Managerial Incentives and Stock Price Manipulation

Lin Peng and Ailsa Röell

This Draft: May 2, 2011

Abstract

We present a rational expectations model of optimal executive compensation in a setting where managers are in a position to manipulate short-term stock prices, and managers’ propensity to manipulate is uncertain. We analyze the tradeoffs involved in conditioning pay on long- versus short-term performance and show how manipulation, and investors’ uncertainty about it, affects the equilibrium pay contract and the informativeness of asset prices. Characteristics of firms and managers determine the optimal compensation scheme: the strength of incentives, the pay horizon and the use of options. We consider the role of corporate governance regulations and disclosure policies in enabling better contracts.
1 Introduction

Is improperly structured incentive compensation a key factor underlying the recent financial crisis and the recurrent corporate fraud scandals of the past decade? A recent survey reports that 98% of responding banks "believe the compensation structures were a factor underlying the crisis." Policy makers are similarly concerned about the impact of short-term pay, especially in the financial sector: US Treasury Secretary Geithner states that "This financial crisis had many significant causes, but executive compensation practices were a contributing factor ... Some of the decisions that contributed to this crisis occurred when people were able to earn immediate gains without their compensation reflecting the long-term risks they were taking for their companies and their shareholders ... Companies should seek to pay top executives in ways that are tightly aligned with the long-term value and soundness of the firm." Meanwhile, a growing body of empirical evidence identifies the increase in stock- and option-based compensation since the late 1980s as a central contributor to the spate of corporate scandals exemplified by WorldCom and Enron, as well as the exponential increase in accounting restatements reported by the GAO (2002, 2006). The evidence suggests that there is a downside to short-term performance-based pay: it can encourage management to focus excessively on enhancing the short term stock price, jeopardizing the creation of long-term value. This raises the issue of how compensation contracts can be designed to balance incentives for productive effort against incentives for wasteful manipulation of short term stock prices.

This paper provides a rational expectations model that characterizes optimal executive compensation in a setting where managers are in a position to manipulate short-term stock prices. In the model, pay that depends on the short-term stock price elicits both effort and manipulation. Long-term pay does not elicit wasteful manipulation, but has the disadvantage of imposing some additional risk on the manager. We analyze the trade-offs involved in conditioning pay on long- versus short-term performance and characterize a second-best optimal compensation scheme. The analysis provides guidance for pay practices, by characterizing circumstances under which features like short-term pay and/or the use of options would be

---

3The empirical studies use a variety of indicators of performance report manipulation: earnings management as measured by discretionary accruals, SEC accounting enforcement actions, accounting restatements and shareholder class action litigation (Bergstresser and Philippon, 2006, Burns and Kedia, 2006, Denis, Hanouna and Sarin, 2006, Peng and Röell, 2008a, and Johnson, Ryan and Tian, 2009).
more or less appropriate. It also shows how public policy regarding corporate governance and financial disclosure standards can be used to improve the contracting environment.

A key feature of the model is that the managers’ propensity to manipulate is uncertain: this points to an important new dimension of incentive contracting that has not previously received attention in the literature on manipulation. In particular, the strength of incentives is determined by the product of the elasticity of pay to the stock price and that of the stock price to reported firm performance. Then even if pay is very sensitive to the stock price, incentives for effort may be weak if the stock price is unresponsive to reported performance - which would be the case if investors suspect that performance reports are inflated to an uncertain degree, and therefore unreliable. For purposes of empirical measurement of the strength of incentives, this means that an exclusive focus on the stock price sensitivity (or elasticity) of pay can be misleading.

We show that, paradoxically, an increase in manipulation uncertainty usually calls for pay to be more sensitive to the short-term stock price. Thus for firms, industries or CEO traits where the degree of manipulation is more uncertain, higher powered incentive contracts are likely to emerge, consistent with empirical evidence that startup firms and high-tech, high-growth, intangible asset-intensive industries typically feature extremely stock price-sensitive pay with generous use of options. The model also predicts that younger managers, who do not have a track record regarding the degree to which their reports are inflated, will similarly receive more option based pay (in contrast, ex ante uncertainty about their skills would generate less use of options).

In policy terms, the analysis shows that regulations or policies that help to reduce manipulation uncertainty (such as tighter accounting standards) can improve contracting efficiency and make it possible to induce effort more effectively, enhancing firm value. We compare and contrast manipulation uncertainty with other sources of noise in short-run performance reports, and discuss how policy remedies differ.

Uncertainty about manipulation costs is a distinguishing feature of our model: in equilibrium even fully rational investors are deceived, causing disparities between their beliefs based on firms’ reports and the true state of affairs. This describes many real-world cases of corporate fraud, where it becomes clear from investigations ex post that prices were out of line, not just due to random unforeseen events, but as a result of deliberate misrepresentation. If all managers always had the same propensity to exaggerate their reported performance, a rational investor would be able to back out the degree of manipulation and
a correct assessment would be reflected in the share price as, for example, in the signal-jamming models of Narayanan (1985), Stein (1989) and Goldman and Slezak (2006). In such a setting, manipulation can be costly and wasteful, but it does not affect the accuracy of stock market price discovery as the investors are fully aware of the true state of the company. In contrast, uncertainty about the degree to which reports are inflated imposes an additional source of risk on investors, resulting in less informative stock prices and less effective contracting, even in settings where the direct costs of manipulation are modest or merely involve a transfer of relatively small amount of money from the shareholders to the managers. As we have argued, such uncertainty about the degree of manipulation typically leads to a stronger link between pay and stock price, in contrast to the weaker link that emerges from existing models that consider manipulation.

In the long run the truth will come out, even if in the short run stock price can be manipulated. Then long term pay can align the manager’s personal objectives with long term shareholder value, mitigating the waste of valuable managerial time and resources on manipulation associated with short term incentives. We therefore investigate the optimal mix of long and short term incentives. We find that in general, when long term incentives are allowed, at least part of the incentive pay is shifted from the short term to the long term, resulting in higher equilibrium effort choice and enhanced firm value. Long-term compensation is particularly desirable when short-term information is clouded by manipulation uncertainty or other sources of noise. Empirical work by Chi and Johnson (2009) confirms that long-term incentives have a significant impact on firm value, indeed more so than short term incentives. Bizjak, Brickley and Coles (1993) and Cadman, Rusticus and Sunder (2010) find that long term incentives are used more in industries where short term performance is a poor guide to long run value. But long term incentive pay has its own downside: it introduces extra risk from longer term shocks that are outside the current manager’s control into his pay. This is costly to the firm because of the need to compensate the manager for bearing the additional risk. Thus short run pay should remain part of the optimal contract if long term volatility is substantial or if the manager is very risk averse.

The intertemporal tradeoffs involved make optimal incentives depend on firm and manager characteristics in a surprisingly non-monotonic manner. For example, the larger the firm, the stronger the long term incentives, while short term incentives peak at an intermediate firm size. In contrast, in a purely short-term model such as that of Peng and Röell (2008b), incentives are straightforwardly monotonic in determinants such as firm size, man-
agential skill or risk aversion because the firm cannot uncouple the incentives for effort and manipulation by shifting incentives toward the long run.

Other papers have investigated the tensions between short- and long-run incentives. Holmström and Tirole (1993) do not consider manipulation, but focus on how the firm can manage the informativeness of the short term stock price by using the free float of shares in the hands of outside investors to incentivize information gathering. A very different approach is taken by Bolton, Scheinkman and Xiong (2005 and 2006), who drop the assumption that the principal’s goal is long-term value maximization, and explore the conflict of interest between future shareholders and short-termist current shareholders, who design a contract that encourages ramping up the short-term share price. Axelton and Baliga (2009) argue that scope for manipulation that obscures short-term value can be beneficial in discouraging ex post efficient but ex ante inefficient renegotiation.

We will assume manager preferences with constant relative risk aversion and Cobb-Douglas substitutability between wealth (consumption) and leisure, with effort modeled as a time cost that increases firm value proportionately, and firm value distributed lognormally, as in the multiplicative model of Peng and Röell (2008b). Compensation schemes are limited to a loglinear form in order to keep the analysis tractable and generate closed-form solutions. This framework has the attractive feature that it makes direct predictions about the elasticity of pay (rather than the $-for-$ sensitivity of pay to firm value as in the traditional model), providing testable comparative static insights that are more closely aligned with the empirical literature: in his survey of empirical work on executive compensation, Murphy (1999) argues that in comparison with the sensitivity approach, the elasticity approach generally produces a better empirical "fit" in cross-sectional analysis of the relationship between pay and firm value and has the desirable property that it can be better compared across firms of different size. The traditional basic agency model of executive compensation (normally distributed firm value, effort with an additive impact, a constant absolute risk averse manager whose cost of effort is independent of his wealth, and pay that is linear in firm value) may be a poor descriptor of reality and thus hard to reconcile with the data: a concern explored, for example, by Baker and Hall (2004). And our multiplicative framework permits a richer exploration of comparative statics than the alternative multiplicative model proposed by Edmans et al. (2009), which is more tractable but restricted to risk neutral CEOs making a binary effort choice.

We find that option pay can be a part of the optimal pay package. In the model, effort
is determined directly by the elasticity of pay, and the optimal value of this elasticity may be very large, especially for firms that are large and subject to very uncertain manipulation of short run prices, and for poor and skillful CEOs who are not too risk averse. Including options in the pay contract makes it possible to attain highly elastic pay in a way that cannot be achieved through a stock grant. Thus options may be called for, and if it is long-term incentives that need to be strengthened, it would be appropriate to impose a suitably long vesting period on the options.

A final feature of the model that deserves mention is the formulation of the cost of manipulation as a time cost. In the model, all managers need to spend some time to convince the stock market that their company has value: the term "manipulation" is perhaps too pejorative, since even a manager who wishes to convey the truth to investors needs to devote time to investor relations to convey the company’s worth. This is time taken away from productive effort directed at enhancing the firm’s long term value and therefore entails a real distortionary cost. In reality, managerial time is a scarce resource and managers routinely mention the significant amount of time and attention they are forced to devote to public relations and reassuring the stock market. For example, in Europe prominent business leaders have complained that the threat of a takeover, now that corporate control is more contestable than it used to be, has the unfortunate side effect of distracting management from running the underlying business. This time cost comes out clearly in the London Stock Exchange’s *A Practical Guide to Listing*, as well as in reports on the drawbacks of going public in the U.S.

Our model of the cost of manipulation as a demand on managerial time is similar in spirit to some of the existing literature. Narayanan (1985), Stein (1989) and Bolton et al.

---

4Both the flotation process itself and the continuing obligations – particularly the vital investor relations activities – use up significant amounts of management time which might otherwise be directed to running the business. It is vital that you maintain your company’s profile, and stimulate interest in its shares on a continuing basis. Many listed companies, even relatively small ones, employ specialist financial public relations and investor relations advisors on a retainer basis to keep the business on the financial pages and in the minds of investors. However, you cannot leave press or investor relations to your advisers. Top executives will commonly devote at least a couple of days a month to developing and nurturing such contacts. This commitment will increase sharply around regular announcements, at the launch of a new product or strategy, or at times when the business or its profile have been hit by adverse events. This must be regarded as time well-spent. As a publicly-quoted company, it is a core element of running your business properly and responsibly.” (pp. 11, 47-48). Available at http://www.londonstockexchange.com.

5E.g. "There are some ... compelling reasons to stay private. The regulatory cost of being a public company can be onerous, stockholder expectations can hinder long-term plans ... Are these things that management wants to be spending its time on? ... Depending on where they are in the growth cycle of the company, is it appropriate to be putting so much of their time and resources into going and staying public as opposed to growing their business and brand?" Second Thoughts On Going Public, *Wall Street Journal*, 13 September 2010.
(2005, 2006) view the main cost as a distortion of investment decisions towards projects that give palpable results in the short run. Kedia and Philippon (2009) and Bemmelich, Kandel and Veronesi (2009) argue that overinvestment and/or excessive employment are the real underlying cost of manipulation. Similarly, we model manipulation as a distortionary shift of real resources away from long-term value creation to activities that contribute mainly in the short run. Such manipulation need not just be window dressing: it can distort real investment and project choice decisions.

The paper is organized as follows: In Section 2 we describe the model setup and derive the equilibrium stock price, the manager’s actions and utility, and expected firm value for any given pay contract. Section 3 characterizes the optimal pay contract that trades off short- and long- term incentives in the presence of manipulation uncertainty. Section 4 discusses the empirical predictions of the model and addresses policy implications. Section 5 concludes. Proofs of all propositions are collected in the Appendix.

2 The model

In this section we describe and motivate the basic model and solve it for the equilibrium stock price, expected managerial utility and pay, and expected firm value, for any given managerial pay contract. These results will be drawn upon in subsequent sections to characterize optimal executive pay contracts in a variety of settings.

2.1 Model assumptions

We analyze a multiperiod model with decisions taken at dates 0 and 1 and final payoffs established at date 2. At time 0, the manager and the firm’s shareholders sign a contract \textit{ex ante}, before the cost of manipulation is observed. The shareholders are risk neutral: their objective is to maximize expected firm value net of managerial compensation. The manager is assumed to be risk averse. Between dates 0 and 1, the manager privately finds out the cost of manipulation. He then chooses his level of productive effort as well as his degree of manipulation; the latter is modeled as a factor by which he inflates a report regarding firm value at date 1. The true underlying value of the firm at date 1 is determined by an exogenous shock and the manager’s level of effort. The performance report released by the manager at this point conflates the true value of the firm at date 1 and his degree of manipulation. The stock price at time 1 is based on the manager’s report. At date 2, the true long-term value of the firm, which may incorporate a further exogenous shock, is
revealed. For simplicity, we set the interest rate to zero.

