once households have access to LIPs. The reason that the interest rate on CPMs falls is that the households that were most likely to default in the economy with only CPMs now choose LIPs. As Figure 5 illustrates, it is primarily middle-aged households that opt for CPMs once LIPs are available. In the first period of life, all households that choose home ownership opt for LIPs. The fraction of home owners using LIPs falls steadily until the retirement age; no households aged 13 use LIPs. After retirement, the fraction of home owners using LIPs gradually rises with age until it reaches 80% in the last period before households are sure to die. Young households, who are responsible for most of the default in the economy, almost exclusively
Figure 4: Home Ownership Rates in the Hyperbolic Economies

Note: Home Ownership Rates at Start of Period.
finance their home with LIPs such that the default rate on CPM mortgages falls.

At 9.59%, the three-year foreclosure rate in the economy with CPMs and LIPs is more than double that of the economy in which households only have access to CPMs. The high foreclosure rate results in an equilibrium interest rate of 23.5% for LIPs. The average mortgage rate (averaged over the number of mortgage originations) rises to 18.28% although the interest rate on CPMs falls to 15.25% when LIPs are introduced from 16% in the economy with only CPMs. The interest rate on CPMs falls because the riskiest households now choose CPMs.

The average mortgage rate in the hyperbolic economy with LIPs is about 200 basis points higher than the average mortgage rate in the exponential economy with LIPs. This is largely because more households in the hyperbolic economy choose an LIP than in the exponential economy with LIPs. In the economies with LIPs, the CPM mortgage rate in the exponential economy is only 25 basis points lower than the CPM rate in the hyperbolic economy; the LIP mortgage rate in the exponential economy is actually 25 basis points higher in the exponential economy. Because many fewer households choose LIPs in the exponential
Table 3: Expected Lifetime Utility at Age 0 by Income in Hyperbolic Economies

<table>
<thead>
<tr>
<th>Income</th>
<th>Only CPMs Available</th>
<th>CPMs and LIPs Available</th>
<th>% Increase from LIP Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest</td>
<td>64.64</td>
<td>64.69</td>
<td>0.07%</td>
</tr>
<tr>
<td>Middle</td>
<td>68.77</td>
<td>68.82</td>
<td>0.07%</td>
</tr>
<tr>
<td>Highest</td>
<td>70.49</td>
<td>70.63</td>
<td>0.20%</td>
</tr>
</tbody>
</table>

economy, the foreclosure rate in the exponential economy rises more modestly to 4.82% from 2.80%.

Average expected lifetime utility at age 0 is higher in the economy in which the household has access to LIPs. The difference is not large, however; average expected utility is higher by only 0.1%. This is approximately half the difference between expected lifetime utility in the hyperbolic economy with only CPMs and the exponential economy with only CPMs. The increase in welfare is not because the availability of LIPs allows low income earners alone to enter home ownership. Only 13% of low income households that are home owners finance their purchase with an LIP. 22% of middle and high income home owners finance home ownership with an LIP. As Table 3 shows, the broad-based use of LIPs across income groups raises expected lifetime utility at age 0 for all three income groups. Perhaps surprisingly, the welfare gain is largest among households that begin life in the highest income category. Expected lifetime utility is higher by 0.2% for the highest earning households at birth while the availability of LIPs raises expected lifetime utility by only 0.7% for households that start life as middle or low income earners.

Households accrue the benefit from the introduction of LIPs in the first period of life as Figure 6 illustrates. Figure 6 shows average expected lifetime utility in the hyperbolic economies with and without LIPs by age. The increase in lifetime utility at age 0 is almost exclusively because LIPs enable very young households to enter home ownership without restricting their consumption to save for a down payment. The down payment requirement in the economy with only CPMs prevents young households from entering home ownership. The welfare benefit accrues exclusively to young households. After age 0, average expected lifetime utility is slightly lower in the economy in which households have access to LIPs.

Figures 7 and 8 illustrate that the welfare gain from LIPs is not because introducing LIPs
Figure 6: Ratio of Average Expected Lifetime Utility by Age (CPMs and LIPs / CPMs Only) in the Hyperbolic Economies

Note: Expected utility at start of period.
raises average savings. The availability of LIPs exacerbates the undersaving of hyperbolic households of all ages. Figure 7 compares average net worth to income ratios by age in the hyperbolic economy in which households only have access to CPMs and the hyperbolic economy in which households have access to both CPMs and LIPs. Net worth at every age is lower once households have access to LIPs than when they can only take on CPMs. The gap is largest for retirees where the undersaving problem is most acute. The intuition behind this result is that, rather than encouraging households to save, the higher home ownership rate can be sustained with less savings. In the economy with CPMs, households had to have savings at least equivalent to the down payment if they wanted to own a home. In contrast, in the LIP economy, many households save very little and yet are still able to be home owners. Because many retirees in the hyperbolic economy with LIPs finance home ownership with LIPs, many retirees save far less once they have access to LIPs.

Figure 8 shows that the undersaving problem is also more acute after the introduction of LIPs measured relative to what households would save in an exponential economy. The figure shows the ratio of average net worth by age in the hyperbolic economy as a percent of average net worth in the exponential economy when households only have access to CPMs and when both CPMs and LIPs are available. Savings in the hyperbolic economy are lower than savings in the exponential economy when LIPs are present.
Figure 7: Net Worth / Income with CPMs and with CPMs and LIPs (Hyperbolic Discounting)

Note: Net worth at start of period

Figure 8: Ratio of Average Net Worth to Income, Hyperbolic Economy / Exponential Economy

Note: Net Worth at start of period.
Part of the reason is that the introduction of LIPs raises expected lifetime utility is because it raises the home ownership rate. Recall that the benchmark calibration assumes that households receive a utility premium of 0.1 in every period they are home owners relative to when they are renters. However, the increase in the home ownership rate from the introduction of LIPs is not much larger when households discount the future hyperbolically than when households discount the future exponentially suggesting, again, that the increase in welfare has little to do with allowing households access to the commitment mechanism home ownership may provide to households that suffer from temptation problem.

5 Conclusions

This paper has studied how hyperbolically discounting households choose between a traditional fully amortizing mortgage that requires a substantial down payment and a product that offers low initial payments in an equilibrium life cycle framework. The LIPs in the model have similar features to popular subprime mortgage products. I find a strong age dependence in which households choose an LIP; LIPs are chosen primarily by young households and retirees. I also find that allowing households access to LIPs exacerbates the undersaving problem that arises when households behave hyperbolically. However, the welfare of a newly born household is higher when access to LIPs is unrestricted. Introducing LIPs doubles the foreclosure rate and raises the home ownership rate by about 11 percentage points. Perhaps surprisingly, the home ownership rate is always lower when households discount the future hyperbolically than when they discount the future exponentially.

References


