Intrinsic and Time Values of Call Option

- **Intrinsic value of option**
  - The option payoff from immediate exercising an in-the-money call option \( IV = S_0 - X \)
  - Zero for an out-of-money call

- **Time value of option**
  - The difference between the call price and the intrinsic value
  - Comes from “volatility value”
Decomposition of Option Value

- **Intrinsic Value vs. Time Value**

<table>
<thead>
<tr>
<th>Value</th>
<th>Option Value</th>
<th>Time Value</th>
<th>Intrinsic Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Factors affecting *Call* Option Premium

<table>
<thead>
<tr>
<th>Factor</th>
<th>Effect on Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock price ($S_t$)</td>
<td>+</td>
</tr>
<tr>
<td>Exercise price ($X$)</td>
<td>-</td>
</tr>
<tr>
<td>Volatility of stock price</td>
<td>+</td>
</tr>
<tr>
<td>Time to expiration</td>
<td>+</td>
</tr>
<tr>
<td>Dividend payout rate</td>
<td>-</td>
</tr>
<tr>
<td>Interest rate</td>
<td>?</td>
</tr>
</tbody>
</table>

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Binomial Option Pricing

- Assume stock price can take two values in a year
- Can borrow or lend at rate $r_f$

\[ S = 100 \quad S^+ = 200 \quad S^- = 50 \]

\[ C = \begin{cases} \text{C}^+ = 75 \\ \text{C}^- = 0 \end{cases} \]

Stock

Call Option with Strike price $X = 125$

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Binomial Option Pricing

- Binomial option pricing is derived based on replication strategy and arbitrage principle

Buy 1/2 share of stock at $50
Borrow $25/(1+8\%)$ at $23.15$
Net cost today $26.85$

\[
\begin{array}{cccc}
\text{Payoff at time } T: & \text{Down} & \text{Up} \\
\text{Value of Stock} & 25 & 100 \\
\text{Repay loan} & -25 & -25 \\
\text{Net Payoff} & 0 & 75 \\
\end{array}
\]

Payoff of the portfolio $\frac{1}{2} S - PV(25)$ is exactly the same as the payoff of the Call

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Binomial Option Pricing

- Cost of the replicating portfolio should be the same as the call option premium
  \[ C = \$26.85 \]

Another View of Binomial Replication

- Construct the following portfolio
  - Buy 1/2 share of stock and
  - Sell (write) 1 call with strike price \( X = 125 \)
  - Cost today will be \( \frac{1}{2} S_t - C = 50 - C \)

- Portfolio is perfectly hedged
  - Payoff at time \( T \) is certain and independent of \( S_T \)

<table>
<thead>
<tr>
<th></th>
<th>down</th>
<th>up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Value</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>Call Obligation</td>
<td>0</td>
<td>-75</td>
</tr>
<tr>
<td>Net payoff</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

- Hence \( 50 - C = \frac{25}{1+r_t} = 23.15 \), or \( C = 26.85 \)
Hedge Ratio

- How many shares of stock to hold to hedge one call option written and sold?

\[ H = \frac{C^+ - C^-}{S^+ - S^-} = \frac{75 - 0}{200 - 50} = 0.5 \]

- \( H \) is called option delta
- An example: price the same option as before with a strike price of $100

Steps to Price an Call Option

1. Find the hedge ratio (number of shares of stocks need to buy to hedge the option);
2. Find the certain cash flow next period from a portfolio of long stock and short call;
3. Find the present value of the certain cash flow;
4. Set the value of the stock-call portfolio to cost from step 3;
5. Solve for call option value
Black-Scholes Model

- The option price is equal to
  \[ C_0 = S_0 N(d_1) - X e^{-rT} N(d_2) \]
  \[ d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma \sqrt{T}} \]
  \[ d_2 = d_1 - \sigma \sqrt{T} \]

- Interpretation
  - \( N(d_1) \) is a cumulative normal function
  - \( N(d_2) \) measures the risk-neutral probability of the option getting into money given initial stock price.
  - \( X e^{-rT} N(d_1) \) is the present value of expected payable
  - \( S_0 N(d_1) \) is the present value of expected receivable

- Hedge Ratio (delta)
  - number of shares to replicate the option
  \[ H = \frac{\partial C}{\partial S} = N(d_1) \]
  - number of shares need to hold hedge the option written and sold

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**Q:** if \( S = 100, X = 95, T = 1, r = 0.1, \sigma = 0.2 \)

- What is the call / put option price?
- What is the hedge ratio for the call?

**A:** the prices are estimated as follows

- The call price
  \[ C_0 = S_0 N(d_1) - X e^{-rT} N(d_2) = 16.38 \]

- The put price using put-call parity relationship
  \[ P_0 = C_0 - S_0 + X e^{-rT} = -S_0 N(-d_1) + X e^{-rT} N(-d_2) = 2.34 \]

- Hedge ratio
  - The portfolio with 1 call and -0.76 shares will be risk free
  \[ H = \frac{\partial C}{\partial S} = N(d_1) = 0.76 \]
Implied Volatility

- Option price is observable but volatility is not.
  - Use historical volatility to calculate model price
  - Set market price = model price and back out the volatility
    - this is called implied volatility

- Market often quotes option volatility instead of option premium

- Over/under valuation
  - If the implied volatility > real volatility, option is overvalued
  - If the implied volatility < real volatility, option is undervalued

Wrap-up

- The most important determinant parameter for option value is volatility
  - It is also the only parameter that is not observable

- There are two prominent option pricing models:
  - Binomial model (discrete time)
  - Black-Scholes model (continuous time)
  - Equivalent in the limit

- Option value depends critically on hedging/replication strategy