The true value of the company is subject to random shocks, $\epsilon_1$ and $\epsilon_2$, at dates 1 and 2 respectively. It is assumed that the firm’s value in the short run, $V_1$, and in the long run, $V_2$, is multiplicative in the scale of the firm, $X$ and the manager’s productive effort $E$:

$$V_1 = E \cdot X \cdot \epsilon_1 \tag{1}$$
$$V_2 = V_1 \cdot \epsilon_2$$

where multiplicative value shocks $\epsilon_1$ and $\epsilon_2$ are independent mean-one lognormally distributed, $\epsilon_t \equiv \ln \epsilon_t \sim N(-\frac{1}{2} \Sigma_t, \Sigma_t)$ for $t = 1, 2$.

At time 1, the manager sends a report $S$ that portrays the firm’s true value $V_1$ as observed by him at time 1, factored up by a manipulation multiple $M$:

$$S = MV_1 = MEX\epsilon_1 \tag{2}$$

where $M \geq 0$ is unproductive effort devoted to upward manipulation of the firm’s perceived value.

Manipulation should be interpreted broadly, to include a variety of channels for conveying positive information: managing earnings or financial statements by exploiting any leeway in accounting rules, corporate publicity campaigns, optimistic projections of future sales and technology, as well as real activities that bolster short-run earnings but are actually negative-NPV such as deferral of necessary maintenance or R&D expenditure, investments in projects that initially look good to outside observers, etc.\(^6\)

The manager is constant relative risk averse, with a constant relative risk aversion coefficient of $1 - \phi$, and a preferences that are Cobb-Douglas in leisure and money:

$$U = \frac{1}{\phi} \left[ (\bar{L} - C_E E^{1/\alpha} - C_M M^{1/\beta})^\Psi W \right]^\phi \tag{3}$$

where $0 < \alpha, \beta \leq 1$, $\Psi > 0$ and $\phi < 1$

where $\bar{L}$ is his time endowment, and $W$ is the manager’s wealth, derived from his employment at the firm.\(^7\) The manager’s personal time cost for both effort and manipulation is taken

\(^6\)More generally, $M$ need not be interpreted as a complete waste of resources: setting $S = EM\epsilon_1$ and $V_2 = EM^{\lambda \epsilon_1 \epsilon_2}$ for $0 \leq \lambda < 1$, $E$ can be interpreted as effort devoted to projects that are equally valuable in the short and the long run, while $M$ is effort devoted to projects that look more valuable in the short run than they are in the long run. For simplicity, we focus on the case where $\lambda = 0$, since the qualitative results are the same as for any $\lambda$ strictly less than 1.

\(^7\)An alternative model of manipulation cost regards it as a psychological cost for the manager, that is:
to be a convex function parameterized by a constant that scales the time used ($C_E$ and $C_M$ respectively) and a convexity parameter (the exponents, $\alpha$ and $\beta$ respectively).

For convenience we assume that $C_E$ is a fixed, known cost of effort that is identical for all managers, but that the manipulation cost parameter $C_M$ is random, with the following log normal distribution:

$$\ln C_M \equiv c_M \sim N(\tau_M, \Omega) \quad (4)$$

We model $C_M$ as a random variable to capture the idea that, in practice, investors often cannot tell to what degree the information released by managers is over-optimistic: it depends on unknown factors such as the manager’s ethical compass, his ability to project a (possibly unjustified) aura of success, his salesmanship in portraying the merits of an unfamiliar project, and the susceptibility of the firm’s business opportunities to hype.

The parameters of the distribution, $\bar{c}_M$ and $\Omega$, are determined by observable characteristics of the firm, the industry and the executive as well as by the regulatory environment governing disclosure of price-relevant information. We will find that the average level of (the log of) manipulation cost $\tau_M$ has no real impact on employment contracts because in our Cobb-Douglas model, the total share of time wasted on manipulation is independent of $\bar{c}_M$, and moreover, if all reports are inflated by a known common factor the stock market will simply correct for that by scaling down the price appropriately.

But what does matter for the nature of employment contracts is the degree of uncertainty about manipulation $\Omega$. This parameter differs across observable characteristics of people, firms and industries; and it also depends on the regulatory environment. Regarding personal traits, the uncertainty $\Omega$ is likely to be higher for young managers who have a relatively short track record, or whose cultural norms are unfamiliar, because the stock market would find it harder to assess the degree of inflation in their reports. But the manipulation uncertainty is not solely determined by the personal traits of the manager. Some firms may have far-flung operations that provide more scope for shading their results. And some industries provide a more uncertain environment in which there is more scope for differences of interpretation of the facts. Examples are IT companies that develop large but infrequent customer-specific

$$U = \frac{1}{\phi} \left[ (I - C_E E)^\psi (R - C_M M)^\Theta W \right]^\phi.$$  

where $R$ is an endowment of "self-respect". This formulation yields very similar results to the version used in this paper.
software projects (for which the timing of revenue recognition is a major issue) and financial services firms that are able book current fee income by taking on complex risky positions whose ultimate value will only emerge in the future. The variance \( \Omega \) is likely to be high in high growth, high tech industries, with more intangible assets such as patents, and where new, complex and hard to value products are introduced, and generally industries where current earnings are a poor guide to the future. If company and industry fundamentals are easy to understand and key indicators of performance are publicly observable, there will not be much room for variation in managers’ reports no matter how different their personalities in terms of willingness to make exaggerated claims for personal benefit. Lastly, a regulatory environment that imposes thorough and informative disclosures will limit the amount of leeway that managers have: for example, quarterly sales reporting under tight accounting norms regarding sales recognition reduces the scope for both exaggerated and overly modest claims about how well products are selling. Thus the characteristics of people, their firm, the industry and the disclosure rules all interact to determine the level of manipulation uncertainty.

Participants in the stock market have rational expectations: they understand managers’ incentives to work and to manipulate, but they do not know the realized value of \( c_M \) and thus of \( M \) and cannot observe \( V_1 \). They establish the market price at time 1 based on the manager’s report:

\[
P_1 = \text{E}[V_1 | S] \tag{5}
\]

where for convenience we define \( P_1 \) as the gross-of-pay stock price, that is, the gross expected value of the firm, \textit{before} executive pay is deducted: the actual stock market capitalization would be equal to this expected gross value of the firm, minus expected executive pay.

We conjecture that \( P_1 \) takes the following log-linear price form:

\[
P_1 = \pi X^{1-\gamma} \cdot S^\gamma \tag{6}
\]

for some values of the parameters \( \pi \) and \( \gamma \) that are to be determined in equilibrium.

We restrict attention to a three-parameter pay contract where the manager’s compensation package (parameterized by \( \{\omega, \mu, \eta\} \)) is constant-elastic in the firm’s long- and short-term value, that is, the pay contract takes the log-linear form:

\[
W = \omega P_1^\mu V_2^\eta \tag{7}
\]
This multiplicative functional form is chosen as a convenient approximation that yields closed-form solutions. A novel aspect of this specification is that performance at dates 1 and 2 does not enter additively into pay, but in mutually reinforcing fashion. This is not unrealistic. The pay contract can be thought of as one in which short-run performance as represented by the stock price at time 1 determines the size of a package of fixed pay, stock and options to be awarded to the manager; if the firm outperforms in the long run, this package will increase further in value.\footnote{Rewriting the functional form as $\omega P_1^{\mu+\eta} \left( \frac{V_2}{P_1^2} \right)^\eta$, pay can be interpreted as an anticipated reward for short term performance, $\omega P_1^{\mu+\eta}$, multiplied by a factor, $\left( \frac{V_2}{P_1^2} \right)^\eta$, that penalizes (rewards) any short term overpricing (underpricing, respectively).} The chosen three-parameter constant-elastic functional form is tractable and flexible, and yields direct insights into the optimal elasticity of pay to long- and short-run firm performance.

Note that the manager’s pay does not depend directly on his report $S$: such a report is presumed to be unverifiable and too complex to summarize into a form that a pay contract can be based upon. It may include predictions about market share, product quality, earnings, the business climate, the competence and health of the management team, etc. It is left to the impersonal judgment of stock market participants to distil this information into a summary judgment about the firm’s underlying value, captured by $P_1$.

### 2.2 The manager’s problem

After signing the compensation contract at date 0, the manager discovers $C_M$, that is, how costly it is to manipulate the performance signal. He then chooses the level of effort $E$ that he will exert and the manipulation factor $M$ by which he will scale up his report of the firm’s value at time 1:

\[
\max_{E, M} \mathbb{E}_1 \left[ \frac{1}{\varphi} \left\{ \left( \bar{L} - C_E E^{1/\alpha} - C_M M^{1/\beta} \right)^\psi W \right\}^\phi \right] \tag{8}
\]

Substituting out $P_1$ and $V_2$ in equation (7) using equations (1) and (6), the manager’s compensation can be expressed as:

\[
W = \omega P_1^{\mu} V_2^{\eta} = \omega \pi^\mu X^{\mu+\eta} M^{\gamma+\eta} E^{\gamma+\eta} \epsilon_1^{\gamma+\eta} \epsilon_2^{\eta} \tag{9}
\]
The manager’s optimal choice of effort, manipulation and leisure is then given by:

\[
E = \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha}
\]

(10)

\[
M = \left( \frac{L}{C_M \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\beta}
\]

(11)

\[
L = T - C_E E^{1/\alpha} - C_M M^{1/\beta} = \frac{\Psi}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} L
\]

(12)

Note that, from equations (10) and (11), the time cost of manipulation and the time devoted to effort are:

\[
C_M M^{1/\beta} = \frac{\beta \gamma \mu}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu}
\]

(13)

\[
C_E E^{1/\alpha} = \frac{\alpha (\gamma \mu + \eta)}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu}
\]

(14)

This means that manipulation is determined by both the elasticity of the stock price to the performance report (\(\gamma\)) and the price elasticity of pay in the short run (\(\mu\)), while productive effort also depends on the long run pay elasticity (\(\eta\)).

Thus management chooses to devote only a fraction of its time at the firm on productive effort; the rest is devoted to finding ways to convey a positive short-term view of the firm. Manipulation has real consequences and detracts directly from long-run firm value: the firm is paying management to spend time overseeing the company’s activities, and any time spent boosting short-run performance indicators is modeled as time and attention taken away from more productive, long term value-enhancing effort \(E\).

Observe that for the Cobb-Douglas formulation of preferences used in this paper, the choice of effort and manipulation is independent of the exogenous firm value shocks \(\epsilon_1\) and \(\epsilon_2\). We have presented the manager’s time allocation decision as taking place before he becomes aware of the outcome of these random shocks, but the model can alternatively be regarded as one where the manager observes the output shocks before he decides how much to manipulate.

2.3 The short run stock price

At time 1, the stock market observes only the report sent by the manager, not the true value of the firm. Market participants do observe the pay contract signed at time 0 and understand the manager’s incentives to inflate the performance report. Thus they correctly
back out the optimal level of effort exerted by the manager and form estimates of the firm’s value and the level of manipulation based on the \textit{ex-ante} distribution of the manipulation cost $C_M$ and the shock to the firm’s value $\epsilon_1$. The rational-expectations short term stock price (gross of expected compensation) at time 1 is the conditional expectation of the firm value given the manager’s report $S$, set down in the following proposition.

**Proposition 1** The short term gross-of-pay stock price is given by

$$P_1 = \pi X^{1-\gamma} S^\gamma$$  \hspace{1cm} (15)

where

$$\gamma = \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1}$$ \hspace{1cm} (16)

and

$$\pi = \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha(1-\gamma)} \left( \frac{L}{\bar{C}_M \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{-\beta \gamma}$$  \hspace{1cm} (17)

$$= E^{1-\gamma \frac{1}{M^{-\gamma}}}$$ \hspace{1cm} (18)

and $\bar{C}_M$ denotes $\exp \bar{c}_M$.

The short term stock price is a weighted geometric average of the investors’ \textit{ex ante} expected gross firm value, $P_0 \equiv XE$, and the manager’s report, $S$ discounted by a factor equal to the median level of manipulation $\bar{M} \equiv \left( \frac{L}{C_M \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\beta}$:

$$P_1 = E^{1-\gamma \bar{M}^{-\gamma}} X^{1-\gamma} S^\gamma = P_0^{1-\gamma} \left( \frac{S}{M} \right)^{\gamma}.$$ \hspace{1cm} (19)

Accounting for the possibility that the report $S$ may be manipulated, the stock market attaches less weight $\gamma$ to the report, the more uncertain it is about the degree of manipulation ($M$); the basic idea that noise in managerial reporting bias reduces the sensitivity of the stock price to reported performance was articulated by Fischer and Verrecchia (2000). The uncertainty, as captured by the variance of $\ln M$ is, from equation (11), given by $\beta^2 \Omega$: the parameter $\Omega$ captures the dispersion in managers’ time cost of manipulating their reports, while the manipulation cost convexity parameter $\beta$ captures the degree to which extreme degrees of manipulation are relatively costly in terms of wasted time. Meanwhile the stock market attaches a greater weight to the report $S$, the greater the fundamental uncertainty about the firm ($\Sigma_1$) (holding constant the noisiness of the report) because updating the estimate of firm value becomes more important.
Based on the distribution of manipulation cost, the investors are able to back out the average degree of manipulation (captured by the median $\overline{M}$) and adjust for it accordingly in setting equilibrium prices, as in the signal jamming models of Stein (1989) and others. However, to the extent that the actual manipulation cost may be higher or lower than average, the stock price is inaccurate. Investors underestimate the degree of manipulation for the managers with a below-average manipulation cost $C_M$, setting a short term stock price that is too high. The manager’s short term compensation is also too high, resulting in a loss of net firm value. Investors will be disappointed in the long run when stock prices eventually revert to fundamental value. For example, in the runup to the crisis of 2008, some financial services firms made negative NPV investments that looked profitable in the short run: they were able to overstate the profitability of their business to an extent that was not anticipated by outside investors, partly as a result of the opacity of the innovative financial transactions used to structure their deals.

