CHAPTER 9: BOND PRICES AND YIELDS

3. The bond callable at 105 should sell at a lower price because the call provision is more valuable to the firm. Therefore, its yield to maturity should be higher.

4. The bond price will be lower. As time passes, the bond price, which is now above par value, will approach par.

9. a. The bond pays $50 every six months.
   
   Current price = \[50 \times \text{Annuity factor}(4\%, 6)\] + \[1000 \times \text{PV factor}(4\%, 6)\]
   
   \[= \frac{50}{0.04} \times [1 - \frac{1}{(1+0.04)^6}] + \frac{1000}{(1+0.04)^6} = 1,052.42\]

   Assuming the market interest rate remains 4% per half year, price six months from now:
   
   \[= \frac{50 \times \text{Annuity factor}(4\%, 5)}{1,052.42} + \frac{1000 \times \text{PV factor}(4\%, 5)}{1,052.42} = 1,044.52\]

   b. Rate of return = \[\frac{50 + (1,044.52 - 1,052.42)}{1,052.42} = \frac{50 - 7.90}{1,052.42} = 0.0400 = 4.00\%\] per six months

10. a. \[40/r *[1-1/(1+r)^40] + 1000/(1+r)^40 = 950\]

   Use Excel Goal Seek function. This implies a bond equivalent yield to maturity of:
   \[(4.26\% \times 2) = 8.52\%\]

   Effective annual yield to maturity = \[(1.0426)^2 - 1 = 0.0870 = 8.70\%\]

   b. Since the bond is selling at par, the yield to maturity on a semi-annual basis is the same as the semi-annual coupon, 4%. The bond equivalent yield to maturity is 8%.

   Effective annual yield to maturity = \[(1.04)^2 - 1 = 0.0816 = 8.16\%\]

   c. Similar to part a, but set price = 1050, we find a bond equivalent yield to maturity of 7.52%, or 3.76% on a semi-annual basis.

   Effective annual yield to maturity = \[(1.0376)^2 - 1 = 0.0766 = 7.66\%\]

14. 

<table>
<thead>
<tr>
<th></th>
<th>0% Coupon</th>
<th>8% coupon</th>
<th>10% coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Current prices</td>
<td>$463.19</td>
<td>$1000</td>
<td>$1134.20</td>
</tr>
<tr>
<td>b. Price one year from now</td>
<td>$500.25</td>
<td>$1000</td>
<td>$1124.94</td>
</tr>
</tbody>
</table>
Price increase $  37.06 $ 0.00 -$  9.26
Coupon income $    0.00 $80.00 $100.00
Income $  37.06 $80.00 $   90.74
Rate of Return 8.00% 8.00% 8.00%

24. a. The bond sells for $1,124.72 based on the 7.0% yield to maturity:

\[
\frac{40}{0.035} \times \left[\frac{1}{1-\left(\frac{1+0.035}{1+0.035}\right)^{60}}\right] + \frac{1000}{(1+0.035)^{60}} = 1124.72
\]

\[n = 60; i = 3.5; FV = 1000; PMT = 40\]

Therefore, \( r \) = yield to call is 3.368% semiannually, 6.736% annually:

\[
\frac{40}{r} \times \left[\frac{1}{1-\left(\frac{1+r}{1+r}\right)^{10}}\right] + \frac{1100}{(1+r)^{10}} = 1124.72
\]

\[n = 10; PV = 1124.72 ; FV = 1100; PMT = 40\]

b. If the call price were $1050, we would set \( FV = 1050 \) and redo part (a) to find that yield to call is 2.976% semi-annually, 5.952% annually. With a lower call price, the yield to call is lower.

\[
\frac{40}{r} \times \left[\frac{1}{1-\left(\frac{1+r}{1+r}\right)^{10}}\right] + \frac{1050}{(1+r)^{10}} = 1124.72
\]

\[n = 10; PV = 1124.72 ; FV = 1100; PMT = 40\]

c. Yield to call is 3.031% semiannually, 6.062% annually:

\[
\frac{40}{r} \times \left[\frac{1}{1-\left(\frac{1+r}{1+r}\right)^{4}}\right] + \frac{1050}{(1+r)^{4}} = 1124.72
\]

\[n = 4; PV = 1124.72 ; FV = 1100; PMT = 40\]

25. The stated yield to maturity equals 16.075%:

\[
\frac{140}{r} \times \left[\frac{1}{1-\left(\frac{1+r}{1+r}\right)^{10}}\right] + \frac{1000}{(1+r)^{10}} = 900
\]

\[n = 10; PV = 900; FV = 1000; PMT = 140\]

Based on expected coupon payments of $70 annually, the expected yield to maturity is 8.526%.

\[
\frac{70}{r} \times \left[\frac{1}{1-\left(\frac{1+r}{1+r}\right)^{10}}\right] + \frac{1000}{(1+r)^{10}} = 900
\]

\[n = 10; PV = 900; FV = 1000; PMT = 70\]
Additional Question (Q8)

a. Market price of bond B can be calculated based on ytm
   \[ P_B = \frac{200}{(1+11.5\%)} + \frac{1200}{(1+11.5\%)^2} = 1144.60 \]

b. What is the implied one-year and two-year discount factors (how much would you pay today for a dollar to be delivered in one or two years?)
   \[ P_A = 800 = d_2*1000 \Rightarrow d_2 = 0.8 \]

   We can find out the implied \(d_1\) by plugging \(d_2\) into the pricing equation for bond B
   \[ P_B = 1144.60 = 200*d_1 + 1200*d_2 \Rightarrow d_1 = 0.923 \]

c. What is the one year and two year spot rates \((s_1\) and \(s_2\)), and one-year forward rate \((f_{1,2})\)?
   \[ d_1 = \frac{1}{(1+s_1)} \Rightarrow s_1 = 8.34\% \]
   \[ d_2 = \frac{1}{(1+s_2)^2} \Rightarrow s_2 = 11.8\% \]

   \[ f_{1,2} = d_1 / d_2 - 1 = 15.4\% \]

d. What should be the current market price of a two-year bond with 10\% annual coupon rate and $1000 face value (Bond C)?

   with \((d_1, d_2)\) or \((s_1, s_2)\) in hand, you can price any two-year cash flow
   \[ P_c = 100*d_1 + 1100*d_2 = 972.30 \]

e. What is the implied YTM for Bond C, based on the market price you just calculated? Is the YTM for Bond C different from that of Bond A or Bond B?

   YTM for different two-year bonds can be different. Use EXCEL goal seek function (under “tools”) to find out YTM for bond C

   \[ \text{YTM}_A = s_2 = 11.8\% \]
   \[ \text{YTM}_B = 11.5\% \]
   \[ \text{YTM}_C = 11.63\% \]

   All three are similar, yet not identical. While each of the two-period cash flows can have a different YTM, there are only two discount factors \((d_1\) and \(d_2)\) or two spot rates \((s_1\) and \(s_2)\) in the economy that can price all of them.