Chapter 5
Risk and Return – Part I

Single Period Return

- Holding Period Return:
  - percentage gain during a period
  \[ HPR = \frac{P_1 + D_t - P_0}{P_0} \]
  - \( HPR \): holding period return
  - \( P_0 \): beginning price
  - \( P_1 \): ending price
  - \( D_t \): cash dividend (or bond coupon)
Probability Distribution of Returns

- Scenario analysis
  - A list of possible economic scenarios, the likelihood and HPR associated with each scenario
  - Probability distribution of HPR describes the chances of returns falling to different levels

- An example of return distribution

<table>
<thead>
<tr>
<th>Economy State (s)</th>
<th>Prob: p(s)</th>
<th>HPR: r(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>0.25</td>
<td>44%</td>
</tr>
<tr>
<td>Normal</td>
<td>0.50</td>
<td>14%</td>
</tr>
<tr>
<td>Bust</td>
<td>0.25</td>
<td>-16%</td>
</tr>
</tbody>
</table>

Risk Measure

- Expected Return (μ)
  - Probability-weighted mean value of HPR
  \[ E[r] = \mu = \sum p(s) r(s) = [0.25 \times 44\% + 0.5 \times 14\% + 0.25 \times (-16\%)] = 14\% \]

- Return Variance (σ²)
  - Probability-weighted squared deviation from mean HPR
  \[ Var[r] = \sigma^2 = \sum p(s)(r(s) - E[r])^2 \]
  \[ = 0.25 \times (.44 - .14)^2 + 0.5 \times (.14 - .14)^2 + 0.25 \times (-.16 - .14)^2 = 0.045 \]

- Standard Deviation (σ)
  - Square root of the variance
  \[ SD[r] = \sigma = \sqrt{Var[r]} = \sqrt{0.045} = 0.2121 = 21.21\% \]
Multi-period Return

- Consider a mutual fund

<table>
<thead>
<tr>
<th>1Q</th>
<th>2Q</th>
<th>3Q</th>
<th>4Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets at the start</td>
<td>1.0</td>
<td>1.2</td>
<td>2.0</td>
</tr>
<tr>
<td>HPR(%)</td>
<td>0.10</td>
<td>0.25</td>
<td>-0.20</td>
</tr>
<tr>
<td>Assets before net inflow</td>
<td>1.1</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Net Inflow</td>
<td>0.1</td>
<td>0.5</td>
<td>-0.8</td>
</tr>
<tr>
<td>Assets in the end</td>
<td>1.2</td>
<td>2.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>

- Net inflow when the fund does well
- Net outflow when the fund does poorly

- Question - how do we characterize the fund’s historical performance over the year?

Multi-period Return

- Arithmetic Average
  - Sum of returns in each period divided by the number of periods
  \[
  r_a = \frac{r_1 + r_2 + \ldots + r_N}{N} = \frac{1}{N} \sum_{i=1}^{N} r_i
  \]
  - \( r_a \): arithmetic return
  - \( r_i \): HPR in the \( i \)-th period
  - \( N \): number of periods
  - Best forecast of future single-period return
  - An estimate of expected return
Example: Arithmetic Average Return

- Calculate the arithmetic average return of the fund

Multi-period Return

- Geometric Average
  - A single period return that gives the same cumulative performance as a sequence of actual returns for a **buy-and-hold** strategy
  - Also called time-weighted average return
    
    \[ r_g = \left( \prod_{i=1}^{N} (1 + r_i) \right)^{\frac{1}{N}} - 1 = \left( \prod_{i=1}^{N} (1 + r_i) \right)^{\frac{1}{N}} - 1 \]

  - \( r_g \): geometric average return
  - \( r_i \): HPR in the \( i \)-th period
  - \( N \): number of periods
  - Better measure for buy-and-hold investing
Example: Geometric Average Return

- Calculate the geometric average return of the mutual fund

\[
r_g = \left[ (1+10\%) \times (1+25\%) \times (1-20\%) \times (1+25\%) \right]^\frac{1}{4} - 1
\]

\[= 8.29\%\]

- If you did not add or withdraw any funds during the year, what is the ending asset value for an initial investment of $1000?

Multi-period Return

- Dollar-weighted return (IRR)
  - The discount rate that sets the present value of the future cash flows equal to the amount of initial investment
  - Same as Internal Rate of Return (IRR)

\[
0 = CF_0 + \frac{CF_1}{1 + IRR} + \frac{CF_2}{(1 + IRR)^2} + \ldots + \frac{CF_N}{(1 + IRR)^N} = \sum_{i=0}^{N} \frac{CF_i}{(1 + IRR)^i}
\]
Multi-period Return

- **Dollar-weighted return (continued)**
  - Take into account changes in the amount of asset under investment
  - Conventions (from investor’s viewpoint)
    - Initial investment as outflow (negative)
    - Ending value from liquidation as inflow (positive)
    - Additional investment as outflow (negative)
    - Withdrawal as inflow (positive)
  - Good measure with intermediate contribution/withdrawal

Example: Dollar-weighted Return

- **Calculate IRR based on our mutual fund data**

<table>
<thead>
<tr>
<th></th>
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<th>2Q</th>
<th>3Q</th>
<th>4Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets at the start</td>
<td>1.0</td>
<td>1.2</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>HPR</td>
<td>10.0%</td>
<td>25.0%</td>
<td>-20.0%</td>
<td>25.0%</td>
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</table>

- **By definition**
  \[ 0 = -1 + \frac{-0.1}{1 + IRR} + \frac{-0.5}{(1 + IRR)^2} + \frac{0.8}{(1 + IRR)^3} + \frac{1.0}{(1 + IRR)^4} \]

- **Using Excel function**

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>IRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF</td>
<td>-1.0</td>
<td>-0.1</td>
<td>-0.5</td>
<td>0.8</td>
<td>1.0</td>
<td>4.17%</td>
</tr>
</tbody>
</table>
Real vs. Nominal Returns

- Historical returns are usually measured in nominal dollar terms
  - 10% nominal return does not mean you can buy 10% more goods and services
  - In the course of the investment, the prices of “consumption basket” might also change
- Inflation \( (i) \) is usually measured by CPI \( (CPI-U) \)
  - A bundle of goods considered representative of consumption pattern of a typical family
- Investors ultimately are interested in purchasing power, i.e. real returns

Real vs. Nominal Returns

- Nominal return measures the growth rate of your money \( (R) \)
- Real return measures the growth rate of your purchasing power \( (r) \)
- An exact formula
  \[ r = \frac{R - i}{1 + i} \]
- An approximate formula
  \[ R \geq r + i \text{ or } r \geq R - i \]
- Tax is levied on nominal returns \( (R) \), not real returns \( (r) \)
  - Good or bad for an investor?
An Example on Return Taxation

- Over the past year, you observed the following:
  - Inflation rate is 10%
  - Your investment generates 14% return
  - You are in the 35% marginal tax bracket

- Q: What is your before-tax real return?

- Q: What is your after-tax real return?

Wrap-up

- Expected return and variance calculation under scenario analysis
- Different summary measures of realized returns over multiple holding period
- Real vs. nominal returns
  - Tax effect