The informativeness of the short term stock price, $P_1$, is determined by the degree to which short term signal is clouded by manipulation uncertainty:

\[
\text{var} \left[ \ln \frac{P_1}{P_0} \right] = \gamma \Sigma_1 \tag{20}
\]

\[
\text{var} \left[ \ln \left( \frac{V_2}{P_1} \right) \right] = (1 - \gamma) \Sigma_1 + \Sigma_2 \tag{21}
\]

The higher the manipulation uncertainty ($\Omega$), the lower the $\gamma$, and therefore the slower the incorporation of value-relevant information into stock prices. If there were no manipulation uncertainty or other noise in the signal, then $\gamma = 1$ and $P_1$ would fully capture the true state of the firm at time 1.

2.4 Manager’s expected utility and net firm value

Given any compensation contract $\{\mu, \omega, \eta\}$, the manager expects to choose optimal levels of effort and manipulation at the interim date 1, and the stock market to respond rationally to his report. Standing at date 0, before the realization of the manipulation cost or any random shocks to the firm’s value, his ex ante expected utility is as given in the following proposition.

**Proposition 2** The manager’s ex ante expected utility, before he is aware of his own propen-
sity to manipulate, is given by:

\[
E_0 [U] = \frac{1}{\phi} \left[ \omega \left( \frac{\Psi}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\Psi} X^{\mu+\eta} \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha(\mu+\eta)} \right]^\phi \cdot \exp \left\{ -\frac{\phi}{2} \left[ (\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2 \right] + \frac{\phi^2}{2} \left[ (\gamma \mu^2 + 2\gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2 \right] \right\}
\]  

where \( \gamma \) is given in Proposition 1 and \( \omega, \mu \) and \( \eta \) are the terms of his compensation contract.

The \textit{ex ante} expected wealth of the manager, obtained by setting \( \phi = 1 \) and removing the term representing leisure in equation (22), is given by:

\[
E_0 [W] = \omega X^{\mu+\eta} \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha(\mu+\eta)} \cdot \exp \left\{ \frac{1}{2} \left[ (-\gamma \mu + \gamma \mu^2 + 2\gamma \mu \eta - \eta + \eta^2) \Sigma_1 + (\eta + \eta^2) \Sigma_2 \right] \right\}
\]  

The expected net-of-pay firm value follows directly as the difference between the gross expected value and the expected payment to the manager.

**Proposition 3** The \textit{ex ante} company expected firm value, net of executive compensation, given a pay contract \( \{\omega, \mu, \eta\} \) is:

\[
E_0 [V_2 - W] = X \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha} \cdot \omega X^{\mu+\eta} \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha(\mu+\eta)} \cdot \exp \left\{ \frac{1}{2} \left[ (-\gamma \mu + \gamma \mu^2 + 2\gamma \mu \eta - \eta + \eta^2) \Sigma_1 + (\eta + \eta^2) \Sigma_2 \right] \right\}
\]  

### 3 The optimal contract

We now characterize the optimal employment contract and show how conditioning pay on long-term performance can mitigate the agency problem arising from the manipulability of short-term performance. The idea is to wait until all the evidence is in before settling the final payout to the manager. In practice, it is much harder to manipulate long-term outcomes, as the truth will emerge in the end; for simplicity we model this by assuming that the manager cannot manipulate long-term value at all.
Net firm value as expressed in equation (24) is maximized over the three parameters of the employment contract \( \{\omega, \mu, \eta\} \), subject to the participation constraint that the manager’s expected utility (equation 22) attains his reservation level \( \bar{U} \), indexed in money terms by a reservation wage \( \bar{W} \) implicitly defined by \( \bar{U} = \frac{1}{\phi} \left( \bar{L}^\Psi \bar{W} \right)^\phi \). If that constraint is binding\(^9\), then substituting out for \( \omega \), net firm value can be written as a function of the short- and long-term wage elasticities \( \{\mu, \eta\} \):

\[
E_0 [V_2 - W] = X \left( \frac{\bar{L}}{C_E} \frac{\alpha (\gamma \mu + \eta)}{\psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^\alpha \\
- \bar{W} \left( \frac{\psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu}{\psi} \right)^\psi \\
\cdot \exp \left\{ \frac{1 - \phi}{2} \left[ (\gamma \mu^2 + 2\gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2 \right] \right\} 
\]

We first establish some general properties of the optimal pay contract.

**Proposition 4** The second-best optimal contract has the following properties:

(i) the long-term incentive parameter, \( \eta \), is strictly greater than zero but the short-term parameter, \( \mu \), is only weakly greater than zero;

(ii) long- and short-term incentives are substitutes in the sense that the short-term incentive parameter \( \mu \) is lower than it would be if long-term incentives were constrained to be absent \( (\eta = 0) \);

(iii) effort is below the first-best optimal level and below the second best optimal level in the absence of manipulation.

Typically, the optimal contract includes long- as well as short-term incentives. Long-term incentives enable the firm to align the manager’s objectives with true firm value, so that less effort is wasted on improving the short-term stock price. On the other hand, they have the disadvantage of introducing extra risk from longer-term exogenous shocks to firm value (captured by \( \Sigma_2 \)) into the manager’s pay. In the model, long-run incentives

\(^9\)In deriving optimal contracts we have assumed that the manager will accept an employment contract that yields utility of at least \( U(W, L) \). We implicitly assume that the manager has no initial wealth, so that his final wealth, \( W \), is equal to his compensation from the firm. If he does have an initial wealth endowment, \( \bar{W} \), then the firm only needs to pay him \( (W - \bar{W}) \) and so \( \bar{W} \) should be added to the above expression for the net-of-pay firm value. To preserve the constant-elastic functional form for his final wealth, it should then be assumed that the manager sinks any initial wealth \( \bar{W} \) that he owns into the firm, in return for a constant-elastic pay contract that gives him expected utility \( U(W, L) \).
may displace the short-run incentives altogether for some parameter configurations; but it is always advantageous to include at least some long-term element in pay to mitigate the waste associated with manipulation.

Compared to a contract that is limited to short term pay, the optimal contract shifts incentives away from the short term, so that the elasticity of pay to the short term stock price is lower. As a result, the amount of time wasted on manipulation is also lower.

Part iii of the proposition implies that the possibility of manipulation leads to an optimal pay contract which calls forth lower effort $E$ than in the case where manipulation is impossible. Intuitively, there are two distinct reasons why.

First, the detrimental side effects tend to reduce the optimal incentive for effort, just as in the multitask model of Holmström and Milgrom (1991). Incentive compensation encourages effort, but brings with it the undesirable side effect of encouraging manipulation, which entails a resource cost in terms of available managerial time. Indeed, the total time needed to produce a unit of productive effort is scaled up by a factor $1 + \frac{\beta \gamma \mu}{\alpha (\gamma \mu + \eta)}$ as can be seen from equations (10) and (11). The time wasted on manipulation is a source of agency cost, making it more costly to incentivize effort.

Second, the uncertainty about the manipulation cost further reduces the equilibrium level of effort. This uncertainty makes it hard to distinguish true performance from managerial hype, and therefore contaminates the value of the short-term stock price as a measure of managerial effort. Managers who do not exaggerate their performance as much as others are unable to persuade the market that they are simply modest, not lazy. This imposes an additional risk on the risk-averse managers *ex ante*, before they learn their type.

How does the optimal contract vary with firm characteristics and managerial traits? Comparative static results regarding firm value and effort can be established analytically:

**Proposition 5** The second-best optimal contract has the following properties:

(i) the maximized net firm value is (weakly) decreasing in risk aversion $(1 - \phi)$, in short-term volatility $\Sigma_1$ (holding constant either $\Omega$ or $\gamma$, regardless), in long-term volatility $\Sigma_2$, and in the manipulation uncertainty $\Omega$;

(ii) effort ($E$) is increasing in firm scale $X$ and the manager’s time endowment $\bar{L}$, and decreasing in the reservation wage $\bar{W}$ and the cost of effort $C_E$: the time spent on effort ($C_E E^{\frac{\mu}{\gamma}}$) is similarly increasing in $X$ and $\bar{L}$, and decreasing in $\bar{W}$.

---

10If the contract is constrained to the short term ($\eta \equiv 0$), then for every minute of effort, $\frac{\beta}{\alpha}$ minutes are wasted on manipulation.
In contrast, the short- and long-run pay elasticities, $\mu$ and $\eta$, are related to the underlying parameters in an interesting and quite complex way: it is not just the overall intensity of incentives that varies, the relative power of long- and short-run incentives varies as well. We will use numerical analysis to illustrate these effects, in the process showing where and why monotone comparative static results can be ruled out.

We present the results in a series of figures, using a set of benchmark parameter values and varying some of these parameters individually to gain insight on their effects. Our qualitative findings are robust to a wide range of reasonable parameter values. We will set the effort cost convexity parameter $\alpha = 1$ throughout. We set the firm scale parameter $X$ to 100 and without loss of generality, normalize $C_E = \bar{L} = \bar{W} = 1$: these four parameters are basically just one as what matters for optimal incentives is the composite ratio $\frac{X \bar{L}}{\bar{W} C_E}$. We set the short run return variance $\Sigma_1$ to 1 and the long run return variance $\Sigma_2$ to 5. The manipulation uncertainty $\Omega$ is set to 1. The value of $\phi$ is set to $-1$, corresponding to a constant relative risk aversion coefficient $1 - \phi$ of 2, a value within the generally accepted range. The coefficient on leisure in the manager’s utility function $\Psi$ is set to 1; a low value for $\Psi$ indicates a workaholic manager, while a high value indicates a slacker. We also set the parameter $\beta$ to 1 in the benchmark case.

4 Discussion and policy implications

4.1 Impact of information uncertainty and disclosure policy

We start by explaining how manipulation uncertainty, or more generally, noise in the short run signal, affects pay contracts and firm value. We then consider the impact of disclosure policy on the contracting environment.

In our model, uncertainty about the propensity to manipulate ($\Omega$) is captured by randomness in the manipulation cost $C_M$. Agency theory predicts that incentives should load less onto a performance signal if it is noisier. Since the short term stock price is based on the manager’s report, it seems natural to expect that an increase in the randomness of the noise in that signal should make the contract less reliant on the short term stock price. But, as illustrated in Figure 1, the opposite is true: an increase in the manipulation noise ($\Omega$) generally leads to an increase in the elasticity of pay with respect to the short term stock price ($\mu$).

[Insert figure 1 about here]
This counterintuitive result can be explained by noting that the observed employment contract is written on the stock price \( P_1 \) and not directly on the manager’s report \( S \). As the managerial report becomes noisier, the stock price becomes less responsive to it: \( \gamma \) is decreasing in \( \Omega \). To restore incentives, pay would need to be more responsive to the stock price. In the optimal contract, incentives are partially restored by an offsetting increase in elasticity of pay to the stock price, and the observed contract loads more onto the price:

\[
\frac{dw}{dp} = \frac{dw}{ds} \bigg| \frac{dp}{ds}
\]

where \( w, p_1 \) and \( s \) denote the logarithms of pay \( W \), the short-run price \( P_1 \) and the manager’s performance report \( S \) respectively. In our model this is precisely what happens: as \( \gamma \) falls, \( \mu \) increases. But the overall incentive \( \gamma \mu \) (and thus effort) still falls, as shown in Figure 1.

As shown in Proposition 5, a reduction in the manipulation uncertainty (\( \Omega \)) unambiguously raises net firm value. As illustrated in Figure 1, in general the improvement stems from the beneficial increase in optimal short-term incentives for effort (\( \gamma \mu \)). But paradoxically, this strengthening of incentives has the unavoidable side effect of leading to a greater waste of resources on manipulation.

The analysis suggests that, if manipulation uncertainty is a major source of concern, any policy intervention that reduces \( \Omega \) improves the contracting environment. In particular, stricter or more comprehensive disclosure requirements, such as tighter accounting standards or enhanced disclosure of off-balance sheet obligations, better internal controls through more board monitoring, would allow investors to make sharper inferences about manipulation and firm value, making possible an optimal contract that elicits more effort. Even if regulation of corporate disclosure does not directly target "real" manipulation, it does deter the misallocation of resources if it enables the investing public to distinguish between productive and less productive activities in determining the stock price. But as

\[\text{Indeed, if the contract is constrained to be short-term (that is, } \eta = 0 \text{ is imposed) then the comparative static result that } \mu \text{ is increasing in } \Omega \text{ follows directly from equation (A28).}\]

A competing interpretation is that, when the signal is very noisy, the volatility of the period 1 stock price is low (see equation 20), thus reducing the risk imposed on the manager by stock-price-driven incentive pay and encouraging a more price-sensitive contract. This leads us to check the robustness of our result concerning the impact of noise on \( \mu \) in a steady state setting where signal noise moves the timing of uncertainty resolution forward but does not impact the return volatility, which is simply a constant fundamental volatility, \( \Sigma \). It is shown in the Appendix A.3 that in this relay model, the impact of manipulation uncertainty (\( \Omega \)) on the price elasticity (\( \mu \)) of the pay contract is not always unambiguous, but still positive in a wide range of settings, in particular in those where the firm is large enough that the optimal incentives need to be powerful.
explained above, such policies, by promoting stronger short term incentives for effort, would also lead to more manipulation, but even so, net firm value is raised, stock prices are more informative and investors’ welfare is improved. In contrast, policies such as the relaxation of mark-to-market requirements for the financial services sector as instituted in early 2009, may give managers too much leeway in reporting and thus increase uncertainty $\Omega$, making the stock price less informative and harming the effectiveness of incentive compensation based on the short-term price.

To gain more insight into how specific policies can help to reduce the uncertainty ($\Omega$) associated with the short term report ($S$), it is useful to distinguish potential sources of this uncertainty. There could be other sources of noise, besides the differences in the propensity to manipulate that we have focused on so far, that contribute to the noisiness of short-run performance reports. Such other sources of random noise would have a qualitatively similar impact on the optimal pay contract in our model. For example, short term performance could contain transitory elements; or new technologies could be hard to interpret; etc. Any such source of noise delays the incorporation of information about performance into the stock price, increases the deadweight cost of providing incentives for effort, and impacts price informativeness and the compensation contract.

The distinction between manipulation uncertainty and other transient noise is of relevance to the current debates about fair value accounting. The 2006 accounting standard FAS 157 (Fair Value Measurements) defines fair value as "the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date". If there is an active liquid market for the asset, then it is marked to market. Illiquid assets whose market value cannot be easily obtained are then "marked to model" – typically, through a NPV calculation based on some assumptions.\textsuperscript{12} Strict enforcement of a mark-to-market rule for illiquid securities will introduce error into the pricing if there are episodes of disorderly markets when securities are temporarily mispriced; relaxing the rule is intended to allow the manager to better convey long-run value, but the judgment calls involved in determining fair value, especially for assets and liabilities that are not or infrequently traded, give the management some scope for manipulation.\textsuperscript{13}

\textsuperscript{12}Because of the subjective nature of marking to model, US GAAP requires that companies provide a footnote disclosure that breaks down the fair values they use in the balance sheet into different levels, based upon the quality of the fair value estimates.

\textsuperscript{13}Michael Lewis describes an instance of such manipulation in The Big Short: "with no one else buying or selling exactly what Michael Burry was buy or selling, there was no hard evidence what these things are worth - so they were worth whatever Goldman Sacks and Morgan Stanley said they were worth ... "Whatever the banks’ net position was would determine the mark," he said. "I don’t think they were looking to the
historic cost accounting may give a very inaccurate picture of current value, but it leaves very little scope for manipulation. The accuracy-manipulability tradeoff discussed here is a neglected dimension of the recent debate on historic versus fair value accounting, which focuses mainly on illiquidity and systemic risk (see Allen and Carletti, 2008, and Plantin, Sapra and Shin, 2008).

These considerations carry over into the related transatlantic debate about rules- versus principles-based accounting. If purely random noise in signals is the problem, then principles-based accounting guidelines would mitigate the problem by allowing management discretion to smooth out any temporary fluctuations in earnings that do not affect the firm’s long run value, thus conveying a more accurate picture to shareholders. In contrast, if deliberate manipulation is the primary concern, then rules-based accounting would be more appropriate so that those managers who are more inclined to exaggerate relative to what investors expect have little latitude to inflate their reports.

The relative importance of manipulation uncertainty and other sources of noise is an empirical issue. The main observable difference between the two concerns the forensic evidence that emerges in the aftermath of large stock price changes. If price corrections are purely random and there is no ex post evidence of any intentional bias in prior reports, the random noise interpretation is more plausible. In contrast, when major price changes are found to be corrections of deliberately misleading prior disclosures (or failures to disclose price-relevant information), there is intentional manipulation. Ex post signs of deliberate manipulation could include SEC investigations, material earnings restatements and unusual accruals, abnormal insider trading activity and non-frivolous private securities litigation. Empirical evidence suggesting that manipulation uncertainty is a real concern includes Bergstresser and Philippon (2006), Burns and Kedia (2006), Peng and Röell (2008a) and Johnson et al. (2009).

An additional set of possible policy interventions directly targets manipulation by focusing on the detection of and penalties for extreme cases of manipulation. Effectively, such policies can be thought of as affecting the shape of the manipulation cost function in our model. As shown in Figure 2, a policy that makes the manipulation cost function more convex (reducing $\beta$) is value-enhancing because it not only reduces the noise in short-term managerial reports and stock prices but also reduces the waste of valuable managerial time spent on manipulation relative to productive effort.\footnote{The lower the $\beta$, the more convex the time cost of manipulation, and therefore the closer together the market for their marks. I think they were looking to their needs.\textsuperscript{a}} Such an increase in convexity could
come from any measure that makes it more costly to make egregiously inflated reports, without increasing reporting costs across the board. Examples include increased efforts to detect and penalize unusually optimistic reports, accounting standards that require managers to give a detailed justification if the reports deviate substantially from those of similar firms or an established benchmark, and extra scrutiny for accounts that exhibit signs of possible manipulation, based on measures of manipulation such as discretionary accruals or extreme stock price changes. The strength of social norms can also influence $\beta$ by increasing the utility cost of extreme deviations.

Interestingly, as shown in the middle panel of Figure 2, such policies to decrease $\beta$ and enhance shareholder value may not reduce the waste of resources associated with manipulation: under the optimal contract the time spent on manipulation can be U-shaped in $\beta$. Two opposing forces determine the amount of manipulation: as $\beta$ decreases, there is less manipulation holding constant the employment contract. But that encourages the use of short-term incentives in the optimal employment contract, thus driving manipulation up.

### 4.2 The pay horizon

Much of the current concern about compensation focuses on situations where short-term outperformance was richly rewarded, while subsequent revelations showed that negatives for long-term value had simply been hidden from view. This is particularly true of the financial services sector in the years leading up to the 2008 crisis. The Council of Institutional Investors November 2010 report on Wall Street pay finds that:

"Little or no Wall Street compensation was linked to long-term future performance measures. This contrasts with compensation at many non-financial companies, where incentive pay was awarded for hitting long-term performance targets." (p.2)
Our model illustrates the advantages of conditioning pay on long-term performance and gives insight into the circumstances where an optimal contract between shareholders and their agents would put especial weight on long-term performance.

A short term contract (with $\eta = 0$) would be optimal under two conditions. Firstly, if there is no noise in the short term performance signal ($\Omega = 0$) so that all relevant information about managerial effort is incorporated in the short term stock price. Secondly, if short term pay does not induce managers to waste resources on manipulation ($\beta = 0$, that is, manipulation is prohibitively costly).

But in circumstances where the short term signal is an imperfect indicator of true performance, or where managers can manipulate performance effectively, conditioning pay on long term performance can be very valuable. As shown in Figure 1 (Panel B), when manipulation uncertainty increases, a purely short term optimal contract will contain weaker incentives ($\gamma\mu$) and shareholder value drops considerably. With a more flexible contract (Panel A), some of the reduced short term incentives can be replaced by long term incentives ($\eta$) and as a result, the negative impact of the noise in the short-term signal is alleviated. Similarly, in Figure 2, as $\beta$ increases (that is, the cost of extreme manipulation falls) and for any given pay scheme the proportion of time wasted on manipulation increases, short term incentives need to be replaced by long term incentives to reduce this wastage.

Our model thus predicts that long term pay is important in businesses that are not easily analyzed by outside investors, that is, where the manager cannot credibly convey the value of his initiatives in the short run, so that the stock price will only reflect his contribution after the passage of time. Examples of such companies are high-technology, "new economy" firms, and financial firms with illiquid, opaque, off-balance sheet assets and/or liabilities, while counterexamples include utilities, retailers and "old-economy" manufacturing firms. This prediction is consistent with empirical work such as that of Bizjak, Brickley and Coles (1993), who find that asymmetry of information (measured by market/book ratios and R&D) between managers and the stock market increases the reliance on long-term incentives relative to short-term incentives. Cadman, Rusticus and Sunder (2010) similarly find that vesting periods are longer for growth companies, where short-term performance indicators are relatively less meaningful.

Other considerations that determine the pay horizon include firm size, fundamental risk and managerial risk aversion. Figure 3 illustrates how the optimal mix of long and short run incentives depends on firm size. At the lower end of the range of firm sizes, short-
term incentives increase and the amount of manipulation increases concomitantly. But the proportion of time spent on manipulation reaches a maximum at an intermediate firm size and then decreases, as relatively more reliance is placed on long-term incentives. Intuitively, as firm size increases effort increases and the manager’s time becomes a relatively more scarce resource. There comes a point beyond which the waste of time spent on manipulation outweighs the risk-reducing advantages of short-term incentives; thus, at the higher end of the firm size scale, incentives shift to the long run. This prediction of our model, relating the pay horizon to firm size, is a new and testable result which merits empirical investigation.

[Insert figure 3 about here]

Meanwhile, as shown in Figure 4, if the manager is less risk averse, long term pay is more appropriate because he is less concerned about the additional risk associated with long term pay; thus, for example, young and male CEOs, should have more long term pay contracts (for evidence that risk aversion increases with age, see for example, Morin and Suarez (1983); and see Eckel and Grossman (2008) for a review of gender and risk aversion). A decrease in the long term risk ($\Sigma_2$) would clearly have a similar effect. For example, a startup pharmaceutical company that is developing a new drug has a considerable amount of short term uncertainty, but the long run uncertainty is low because the drug either succeeds or fails. If it is impossible to credibly convey that the drug is promising in the short run, then pay should be conditioned on the long term outcome.

[Insert figure 4 about here]

Turning to the practical implementation of long-run incentives, they can be voluntarily included in pay contracts, in the form of slow-vesting stock or options or deferred bonus payouts. Such pay contracts can deter a broad spectrum of opportunistic behavior, ranging from overinvestment in projects that enhance short-run performance indicators to downright fraudulent accounting practices, because the compensation automatically reflects long-term value.

But managers may have a preference for receiving pay in the short run if they have liquidity needs and it is costly to borrow on personal account. Then the bulk of pay may have to be paid out before the final outcome is determined, and implementing long-term
incentives would involve some degree of clawback of money that has already been paid out. In practice clawback clauses may not be credible, because it is so costly to prove that a violation has taken place or to identify those responsible, and the legal costs of retrieving any improperly awarded pay from executives *ex post* are prohibitive. If so, there is a role for government intervention to help foster the use of long term incentives.

Recent policy initiatives are a step in that direction. In response to the accounting scandals of the turn of the century, the Sarbanes-Oxley Act of 2002 included a provision (Section 304) to claw back unwarranted pay in the wake of misconduct, to be enforced by the SEC: CEOs and CFOs must reimburse their company for any bonuses, other incentive-based or equity-based compensation, and profits from stock sales in the year following misstated financial reports that involve company misconduct. Very recently, the SEC has become more aggressive in seeking such disgorgement of unwarranted pay. In response to the financial crisis, Section 954 of the new Dodd-Frank Act of 2010 adds a number of new and tougher clawback provisions: issuers themselves are required to recover any excess incentive-based compensation awarded in the three years preceding a restatement from all "executive officers", with no limitation to restatements triggered by misconduct.

These policy measures have ensured that companies can more credibly commit to the clawback of undeserved pay in the wake of long run underperformance. Over time, company boards have responded by adopting policies to defer bonuses and to recoup improperly awarded incentive compensation: by 2009, 73% of Fortune 100 companies had publicly disclosed clawback policies, up from 18% in 2006. Banks and securities firms are also toughening rules that give them power to seize previously awarded pay from employees for a variety of reasons: expected profits not materializing, excessive risktaking, violation of company policies, etc. Regulatory pressure has helped to ensure that such repayments are actively pursued.

But note that the policy reforms discourage outright violations of accounting rules but they do not address all creative accounting, some of which is well within GAAP rules. Nor do they deter real manipulation: the survey conducted by Graham, Harvey and Rajgopal (2005) finds that most managers would rather take real actions that sacrifice long-run value, than manage earnings via accounting adjustments. Thus the effectiveness of legal remedies

---

15 In two recent cases the SEC has even gone after CEOs who are not personally charged with misconduct: the former CEO of automated teller machine maker Diebold Inc. has agreed in a settlement to return some of his bonus awards, and there is a case pending to recover stock-option gains and bonuses from the former CEO of auto-parts retailer CSK Auto Inc.
is limited to special forms of manipulation, where there is an accounting restatement and/or demonstrable misconduct.

4.3 How to measure incentives empirically

Our framework has three major implications for the empirical measurement of incentives. First, the strength of incentives is determined by the elasticity of pay to firm value, not the $-for-$ sensitivity. Second, in measuring short-term incentives, it is not just the elasticity of pay to price, but also the elasticity of price to performance reports, that matters. Third, short- and long- term incentives can differ dramatically in their response to underlying determinants, so that an exclusive focus on the behavior of short run incentives can give a misleading picture.

With regard to the first point, in the model it is the elasticity of managers’ wealth to firm value (percent-for-percent) rather than the sensitivity (dollar-for-dollar) that determines effort. This is a result of Cobb-Douglas managerial preferences and multiplicative impact of effort on firm value assumed in the model, in contrast to the traditional specification used in the literature, which is additive in both respects. In our model effort is determined by the elasticities of pay, \( \mu \) and \( \eta \), as shown in equation (10). For example, consider the case of a manager who has no wealth of his own and is compensated entirely through the long-run appreciation of his stock (i.e., \( \eta = 1 \) and \( \mu = 0 \)): he will put in an equal amount of effort, no matter what proportion of the total stock he holds (\( \omega \)). His stake (\( \omega \)) needs only to be large enough to induce him to accept the job. What matters in eliciting effort is the elasticity of the compensation scheme - the more elastic it is, the more effort is induced.

The elasticity approach aligns well with the empirical evidence on the cross-sectional variation of incentives. Murphy’s (1999) survey of empirical work on executive compensation notes that models phrased directly in terms of elasticities fit better:

"The primary advantage of the elasticity approach is that it produces a better “fit” in the sense that rates of return explain more of the cross-sectional variation of \( \Delta \ln(\text{CEO Pay}) \) than changes in shareholder value explain of \( \Delta(\text{CEO Pay}) \)."

Empirical work often uses $-for-$ pay sensitivity as a measure of incentives, which is appropriate when the manager is constant absolute risk averse and his effort impacts the firm additively. But elasticity-based incentive measures capture incentives better when it is more realistic to assume that the manager is constant relative risk averse and his efforts
impact firm value proportionally. Then Jensen and Murphy’s (1990) finding that prior to the 1990s, CEO wealth increased by $3.25 for every $1000 increase in firm value, is not necessarily a sign of weak incentives. For if the manager is not independently wealthy and his average pay is low relative to the value of the firm, such a low slope for the pay-performance relationship may well entail strong incentives in terms of the elasticity. For example, a 1% increase in the value of a firm worth $100 billion would have led to a $3.25 million increase in executive pay, an increase in wealth by nearly 1/3 for an executive with initial wealth $10 million.

Second, in our model the short term incentive depends on $\gamma \mu$, the product of the price elasticity of pay and the elasticity of the stock price with respect to the performance report from the manager (which is proportional to effort $E$); while empirical studies typically focus on $\mu$, the elasticity of managerial pay with respect to the stock price alone. We have shown in Proposition 1 that in the presence of high manipulation uncertainty or other noise, the stock market is less responsive to managers’ reports ($\gamma$ is low) and therefore a higher powered incentive $\mu$ is necessary to induce optimal effort even though the product $\gamma \mu$ (and thus effort $E$) is actually lower, as argued in Section 4.1. That is, a high pay sensitivity to stock price $\mu$ can be consistent with a low effort $E$ if it is difficult to persuade the stock market that performance is high. The implication for empirical work is that simply using the stock price elasticity of pay to proxy for incentives may give rise to misleading conclusions about the power of incentives. Differences in $\gamma$, the responsiveness of the short term price to managerial performance reports, should also be accounted for.

Third, the total incentive is determined by both short term ($\gamma \mu$) and long term elasticities of pay ($\eta$). In some cases long term and short term incentives may move in opposite directions when the underlying parameter changes. For example, in Figure 1, when manipulation uncertainty increases, long term incentives increase, while short term incentives decrease. Similarly, in Figure 3, the short term incentive is eventually decreasing in firm size, while the long term incentive is increasing throughout; and overall, the total incentive is increasing in firm size. This pattern is obscured in empirical work that restricts attention to short run incentives.

But in practice, empirical work has focused more on the short term incentives because they are more easily measured with readily available data. Measuring long term incentives can be tricky since the performance triggers in the long term incentive plans are not standardized; and some aspects of the plan, such as the conditions for firing a CEO, are
necessarily implicit. Unvested options and restricted stock do give some insight into long term incentives, but have some limitations. In particular, the average vesting period can be quite short – the median vesting period is 2 years for options and 3.3 years for restricted stocks for the ExecuComp firms analyzed by Chi and Johnson (2009). Since 2007, the SEC requires more detailed compensation disclosure, which should enable researchers to better measure how pay responds to short- and long term performance.

4.4 Are options optimal?

There is an open debate about the degree to which stock options should form a part of executive compensation contracts. Options are a way to achieve a convexity in pay and thus to increase its elasticity: restricted stock ownership cannot provide an elasticity that is greater than one. In general, there is no reason why the optimal pay elasticity should be one or below. As shown in Proposition 5, it is optimal to induce more effort if the firm is large relative to managerial wealth, if the manager is productive or if the manager is not too risk averse, and pay needs to be more elastic to elicit such effort. In those circumstances, options are more likely to be part of the optimal contract.

An additional factor that may call for higher pay elasticity is any noise in the short term signal, resulting \textit{inter alia} from manipulation uncertainty. As discussed in Section 4.1 and 4.3 this tends to increase the sensitivity of optimal pay to short-run performance ($\mu$). It also increases the sensitivity to long-run stock performance ($\eta$), as illustrated in Figure 1 Panel A: the long-run pay elasticity is pushed up to mitigate the difficulties with providing short-run incentives. The model thus suggests that for companies, industries or managerial profiles in which the uncertainty about managerial manipulation is high, that is, it is harder to disentangle the true firm value from hype in the manager’s disclosures, the optimal contract should actually be more elastic with respect to price. Thus the model predicts that options, viewed as a means of adding convexity and thereby raising the elasticity ($\mu, \eta$) of the pay scheme, might be more prevalent in such situations.

The prediction of the model is consistent with empirical findings on the use of options. Option compensation is more prevalent in high-technology, "new economy" firms and more generally in growth industries (such as computer, software, and pharmaceutical firms), and less so in utilities, "old-economy" manufacturing firms and low-growth industries in general (Murphy, 1999, Core and Guay, 2001, Ittner, Lambert and Larcker, 2003). The evidence contradicts the traditional agency model intuition that in a riskier industry, the optimal
contract should be less price-sensitive in order to reduce the risk imposed on the managers. But our model can explain the empirical findings: these industries do not just have high fundamental volatility ($\Sigma$), but also high uncertainty in the interpretation of short-run signals ($\Omega$). Then, even though optimal incentives are weakened overall by the short-term noise, pay will typically be more stock price elastic in both the short and the long run.

This mechanism is conceptually distinct from the usual argument in favor of option compensation: that it aligns the interests of managers who are risk-averse with shareholders’ objectives by mitigating their risk aversion, thus encouraging them to choose more volatile projects (though Lambert, Larcker, and Verrecchia (1991), Carpenter (2000), Ross (2004), and Lewellen (2006) all point out that this intuition does not have general validity). Empirically, Coles, Daniel and Naveen (2006) and others do show that options tend to induce risk taking. But these papers do not directly address whether options should be part of an optimal contract, and if so, to what extent. Dittman and Maug (2007) argue that the observed widespread use of option contracts cannot be rationalized by calibrating a standard additive principal-agent model with constant relative risk aversion and lognormal stock prices. Subsequent work by Dittman and Yu (2010) shows that by incorporating the need to provide risk-taking incentives in addition to effort incentives into the calibration, the predicted option holdings match up well to observed compensation practice.

In contrast, our model provides a rationale for the use of options that does not depend on a need to provide risk-taking incentives: once manipulation uncertainty is taken into account, the short-term elasticity of the optimal contract $\mu$ (and hence its convexity) increases. Even for reasonable levels of risk aversion, options may be needed to achieve the optimal degree of convexity, especially if the manipulation uncertainty is high. And even short-term options may be optimal if, say, long-run fundamental uncertainty is relatively high or if the CEO is very risk averse.

In the public debate, option compensation is often held responsible for the manipulation of short-term stock prices. Former SEC chairman Arthur Levitt (2002, p.111) describes the impact of the upsurge in the use of option-based compensation from the late 1990s onwards as follows:

... the options craze created an environment that rewarded executives for managing the share price, not for managing the business. Options gave executives strong incentives to use accounting tricks to boost the share price on which their compensation depended.
Burns and Kedia (2006) and Peng and Röell (2008b) find that accounting manipulation and securities fraud are related to executives’ holdings of options, but not other components of pay such as stock. This evidence suggests that manipulation is a potentially damaging side effect of providing strong incentives, that should be considered when setting a pay contract. But, rather than eliminate options, it may be more efficient to modify their terms, for example by lengthening the vesting period, as discussed in Section 4.2.

4.5 Effort, incentives and firm characteristics

As shown in Proposition 5, the optimal contract elicits effort that is increasing in firm size. The intuition is that the gain in firm value per unit of managerial time devoted to effort is proportional to firm size \((X)\). So, all else equal, a more elastic pay contract providing stronger incentives is optimal for larger firms. In contrast, in traditional linear-additive models, the optimal $-for-$ pay sensitivity and the resulting effort are independent of firm size. It is worth pointing out that this relationship between firm size and the power of incentives is driven by the multiplicative nature of the model rather than by considerations arising from manipulation. This is clear from Appendix A where both the first best (equation A3) and the manipulation-free second best (equation A6) contracts have effort increasing in firm size.

The relationship between the size of the firm and the structure of the optimal contract is illustrated in Figure 3. Panel B depicts the purely short-term contract of Peng and Röell, 2008: the increase in effort is achieved through a monotonic relation between short term incentives and firm size (immediate from equation A8). In contrast, the short-term incentives are nonmonotonic when long-term incentives are available, as shown in Panel A. As firm size increases, optimal effort increases because its impact on firm value is greater. There comes a point beyond which managerial time is so scarce that it becomes important to avoid wasting time on manipulation, and thus incentives shift away from the short run to the long run.

Empirical work by Gibbons and Murphy (1992) and Edmans et al. (2009) finds that the relationship between the pay-performance elasticity and firm size is insignificant. To the extent that empirical work measures the sensitivity of pay to contemporaneous stock

\footnote{Thus even in the first best, optimal effort increases in firm size, as shown in equation (A3) of Appendix A.1.}

\footnote{Note that, even if the elasticity of pay increases with firm size, the average $-for-$ sensitivity of pay can still be decreasing, consistent with empirical findings such as those of Baker and Hall (2004).}
returns mostly captures the short-term incentives, it is consistent with our model that no unambiguous monotonic relationship can be found, as our model predicts a hump-shaped relationship. In our model, equilibrium effort rises with firm size as depicted in the central column of Figure 3 (even in a first best setting this would hold). But this relationship is obscured if long-term incentives are inadequately captured. In contrast, Edmans et al. (2009) interpret their findings as evidence supporting a multiplicative model in which the effort decision is simplified to a binary choice, so that the optimal effort is constant, independent of the scale of the firm.

We turn now to the impact of the fundamental volatility of the firm’s business, both in the short run ($\Sigma_1$) and in the long run ($\Sigma_2$), on the optimal pay contract. The interplay between short run and long run incentives leads to novel comparative static predictions.

Short-term incentives ($\gamma\mu$) decline steadily as short term risk ($\Sigma_1$) increases, as shown in Figure 5, because the cost of compensating the manager for bearing the risk attached to the incentives increases; and there comes a point beyond which it is optimal to provide no short-term incentives ($\mu^* = 0$). On the other hand, as $\Sigma_2$ increases, $\mu$ tends to increase as incentives are shifted to short-term performance because of the extra risk from conditioning payments on long-term performance. Thus if empirical work focuses on short term measures of incentives, it is not always clear that businesses with higher fundamental risk should have reduced incentives – the impact of fundamental risk on short term incentives depends on the timing of the risk. As for predictions concerning the degree of manipulation, managers will devote less time and attention to managing short term stock prices in firms that are more risky in the short term, as a side effect of the weaker short-term incentives.

It would be natural to expect long-term incentives ($\eta$) to decrease with short term risk ($\Sigma_1$) as well, given that the associated total risk ($\Sigma_1 + \Sigma_2$) increases. But, as shown in Figure 5, that is not the case. There is an intermediate range where $\eta$ is actually increasing, presumably as a substitute for the drop in short-term incentives: the risk-reduction advantages of short-term compensation become less salient and the manipulation-cost-saving advantages of long-term compensation dominate. Once short-term incentives have been driven down to zero altogether, there is no further substitution effect and so increases in risk simply lead to decreases in long-term incentives. As for the impact of long term risk ($\Sigma_2$) on long term incentives, the usual result obtains: more risk, weaker incentives.
Comparative static predictions across industries require caution because the underlying parameters of the model may move together. For example, firms with greater fundamental uncertainty (\( \Sigma_1, \Sigma_2 \)) may also have noisier short term signals or greater manipulation uncertainty (\( \Omega \)). The optimal mix between short term and long term incentives then depends on the relative magnitude of the three.

4.6 Incentives and CEO characteristics

In this section, we discuss the impact of CEO characteristics such as personal wealth, ability, length of tenure and risk aversion. These implications of the model are empirically testable, but currently underexplored because personal data on managerial characteristics such as skills and personal wealth are required.

Proposition 5 shows how the personal characteristics of the manager matters in determining the optimal amount of effort elicited. In the model, the manager’s reservation wage (\( W \)), time endowment (\( L \)) and skill (\( 1/C_E \)) enter alongside firm size \( X \) into equations (A28) and (A29), determining the optimal incentives via the expression \( \frac{X}{W} \left( \frac{T}{C_E} \right)^\alpha \): the ratio of the maximum value of the firm (attained if the manager devotes 100% of his time to effort) to his reservation wage. Thus managers who are poorer (so that they attach less money value to extra leisure), have more time (for example, they are not managing other firms in parallel, and they are not primary caregivers for small children) and/or are more skilled will be given stronger incentives for effort, and in particular stronger long-term incentives: the impact is identical to that of an increase in \( X \). Again, these results are driven by the multiplicative nature of the model and are present even in the first best (equation A3) and the manipulation-free second best (equation A6) contracts.

The model also predicts that young, untested, executives should receive stronger incentives through pay. This is because they do not have a track record of disclosures to interpret, and therefore manipulation cost uncertainty (\( \Omega \)) is higher. This prediction is consistent with survey evidence of Graham, Harvey and Puri (2009), suggesting that younger executives and those with short tenure receive more compensation in the form of stock, options and bonuses relative to base pay: if the stock market is more unsure about the meaning of their reports, the short run price is less responsive and so the short-term pay needs to be more price-responsive to elicit adequate effort. Regarding the compensation horizon, the model predicts that young executives should receive pay in a form that depends relatively more on long-term outcomes.
The impact of the managers’s risk aversion \((1 - \phi)\) on the optimal contract, effort-manipulation-leisure choice, and firm value is shown in Figure 4. Normally, one would expect high risk aversion to lead to weaker incentives, and that is indeed what happens when the contract is restricted to just a single pay horizon. In contrast, the incentives under the general contract display interesting and non-monotonic patterns when risk aversion varies: when the risk aversion is low enough, only long-term incentives are used: pay is not linked to the short-term stock price. However, as risk aversion increases, short-term incentives become an attractive way to manage risk and the elasticity of pay to long-term value is reduced; and as a side effect, managers devote part of their time to manipulation. As risk aversion increases further, both long- and short-term elasticities fall due to the costs of bearing risk, and both effort and manipulation decline. The figure thus illustrates that short-term incentives are not monotonic in risk aversion: from zero, they first increase as they replace long-term incentives, and then they decline alongside the long-term incentives as risk aversion increases further.

More generally, how is the amount of manipulation related to managerial attributes? In the pure short-term contract of Peng and Röell (2008), depicted in Panel B of Figure 3, there is no way to elicit more effort without accepting more manipulation; as a result, manipulation increases with managerial skill and decreases with managerial wealth. This means, for example, that more skilled managers will also manipulate more even if there is no correlation between skill and the propensity to manipulate. But if there is scope for long-term contracting (Panel A), manipulation is first increasing and then decreasing in firm size and managerial skill - when the stakes are high enough, long-term incentives will replace the short-term ones.

Although uncertainty about managerial skills (the cost-of-effort parameter \(C_E\)) has not been explicitly discussed so far, any such uncertainty would be equivalent to an increase in the fundamental volatility \(\Sigma_1\), and therefore also be associated with weaker overall effort incentives, especially in the short term. Intuitively, the contract mitigates the exposure of the manager to ex ante risk concerning his skills, just like other sources of risk captured by \(\Sigma_1\). But in contrast to manipulation uncertainty, such skill uncertainty reduces the short-term pay elasticity \(\mu\) and increases the responsiveness of the stock price to the performance report \(\gamma\).

These comparative static predictions so far assume that all else is held constant. But in any equilibrium of the market for executives, there will be matching between firm character-
istics and managerial traits. In recent survey evidence of such matching, Graham, Harvey, and Puri (2009) find that less risk averse managers seek employment in more volatile industries. This implies that cross-industry empirical relationships between either risk aversion or firm volatility and overall incentives will be somewhat attenuated if this matching is ignored. Similarly, Gabaix and Landier (2008) show that more skilled managers are likely to be matched to larger companies; this matching, if skills are omitted from the regression, should magnify the empirically observed link between firm size and incentives. Conversely, if CEOs of large companies also tend to be wealthier, the link would be attenuated.

Lastly, would a firm prefer to hire a poor or wealthy manager, all else equal? If the manager only needs to be compensated for the disutility of his effort and for the risk imposed on him, then a firm would prefer to hire a less wealthy manager because the money value of both his time and his risk premium is proportional to his wealth. This is a consequence of the Cobb-Douglas and CRRA preferences assumed.

5 Conclusion

In the wake of the 2008-2009 financial crisis, the disadvantages of short-term pay contracts have become painfully apparent. Policy makers, investors, and the press are calling for the adoption of executive pay schemes with long vesting periods and "clawback" clauses that postpone the determination of final pay until all the evidence is in.

Our framework is designed to explore these ideas by allowing for long term as well as short term incentives in an agency model with manipulation. Short term stock-based incentives elicit not only productive effort, but also wasteful manipulation, while long term incentives mitigate the economic waste associated with manipulation, but expose the (risk averse) manager to extra risks. We investigate how the optimal mix depends on characteristics of the company, the managers and the business environment. Pay horizons need to be longer in cases where short run information is unreliable, either because it is inherently noisy or because the extent to which it is manipulated is uncertain. Examples include high tech start ups and growth companies with hard to value technologies, private equity firms with illiquid assets, or investment banks with complex positions and trading strategies. We also show that longer term incentives are appropriate for larger firms and less risk averse managers. Practical examples of pay that is de facto sensitive to long term performance include stock and option pay plans with long vesting periods; the structuring of private equity contracts so that any excessive carried interest distributions to the general partners are "clawed back"
at the end of the fund’s life; and deferred compensation, which is unsecured and therefore vulnerable if a company subsequently goes bankrupt or underperforms.

We show how manipulation, and investors’ uncertainty about it, affects the power of incentives and the informativeness of asset prices. When manipulation uncertainty is high, or more generally, when short-term performance information is very noisy, pay actually tends to be more closely linked to the stock price even though the stock price is less informative, while the equilibrium level of effort is lower and the stock price is less sensitive to the manager’s report. The cross-sectional implication is that highly stock price dependent incentive contracts are more likely to be used in situations where there is high uncertainty regarding the propensity to manipulate as identified above: when managers are young or untested, or when firms are high-tech or high-growth or reliant on intangible assets. This suggests that measuring incentives by simply looking at the sensitivity of pay to the contemporaneous stock price return can be misleading, as seemingly powerful incentives may elicit little effort if the full impact of the effort on the stock price is delayed beyond the pay horizon.

Our tractable multiplicative formulation generates empirically testable predictions about the determinants of optimal pay elasticities. Incentives for effort depend directly on the elasticity of pay to firm value rather than the "dollar-for-dollar" sensitivity. This implies that whenever the optimal pay elasticity is high, option pay would be a natural part of the optimal contract (as the elasticity of a pay package consisting of only fixed base pay and stock is necessarily less than one). As we have seen, this is particularly likely when short term information is a poor indicator of long term value, which is often the case in firms with risky fundamentals. This explains why such firms tend to rely more on option pay, even though an agency model that neglects the impact of noise on the responsiveness of stock price to effort would predict the opposite.

We find that the interplay between long- and short-run incentives in eliciting effort whilst mitigating manipulation generates nonmonotonic patterns that can be counterintuitive. For example, optimal short-term incentives (as measured by the elasticity of pay to short-term performance) can be increasing in risk aversion and decreasing in firm size, and long-term incentives can be increasing in the fundamental volatility of the business. The optimal contract design is highly firm/manager specific, and this makes the job of a "pay czar" entrusted with designing such contracts particularly daunting.

Rather than imposing constraints on the contract design, we suggest that public policy can focus on improving the contracting environment. For example, policies that reduce
noise and manipulation uncertainty, such as tighter accounting and disclosure rules, or that increase the convexity of manipulation costs, for example by ex-post penalties on extreme misrepresentation of performance, would allow investors to make better inferences and result in more efficient contracts that enhance shareholder value.

In addition, policy can enable firms to credibly commit to claw back unwarranted pay in the wake of misconduct, given that ex post it is very costly to do so and shareholders might be better off letting bygones be bygones. The clawback clauses imposed in the Sarbanes-Oxley Act of 2002 have been reinforced in the Dodd-Frank Act of 2010. Now they oblige companies to recover excess pay in the event of accounting restatements, and there does not need to be executive wrongdoing involved to trigger the recoupment of pay. While limited to accounting manipulation, these measures are a step in the direction of facilitating the use of long run incentives.

Appendix A Benchmark contracts

In this section, we characterize three benchmark contracts. First, we consider the first-best level of effort and pay in the absence of agency problems. We next characterize the optimal second-best contract in a setting without manipulation. Lastly, we consider a setting with manipulation but without long-term pay. The results will serve as a basis for comparison with the optimal contract in the presence of manipulation that is analyzed in Section 3.

A.1 The first best

In the first best optimum, Pareto-efficient effort and compensation are chosen to maximize the expected wealth of the firm’s owners, subject to the agent’s participation constraint of reaching an expected utility of at least \( U(W, L) \equiv \frac{1}{\phi} \left( \frac{\psi}{\phi} W \right)^{\phi} \), his reservation utility. Of course no wasteful manipulation takes place (\( M^* = 0 \)), the manager is paid a fixed salary \( W^* \), and effort and compensation solve the program:

\[
\max_{\{E,W\}} E \cdot X - W \quad \text{(A1)}
\]

subject to \( \left( L - C E^{\frac{1}{\alpha}} \right)^{\psi} W \geq \frac{\psi}{\phi} W \)

and \( E \geq 0 \).
Taking first-order conditions, the first-best solution for the fixed wage $W^*$ and the fraction of time devoted to effort $f^*$, where $f \equiv \frac{C_E E^2}{L}$, is given by:\(^{18}\)

\[
\frac{f^{*1-\alpha}}{(1-f^*)^{\Psi+1}} = \frac{\alpha X}{\Psi W} \left( \frac{L}{C_E} \right)^\alpha
\]

(A3)

\[
W^* = \left( \frac{1}{1-f^*} \right)^\Psi W
\]

(A4)

Naturally, shareholders would not wish to enter into an employment relationship with the manager unless the ensuing expected firm value is positive net of executive compensation; for this, the reservation wage $\overline{W}$ needs to be low enough relative to the other parameters of the model.\(^ {19}\)

Note that from equation (A3), effort is increasing in firm size ($X$) and decreasing in the reservation utility ($\overline{W}$). Because leisure and money are complementary, the preference formulation has the property that it is efficient for "fat cats" to put in less effort than "lean and hungry" executives. Observe also that firms, and in particular large firms who optimally require more effort, would prefer to hire executives who are not independently wealthy: poor executives are willing to exert the same amount of effort in return for less money.

### A.2 Second-best contract in the absence of manipulation

We now characterize the second-best optimal contract in the absence of opportunities for manipulation for our multiplicative version of the classical agency model. Then $\beta = 0$ (i.e.,

\[^{18}\]For the special case $\alpha = 1$, first-best effort is zero ($E^* = 0$, $f^* = 0$) whenever the nonnegativity constraint on effort binds, that is, if:

\[
\overline{W} \geq \frac{XL}{\Psi C_E}.
\]

(A2)

\[^{19}\]For example, for the special case $\alpha = 1$, the first-best expected firm value net of executive compensation is (assuming the manager contributes personal wealth $\overline{W}$ and needs to attain utility $U(\overline{W}, L)$) is:

\[
\frac{XL}{C_E} \left[ 1 - \frac{\Psi + 1}{\Psi} \left( \frac{C_E \overline{W}}{XL} \right)^{\frac{1}{\Psi+1}} \right] + \overline{W},
\]

which is nonnegative as long as

\[
\overline{W} \leq \frac{\Psi}{(\Psi + 1)^{\frac{1}{\Psi+1}}} \frac{XL}{C_E} \left( 1 + \frac{C_E \overline{W}}{XL} \right)^{\frac{1}{\Psi+1}}.
\]

Thus, if the manager has no initial wealth to contribute($\overline{W} = 0$) and for a consumption-leisure tradeoff parameter $\Psi = 1$, the reservation wage $\overline{W}$ cannot exceed one-quarter of the firm’s potential gross value $\frac{XL}{C_E}$ that would be achieved if the manager were to work full-time.
manipulation is prohibitively costly) and therefore by equation (16), \( \gamma = 1 \). In this case, it is optimal for pay to depend only on the short-term price so that \( \eta = 1 \) and \( \mu \) is chosen to maximize expected net firm value as expressed in equation (25), that is:

\[
\max_{\eta} X \left( \frac{L}{C_E} \frac{\alpha \mu}{\Psi + \alpha \mu} \right)^{\alpha} - \frac{W}{1 - \phi} \exp \left\{ \frac{1}{2} (1 - \phi) \mu^2 \Sigma_1 \right\} \quad \text{(A5)}
\]

**Proposition 6** When managerial pay depends only on the long-term value of the firm, so that there is no opportunity for manipulation, the optimal contract is characterized by an elasticity of pay to the short term firm value, \( \mu \), given by the following first-order condition:

\[
\frac{X}{W} \left( \frac{L}{C_E} \right)^{\alpha} = \frac{(\Psi + \alpha \mu)^{\alpha + \Psi} \mu^{1 - \alpha}}{\alpha^{\alpha + 1} \psi^{1 + \psi}} \left[ \alpha \Psi + (\Psi + \alpha \mu) \mu (1 - \phi) \Sigma_1 \right] \cdot \exp \left\{ \frac{1}{2} (1 - \phi) \mu^2 \Sigma_1 \right\} \quad \text{(A6)}
\]

For \( \alpha > 1 \) there always exists a value of \( \mu > 0 \) that satisfies the first order condition (A6).\(^{20}\)

### A.3 Optimal short-term pay contracts with manipulation

We now turn to a setting where stock price manipulation is possible: pay is linked to the short-term stock price, and that price is based on a manipulable report by the manager. In the current section it is assumed that pay can not be tied to long-term performance (so that \( \eta \equiv 0 \)), as in Peng and Röell (2008b).

Inducing effort by making pay depend on short term performance has the inevitable side effect of encouraging the manager to manipulate the stock price. Setting \( \eta \equiv 0 \) so that pay is constrained to depend on short-run performance only, the price elasticity of pay, \( \mu \), needs to maximize net firm value as expressed in equation (25):

\[
\max_{\mu} X \left[ \frac{L}{C_E} \frac{\alpha \gamma \mu}{\Psi + \alpha \mu} \right]^{\alpha} - \frac{W}{1 - \phi} \left[ \frac{\Psi + (\alpha + \beta) \gamma \mu}{\Psi} \right]^{\psi} \cdot \exp \left\{ \frac{\gamma \mu^2}{2} (1 - \phi) \Sigma_1 \right\} \quad \text{(A7)}
\]

**Proposition 7** When managerial pay depends only on the short-term value of the firm, the optimal contract is characterized by an elasticity of pay to the short term firm value, \( \mu \), given

\(^{20}\)For \( \alpha = 1 \), there is a solution with \( \mu > 0 \) as long as the parameters do not satisfy condition (A2), that is, as long as first-best effort \( E^* \) would be strictly greater than zero.
by the following first-order condition:

\[
\frac{X}{W} \left( \frac{T}{C_E} \right)^{\alpha} \frac{\alpha + 1}{\Psi + (\alpha + \beta) \gamma \mu} \left( \frac{\Psi}{\Psi + 1} \right)^{1-\alpha} = \frac{[\Psi + (\alpha + \beta) \gamma \mu]^{\alpha+\Psi} (\gamma \mu)^{1-\alpha}}{\Psi^{\Psi+1}}.
\]  

(A8)

\[
\cdot \left[ (\alpha + \beta) \Psi + [\Psi + (\alpha + \beta) \gamma \mu] \mu (1 - \phi) \Sigma_1 \right] \cdot \exp \left\{ \frac{\gamma \mu^2}{2} (1 - \phi) \Sigma_1 \right\}
\]

It follows directly that when \( \Omega \) increases, \( \gamma \) falls, \( \mu \) goes up (and \( \gamma \mu \) falls).

In this simplified setting, it is feasible to explore the robustness of this result. We argued in Section 4.1 that the increase in elasticity \( \mu \) is needed to partially offset the reduced sensitivity \( \gamma \) of the stock price to reported performance. The alternative intuition is that the volatility of the period 1 stock price is decreasing in \( \Omega \), thus reducing the risk imposed on the manager by stock-price-driven incentive pay and encouraging a more price-sensitive contract.\(^{21} \)

But this setup only describes a single period of tenure. This adequately describes the situation of a new venture, but may not capture the situation of an ongoing firm, where uncertainty about the achievements of the previous management is not yet fully resolved at time 0 when a new pay contract is signed.

Consider instead a "relay race" model of a long-run steady state, where each period can be thought of as the lifespan of a management team, who pass on their responsibilities to a new team at the end of the period. The firm has already settled into a line of business with associated underlying per-period fundamental volatility \( \Sigma_1 \) and manipulation uncertainty \( \Omega \), and the price volatility within each period is equal to the inherent variance of the fundamentals \( \Sigma_1 \), because in a weak-form efficient market, volatility cannot be made to disappear; it can only be pushed into the future. Then at the date of signing a new management team’s pay contract the full impact of the previous team’s efforts is as yet unknown, so that a proportion \( (1 - \gamma) \Sigma_1 \) of the stock price variance is resolved during the current management’s tenure and incorporated into the stock price at its end. The original model needs to be modified to add an uncorrelated shock of variance \( (1 - \gamma) \Sigma_1 \) impacting stock return; but the model is otherwise unaffected. Then the maximization problem (A7) will no longer carry a factor \( \gamma \) in the exponential terms, but otherwise it will be identical, and the first order condition (A8) changes to:

\footnote{To see this, observe that, using equations (2), (6) and (16), the first-period logarithmic return variance is scaled down by a factor \( \gamma \):

\[
var \left[ \ln \frac{P_1}{P_0} \right] = var \left[ \ln \frac{P_1}{XE} \right] = \gamma^2 (\Sigma_1 + \beta^2 \Omega) = \frac{\Sigma_1^2}{\Sigma_1 + \beta^2 \Omega} = \gamma \Sigma_1
\]  

(A9)}
\[
\frac{X}{W} \left( \frac{L}{C_E} \right)^{\alpha} \alpha^{\alpha+1} = \frac{[\Psi + (\alpha + \beta) \gamma \mu]^{\alpha+\Psi} (\gamma \mu)^{1-\alpha}}{\Psi^{\Psi+1}}.
\]  \hspace{1cm} (A10)

\[
\cdot \left[ (\alpha + \beta) \Psi + \left( \frac{\Psi}{\gamma} + (\alpha + \beta) \mu \right) \mu (1 - \phi) \Sigma_1 \right].
\]

\[
\cdot \exp \left\{ \frac{\mu^2}{2} (1 - \phi) \Sigma_1 \right\}.
\]

The RHS of this equation is not monotonic in \( \gamma \) and so in the steady state setting it is no longer always true that greater manipulation uncertainty \( \Omega \) (reducing \( \gamma \)) will lead to an increase in the incentive parameter \( \mu \). However, there is still a broad range of parameter values for which that is the case. For example, it is sufficient (but not necessary) that \( \gamma \mu \geq \frac{\alpha \Psi}{(\alpha + \Psi) \beta} \), that is, the optimal incentives for effort are higher than a constant determined by the parameters of the manager’s utility function. This condition holds if manipulation uncertainty is not too high (so \( \gamma \) is not too low), and if the firm is large and valuable relative to the manager’s wealth and the manager is skilled, so that strong incentives (high \( \gamma \mu \)) are optimal. For example, the condition holds if the ratio of firm size to managerial wealth \( \frac{X}{W} > 109 \), under the remaining parameter assumptions listed on page 18. Thus even in the steady state, when greater \( \Omega \) has no impact on the risk exposure of the manager, manipulation uncertainty still typically calls forth a more price-elastic pay contract (an increase in \( \mu \)) in order to offset the fact that the price is less responsive to the performance report. Meanwhile, the impact of an increase in manipulation uncertainty on equilibrium effort remains unambiguously negative as in the original model (this can readily be verified by recasting the RHS of equation (A10) in terms of the composite variable \( \gamma \mu \) and \( \gamma \): it is increasing in \( \gamma \mu \) and decreasing in \( \gamma \) when holding constant \( \gamma \mu \)).

**Appendix B  Proofs of propositions**

We use lower-case letters to denote the logarithms of the corresponding upper-case variables throughout. It is useful to introduce the short hand notation

\[
k_E = \ln \left( \frac{\alpha (\gamma \mu + \eta)}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \frac{L}{\bar{L}} \right) \quad \text{and} \quad c_E = \ln C_E
\]

\[
k_M = \ln \left( \frac{\beta \gamma \mu}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \frac{L}{\bar{L}} \right) \quad \text{and} \quad c_M = \ln C_M
\]  \hspace{1cm} (A11)
so that the effort and manipulation choices in equations (10) and (11) can be expressed as:

\[
e = \alpha (k_E - c_E) \tag{A12}
\]
\[
m = \beta (k_M - c_M)
\]

**Proof of Proposition 1.** Equation (2) is rewritten in logarithmic form as:

\[
s = x + \tilde{m} + e + \tilde{\varepsilon}_1 \tag{A13}
\]

where \(x, \tilde{m}, e,\) and \(\tilde{\varepsilon}_1\) are the logarithms of \(X, \tilde{M}, E,\) and \(\varepsilon_1\). The joint distribution of \(\tilde{m}\) and \(\tilde{\varepsilon}_1\) is as follows:

\[
\begin{pmatrix}
m \\
\varepsilon_1
\end{pmatrix} \sim N\left( \begin{pmatrix}
\beta (k_M - \varepsilon_M) \\
-\frac{1}{2} \Sigma_1
\end{pmatrix}, \begin{pmatrix}
\beta^2 \Omega & 0 \\
0 & \Sigma_1
\end{pmatrix} \right) \tag{A14}
\]

Then we have

\[
E[v_1|s] = E[x + e + \tilde{\varepsilon}_1 | s = x + \tilde{m} + e + \tilde{\varepsilon}_1]
\]

\[
= x + \alpha (k_E - c_E) - \frac{1}{2} \Sigma_1 + \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1} \left( s - \left( x + \alpha (k_E - c_E) + \beta (k_M - \varepsilon_M) - \frac{1}{2} \Sigma_1 \right) \right)
\]

\[
= x \frac{\beta^2 \Omega}{\beta^2 \Omega + \Sigma_1} + s \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1} + \left[ \alpha (k_E - c_E) - \frac{1}{2} \Sigma_1 \right] \frac{\beta^2 \Omega}{\beta^2 \Omega + \Sigma_1} - \beta (k_M - \varepsilon_M) \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1}
\]

\[
and
\]
\[
\text{var}[v_1|s] = \frac{\beta^2 \Omega \Sigma_1}{\beta^2 \Omega + \Sigma_1} \tag{A16}
\]

From (5), the logarithm of the (gross-of-expected-pay) market price is given by:

\[
p_1 = \ln P_1 = \ln E[V_1|S] = \ln E[\exp(v_1)|s] = E[v_1|s] + \frac{1}{2} \text{var}[v_1|s] \tag{A17}
\]

\[
= x \frac{\beta^2 \Omega}{\beta^2 \Omega + \Sigma_1} + s \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1} + \left[ \alpha (k_E - c_E) - \frac{1}{2} \Sigma_1 \right] \frac{\beta^2 \Omega}{\beta^2 \Omega + \Sigma_1} - \beta (k_M - \varepsilon_M) \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1}
\]

\[
= x \frac{\beta^2 \Omega}{\beta^2 \Omega + \Sigma_1} + s \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1} + \alpha (k_E - c_E) \frac{\beta^2 \Omega}{\beta^2 \Omega + \Sigma_1} - \beta (k_M - \varepsilon_M) \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1}
\]

\[
= (x + e_1) (1 - \gamma) + (s - \tilde{m}) \gamma
\]

where:

\[
\gamma = \frac{\Sigma_1}{\beta^2 \Omega + \Sigma_1} \tag{A19}
\]
\[
\ln \pi = \alpha (1 - \gamma) (k_E - c_E) - \beta \gamma (k_M - \varepsilon_M) \tag{A20}
\]
Proof of Proposition 2. Rewriting manager’s pay in equation (9) in logarithmic form and substituting out for \(m\) and \(e\) using equation (A12) and for \(\pi\) using equation (A20):

\[
\tilde{w} = \ln \omega + \mu \ln \pi + (\mu + \eta) x + \gamma \mu \tilde{m} + (\gamma \mu + \eta) e + (\gamma \mu + \eta) \tilde{e}_1 + \eta \tilde{e}_2 \tag{A21}
\]

\[
= \ln \omega + (\mu + \eta) x + \gamma \mu \beta (\tau_M - \tilde{\tau}_M) + (\mu + \eta) \alpha (k_E - c_E) + (\gamma \mu + \eta) \tilde{e}_1 + \eta \tilde{e}_2
\]

and thus the expected value of the logarithm of the manager’s terminal wealth is:

\[
E_0[\tilde{w}] = \ln \omega + (\mu + \eta) [x + \alpha (k_E - c_E)] - \frac{1}{2} [(\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2] \tag{A22}
\]

and its variance is

\[
\text{Var}(\tilde{w}) = (\gamma \mu + \eta)^2 \Sigma_1 + \eta^2 \Sigma_2 + \gamma^2 \mu^2 \beta^2 \Omega \tag{A23}
\]

The logarithm of the expected utility of the manager (after finding out his effort cost \(C_M\) but before observing the firm value shocks \(\varepsilon_1\) and \(\varepsilon_2\)) is then:

\[
\ln E_0 \left[ \tilde{U} \right] = \ln E_0 \left[ \frac{1}{\phi} \left( L^\Psi \tilde{W} \right)^\phi \right] \tag{A24}
\]

\[
= - \ln \phi + \phi \Psi \ln L + \phi E[\tilde{w}] + \frac{1}{2} \phi^2 \text{Var}(\tilde{w})
\]

\[
= - \ln \phi + \phi \Psi \ln \frac{L}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu}
+ \phi \{\ln \omega + (\mu + \eta) [x + \alpha (k_E - c_E)]\}
- \frac{\phi}{2} [(\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2] + \frac{\phi^2}{2} [(\gamma \mu + \eta)^2 \Sigma_1 + \eta^2 \Sigma_2 + \gamma^2 \mu^2 \beta^2 \Omega]
\]

where the manager’s choice for leisure \(L\) is substituted in from equation (12). Using the definition of \(\gamma\) to substitute out for \(\Omega\) using \(\beta^2 \Omega = \Sigma_1^{1-\gamma}\), we obtain:

\[
\ln E_0 \left[ \tilde{U} \right] = - \ln \phi + \phi \Psi \ln \frac{L}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu}
+ \phi \{\ln \omega + (\mu + \eta) [x + \alpha (k_E - c_E)]\}
- \frac{\phi}{2} [(\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2]
+ \frac{\phi^2}{2} [(\gamma \mu^2 + 2 \gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2] \tag{A25}
\]
Proof of Proposition 3. The expected payment to the manager is:

\[
E_0 \left[ \tilde{W} \right] = \exp \left\{ E_0 [\tilde{w}] + \frac{1}{2} \text{Var} (\tilde{w}) \right\} \\
= -\omega X^{\mu+\eta} \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha(\mu+\eta)} \\
\cdot \exp \left\{ \frac{1}{2} \left[ (-\gamma \mu + \gamma \mu^2 + 2\gamma \mu \eta - \eta + \eta^2) \Sigma_1 + (-\eta + \eta^2) \Sigma_2 \right] \right\}
\]

since the wage is lognormally distributed, and using equations (A22) and (A23).

The \textit{ex ante} company expected value is then:

\[
E_0 \left[ \tilde{V}_2 - \tilde{W} \right] = E_0 \left[ \tilde{V}_2 \right] - E_0 \left[ \tilde{W} \right] \\
= E_0 [X \cdot E \cdot \tilde{e}_1 \cdot \tilde{e}_2] - E_0 \left[ \tilde{W} \right] \\
= X \cdot E - E_0 \left[ \tilde{W} \right] \\
= X \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha} \\
- \omega X^{\mu+\eta} \left( \frac{L}{C_E \Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\alpha(\mu+\eta)} \\
\cdot \exp \left\{ \frac{1}{2} \left[ (-\gamma \mu + \gamma \mu^2 + 2\gamma \mu \eta - \eta + \eta^2) \Sigma_1 + (-\eta + \eta^2) \Sigma_2 \right] \right\}
\]

If the manager’s participation constraint is binding, \( \omega \) can be substituted out of equation (24) by setting his expected utility, given by expression (22), to its reservation value \( U (\tilde{W}, L) \equiv \frac{1}{2} \left( L^\Phi \tilde{W} \right)^\circ \).

\[ \blacksquare \]

Proof of Proposition 4.

Following equation (25), the first order conditions are:

\[
\text{FOC w.r.t. } \mu: \quad \frac{X}{\tilde{W}} \left( \frac{L}{C_E} \right)^{\alpha} \alpha^{\alpha+1} \\
\quad = \frac{[\Psi + (\alpha + \beta) \gamma \mu + \alpha \eta]^{\alpha+\Psi} (\gamma \mu + \eta)^{1-\alpha}}{\Psi^{\Psi} (\Psi - \beta \eta)} \\
\quad \cdot \left\{ (\alpha + \beta) \Psi + [\Psi + (\alpha + \beta) \gamma \mu + \alpha \eta] (\mu + \eta) (1 - \phi) \Sigma_1 \right\} \\
\quad \cdot \exp \left\{ \frac{1 - \phi}{2} \left[ (\gamma \mu^2 + 2\gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2 \right] \right\}
\]

(or \( \mu = 0 \) and \( \leq \) replaces = in the FOC).
FOC w.r.t. $\eta$: \[
\frac{X}{W} \left( \frac{L}{C_E} \right)^\alpha \alpha^{\alpha+1} = A29
\]
\[
= \left[ \psi + (\alpha + \beta) \gamma \mu + \alpha \eta \right]^{\alpha+\psi} (\gamma \mu + \eta)^{1-\alpha} \cdot \frac{\psi \psi \psi (\psi + \beta \gamma \mu)}{\psi \psi \psi (\psi + \beta \gamma \mu)} \cdot \left\{ \alpha \psi + \left[ \psi + (\alpha + \beta) \gamma \mu + \alpha \eta \right] (1 - \phi) \left[ \gamma \mu \Sigma_1 + \eta (\Sigma_1 + \Sigma_2) \right] \right\} \\
\cdot \exp \left\{ \frac{1 - \phi}{2} \left[ (\gamma \mu^2 + 2 \gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2 \right] \right\}
\]

(i) An optimal contract cannot have just short-term but not long-term incentives. Suppose not, that is, suppose the solution has $\eta = 0$ and $\mu \neq 0$. The relevant FOCs then simplify to:

FOC w.r.t. $\mu$: \[
\frac{X}{W} \left( \frac{L}{C_E} \right)^\alpha \alpha^{\alpha+1} = A30
\]
\[
= \frac{[\psi + (\alpha + \beta) \gamma \mu + \alpha \eta]^{\alpha+\psi} (\gamma \mu)^{1-\alpha}}{\psi \psi \psi \psi + \psi} \cdot \left\{ (\alpha + \beta) \psi + [\psi + (\alpha + \beta) \gamma \mu] \mu (1 - \phi) \Sigma_1 \right\} \cdot \exp \left\{ \frac{1 - \phi}{2} \gamma \mu^2 \Sigma_1 \right\}
\]

FOC w.r.t. $\eta$: \[
\frac{X}{W} \left( \frac{L}{C_E} \right)^\alpha \alpha^{\alpha+1} \leq A31
\]
\[
= \frac{\psi \psi}{\psi + \beta \gamma \mu} \cdot \left\{ \alpha \psi + \left[ \psi + (\alpha + \beta) \gamma \mu \right] (1 - \phi) \gamma \mu \Sigma_1 \right\} \cdot \exp \left\{ \frac{1 - \phi}{2} \gamma \mu^2 \Sigma_1 \right\}
\]

Clearly these two conditions cannot be met simultaneously because the RHS of the first FOC is strictly greater than that of the second one (both due to a factor $\beta$ on $\psi$ in the FOC w.r.t. $\mu$, and the appearance of $\gamma \leq 1$ in two extra places reducing the RHS of the FOC w.r.t. $\eta$). Contradiction.

On the other hand it is conceivable that the solution has $\eta > 0$ but $\mu = 0$ simultaneously:

FOC w.r.t. $\mu$: \[
\frac{X}{W} \left( \frac{L}{C_E} \right)^\alpha \alpha^{\alpha+1} \leq A32
\]
\[
= \frac{[\psi + \alpha \eta]^{\alpha+\psi} \eta^{1-\alpha}}{\psi \psi (\psi - \beta \eta)} \cdot \left\{ (\alpha + \beta) \psi + \eta (\psi + \alpha \eta) (1 - \phi) \Sigma_1 \right\} \cdot \exp \left\{ \frac{1 - \phi}{2} \eta^2 (\Sigma_1 + \Sigma_2) \right\}
\]

FOC w.r.t. $\eta$: \[
\frac{X}{W} \left( \frac{L}{C_E} \right)^\alpha \alpha^{\alpha+1} = A33
\]
\[
= \frac{[\psi + \alpha \eta]^{\alpha+\psi} \eta^{1-\alpha}}{\psi \psi \psi} \cdot \left\{ \alpha \psi + \eta (\psi + \alpha \eta) (1 - \phi) (\Sigma_1 + \Sigma_2) \right\} \cdot \exp \left\{ \frac{1 - \phi}{2} \eta^2 (\Sigma_1 + \Sigma_2) \right\}
\]

44
In particular such a contract may be optimal if $\Sigma_2$ is small enough relative to $(\Sigma_1 + \Sigma_2)$ so that the extra risk attributable to long-term incentives is sufficiently small:

$$\frac{\Psi}{\Psi - \beta \eta} \{ (\alpha + \beta) \Psi + \eta (\Psi + \alpha \eta) (1 - \phi) \Sigma_1 \} > \alpha \Psi + \eta (\Psi + \alpha \eta) (1 - \phi) (\Sigma_1 + \Sigma_2)$$

(A34)

i.e. $\beta \Psi > \eta (1 - \phi) [\Psi \Sigma_2 - \beta \eta (\Sigma_1 + \Sigma_2)]$

(A35)

Fixing $\Sigma_1 + \Sigma_2$ and thus $\eta$, this condition is satisfied if $\Sigma_2$ is small enough.

(ii) If we allow long term incentives, i.e. $\eta > 0$, then short term incentives are lower i.e. $\mu$ is reduced and a smaller proportion of time is devoted to manipulation. This is immediately obvious from the FOC with respect to $\mu$: the RHS is increasing in both $\eta$ and $\mu$ so holding all else constant an increase in the one must be offset by a decrease in the other. This also implies there will be less time spent on manipulation as follows directly from equation (11).

(iii) From equation (10), effort is determined by:

$$E = \left( \frac{L}{C_E} \frac{\alpha (\gamma \mu + \eta)}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^\alpha$$

and first best effort ($E^*$) is determined in equation (A3) as

$$\left( \frac{L - C_E E^* \frac{1}{\alpha}}{C_E E^* \frac{1}{\alpha} - 1} \right)^{\Psi+1} = \frac{L^\psi W^\Psi}{X^\alpha}$$

Since the left hand side of the above equation is decreasing in $E$, to show that $E \leq E^*$, we need to show that

$$\frac{L - C_E E^{\frac{1}{\alpha}}}{C_E E^{\frac{1}{\alpha} - 1}} \geq \frac{L^\psi W^\Psi}{X^\alpha}.$$

Equivalently, using equation (10) to replace effort $E$ by its determinants, it needs to be shown that:

$$\left( \frac{L}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right)^{\Psi+1} \geq \frac{L^\psi W^\Psi}{X^\alpha} C^\alpha_E (\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu)^{1-\alpha}$$

i.e.

$$\frac{X}{W} \left( \frac{L}{C_E} \right)^{\alpha} \geq \frac{[\Psi + (\alpha + \beta) (\gamma \mu + \alpha \eta)^{\alpha + \Psi} (\gamma \mu + \eta)^{1-\alpha}] \left( \frac{\Psi}{\Psi + \beta \gamma \mu} \right)^{\Psi+1}}{\Psi (\Psi + \beta \gamma \mu)}$$

But the expression on the RHS satisfies:
where the last equality follows from equation (A29), the FOC with respect to $\eta$. This proves the claim that effort is below first-best.

Taking the right hand side of equation (A28):

$$\frac{[\Psi + (\alpha + \beta) \gamma \mu + \alpha \eta]^{\alpha+\Psi} (\gamma \mu + \eta)^{1-\alpha}}{\Psi^{\Psi/\Psi + \beta \gamma \mu}} \cdot \{\alpha \Psi + \Psi + (\alpha + \beta) \gamma \mu + \alpha \eta\} (1 - \phi) [(\gamma \mu \Sigma_1 + \eta (\Sigma_1 + \Sigma_2))] \cdot \exp \left\{ \frac{1 - \phi}{2} [((\gamma \mu^2 + 2 \gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2] \right\} \\
= \frac{X}{W} \left( \frac{L}{C_E} \right)^{\alpha} \cdot \alpha^{\alpha+1}$$

since $\beta > 0$ and $\gamma \leq 1$. Thus $\gamma \mu + \eta$ in equation (A28) needs to be lower than $\mu$ is in equation (A6) (the second best contract without manipulation). Since effort in equation (10) is increasing in $\gamma \mu + \eta$ and moreover, lowered if $\beta$ is not zero, effort is below the second best level without manipulation.

Proof of Proposition 5.

(i) Inspecting the optimization problem (25), it is immediately obvious that for any given values of $\mu$ and $\eta$, the maximand increases if CRRA $(1 - \phi)$ falls, long-term risk $\Sigma_2$ falls, or $\Sigma_1$ falls whilst $\gamma$ remains constant ($i.e.$ $\Omega$ falls in proportion to $\Sigma_1$). It is also decreasing in $\Omega$ because a fall in $\Omega$ means an increase in $\gamma$; and an increase in $\gamma$, when $\mu$ varies in such
a way as to keep $\mu \gamma$ constant, entails a decrease in $\mu$ which decreases the $\gamma \mu^2$ term in the exponent, increasing the maximand. Lastly, when $\Sigma_1$ falls holding $\Omega$ constant, $\gamma \equiv \frac{\Sigma_1}{\Sigma_1 + \beta^2 \Omega}$ falls. But suppose that you vary $\mu$ in such a way as to keep $\gamma \mu$ unchanged, then the first term in the exponent is $(\gamma \mu)^2 \Sigma_1 / \gamma = (\gamma \mu)^2 (\Sigma_1 + \beta^2 \Omega)$ and thus decreasing along with $\Sigma_1$, so that the entire maximand increases. All these arguments show that the maximand can be increased, even when the endogenous parameters $\mu$ and $\eta$ are not adjusted in an optimal manner; naturally it would increase even more if they were.

(ii) Following equation (25), the optimization problem is:

$$\max_{\{\mu, \gamma\}} X \cdot E(\mu, \gamma) - W(\mu, \gamma) \quad (A36)$$

where effort $E$ and the expected pay $W \equiv E_0 [W(\mu, \gamma)]$ are written as functions of the parameters of the pay contract.

Suppose $\{\mu_1^*, \gamma_1^*\} = \arg \max X_1 \cdot E(\mu, \gamma) - W(\mu, \gamma)$ and $\{\mu_2^*, \gamma_2^*\} = \arg \max X_2 \cdot E(\mu, \gamma) - W(\mu, \gamma)$. Denote $E_1 = E(\mu_1^*, \gamma_1^*), W_1 = W(\mu_1^*, \gamma_1^*), E_2 = E(\mu_2^*, \gamma_2^*), W_2 = W(\mu_2^*, \gamma_2^*)$. Since $E_1$ and $W_1$ are optimal for firms with a scale of $X_1$, and $E_2$ and $W_2$ are optimal for firms with a scale of $X_2$, we have:

$$X_1 E_1 - W_1 \geq X_1 E_2 - W_2 \quad (A37)$$

and

$$X_2 E_1 - W_1 \leq X_2 E_2 - W_2 \quad (A38)$$

Subtract equation (A38) from equation (A37), we have:

$$(X_1 - X_2) E_1 \geq (X_1 - X_2) E_2 \quad (A39)$$

Therefore, if $X_1 > X_2$, then $E_1 \geq E_2$.

Because in equations A28 and A29, $W, L, C_E$ only enter alongside $X$ in the ratio $\frac{X}{W} \left( \frac{L}{C_E} \right)^\alpha$ on the LHS, it follows that optimal effort is increasing in $L$, and decreasing in $W$ and $C_E$.

References


This figure illustrates how the optimal contract, effort-manipulation-leisure choice, and firm value vary with the uncertainty about manipulation, $\Omega$. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager’s report. The areas labeled $E$, $M$, and $L$ represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V - W]$ is the ex-ante expected net firm value after substracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\overline{L} = 1$, $\overline{W} = 1$, $X = 100$, $\alpha = \beta = 1$, $\Sigma_1 = 1$, $\Sigma_2 = 5$, $\phi = -1$, and $\Psi = 1$. 

Figure 1: Optimal contract as a function of manipulation uncertainty.
Figure 2: Optimal contract as a function of the convexity of manipulation cost ($\beta$).

This figure describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with $\beta$. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager’s report. The left hand graph has two vertical axes, with the left one for $\eta$ and the right one for $\gamma \mu$. The areas labeled $E$, $M$, and $L$ represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V - W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\bar{L} = 1$, $\bar{W} = 1$, $X = 100$, $\alpha = 1$, $\phi = -1$, $\Sigma_1 = 1$, $\Sigma_2 = 5$, $\Omega = 1$, and $\Psi = 1$. 
This figure describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with $X$, which parameterizes the scale of the firm. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager’s report. The areas labeled $E$, $M$, and $L$ represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V - W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\bar{L} = 1$, $\bar{W} = 1$, $\alpha = \beta = 1$, $\Sigma_1 = 1$, $\Sigma_2 = 5$, $\Omega = 1$, $\phi = -1$, $\Psi = 1$, and thus $\gamma \equiv 0.5$ throughout.

Figure 3: Optimal contract as a function of firm size.
Figure 4: Optimal contract as a function of manager’s relative risk aversion.

This figure describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with $1 - \phi$, the relative risk aversion coefficient of the manager. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager’s report. The left hand graph has two vertical axes, with the left one for $\eta$ and the right one for $\gamma \mu$. The areas labeled $E$, $M$, and $L$ represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V - W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1, \bar{L} = 1, \bar{W} = 1, X = 100, \alpha = \beta = 1, \Sigma_1 = 1, \Sigma_2 = 5, \Omega = 1, \Psi = 1$, and thus $\gamma \equiv 0.5$ throughout.
This figure illustrates how the optimal contract, effort-manipulation-leisure choice, and firm value vary with $\Sigma_1$, the short term uncertainty about firm fundamentals. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager’s report. The upper righthand graph has two vertical axes, with the left one for $\gamma \mu$ and the right one for $\eta$. The areas labeled "Effort", "Manipulation" and "Leisure" represent the proportion of time the manager spends on effort, manipulation and leisure. "Expected gross firm value" is the ex-ante expected gross firm value and "Expected net firm value" is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $L = 1$, $\bar{W} = 1$, $X = 100$, $\alpha = \beta = 1$, $\Sigma_2 = 5$, $\Omega = 1$, $\phi = -1$, and $\Psi = 1$. 

Figure 5: Optimal contract as a function of short term uncertainty about firm fundamentals.