CHAPTER 5: RISK AND RETURN

1. \[ V(12/31/2004) = V(1/1/1998) \times (1 + r_g)^7 = 100,000 \times (1.05)^7 = \$140,710.04 \]

5. a. The holding period returns for the three scenarios are:
   
   Boom: \[ \frac{50 - 40 + 2}{40} = 0.30 = 30.00\% \]
   Normal: \[ \frac{43 - 40 + 1}{40} = 0.10 = 10.00\% \]
   Recession: \[ \frac{34 - 40 + 0.50}{40} = -0.1375 = -13.75\% \]
   
   \[ E(HPR) = \frac{1}{3} \times 30\% + \frac{1}{3} \times 10\% + \frac{1}{3} \times (-13.75\%) = 8.75\% \]
   
   \[ \sigma^2(HPR) = \frac{1}{3} \times (30 - 8.75)^2 + \frac{1}{3} \times (10 - 8.75)^2 + \frac{1}{3} \times (-13.75 - 8.75)^2 \]
   \[ = 319.79 \]
   
   \[ \sigma = \sqrt{319.79} = 17.88\% \]
   
   b. \[ E(r) = (0.5 \times 8.75\%) + (0.5 \times 4\%) = 6.375\% . \]
   \[ \sigma = 0.5 \times 17.88\% = 8.94\% \]

6. c. [For each portfolio: Utility = \( r_{ce} = E(r) - (0.5 \times 4 \times \sigma^2) \)]
   We choose the portfolio with the highest utility value.

7. d. [When an investor is risk neutral, \( A = 0 \), so that the portfolio with the highest utility is the portfolio with the highest expected return.]
14. a. Time-weighted average returns are based on year-by-year rates of return.

\[
\text{Return} = \frac{\text{capital gains} + \text{dividend}}{\text{price}}
\]

<table>
<thead>
<tr>
<th>Year</th>
<th>( \frac{(110 - 100 + 4)/100}{100} = 14.00% )</th>
<th>( \frac{(90 - 110 + 4)/110}{100} = -14.55% )</th>
<th>( \frac{(95 - 90 + 4)/90}{100} = 10.00% )</th>
</tr>
</thead>
</table>

Arithmetic mean: 3.15%
Geometric mean: 2.33%

b. Time Cash flow Explanation

<table>
<thead>
<tr>
<th>Date:</th>
<th>1/1/02</th>
<th>1/1/03</th>
<th>1/1/04</th>
<th>1/1/05</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-208</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>110</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>396</td>
<td></td>
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</tbody>
</table>

Purchase of three shares at $100 per share (outflow)
Purchase of two shares at $110, plus dividend income on three shares held (net outflow)
Dividends on five shares, plus sale of one share at $90 (net inflow)
Dividends on four shares, plus sale of four shares at $95 per share (inflow)

Dollar-weighted return = Internal rate of return = –0.1661%.
18. a. The expected cash flow is: 
\((0.5 \times $50,000) + (0.5 \times $150,000) = $100,000\). With a risk premium of 10%, the required rate of return is 15%. Therefore, if the value of the portfolio is \(X\), then, in order to earn a 15% expected return:
\[X \times (1 + 5\% + 10\%) = $100,000 \Rightarrow X = $86,957\]

b. If the portfolio is purchased at $86,957, and the expected payoff is $100,000, then the expected rate of return, \(E(r)\), is:
\[\frac{$100,000 - $86,957}{$86,957} = 0.15 = 15.0\%\]
The portfolio price is set to equate the expected return with the required rate of return.

c. If the risk premium over T-bills is now 15%, then the required return is: 5% + 15% = 20%. The value of the portfolio (\(X\)) must satisfy:
\[X \times (1.20) = $100,000 \Rightarrow X = $83,333\]

d. For a given expected cash flow, portfolios that command greater risk premia must sell at lower prices. The extra discount from expected value is a penalty for risk.

19. a. \(E(r_p) = (0.3 \times 7\%) + (0.7 \times 17\%) = 14\% \text{ per year}\)
\(\sigma_p = 0.7 \times 27\% = 18.9\% \text{ per year}\)

b. 

<table>
<thead>
<tr>
<th>Security</th>
<th>Investment Proportions</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Bills</td>
<td>30.0%</td>
</tr>
<tr>
<td>Stock A</td>
<td>0.7 \times 27% = 18.9%</td>
</tr>
<tr>
<td>Stock B</td>
<td>0.7 \times 33% = 23.1%</td>
</tr>
<tr>
<td>Stock C</td>
<td>0.7 \times 40% = 28.0%</td>
</tr>
</tbody>
</table>

c. Your Reward-to-variability ratio = \(S = \frac{17 - 7}{27} = 0.3704\)
Client's Reward-to-variability ratio = \(\frac{14 - 7}{18.9} = 0.3704\)

d. See following graph.
20. a. Mean of portfolio = \((1 - y)r_f + y r_P = r_f + (r_P - r_f) y\) = 7 + 10y

If the expected rate of return for the portfolio is 15%, then, solving for y:

\[ 15 = 7 + 10y \Rightarrow y = \frac{15 - 7}{10} = 0.8 \]

Therefore, in order to achieve an expected rate of return of 15%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b. Security | Investment Proportions
---|---
T-Bills | 20.0%
Stock A | 0.8 \times 27\% = 21.6%
Stock B | 0.8 \times 33\% = 26.4%
Stock C | 0.8 \times 40\% = 32.0%

c. \(\sigma_P = 0.8 \times 27\% = 21.6\%\) per year

21. a. Portfolio standard deviation = \(\sigma_P = y \times 27\%\)

If the client wants a standard deviation of 20%, then:

\[ y = \frac{(20\%)/27\%}{27\%} = 0.7407 = 74.07\% \] in the risky portfolio.

b. Expected rate of return = 7 + 10y = 7 + (0.7407 \times 10) = 7 + 7.407 = 14.407\%
22. a. Slope of the CML = \( \frac{13 - 7}{25} = 0.24 \)

See the diagram below.

b. My fund allows an investor to achieve a higher expected rate of return for any given standard deviation than would a passive strategy, i.e., a higher expected return for any given level of risk.

23. a. With 70% of his money in my fund's portfolio, the client has an expected rate of return of 14% per year and a standard deviation of 18.9% per year. If he shifts that money to the passive portfolio (which has an expected rate of return of 13% and standard deviation of 25%), his overall expected return and standard deviation would become:

\[
E(r_C) = r_f + 0.7(r_M - r_f)
\]

In this case, \( r_f = 7\% \) and \( r_M = 13\% \). Therefore:

\[
E(r_C) = 7 + (0.7 \times 6) = 11.2\%
\]

The standard deviation of the complete portfolio using the passive portfolio would be:

\[
\sigma_C = 0.7 \times \sigma_M = 0.7 \times 25\% = 17.5\%
\]

Therefore, the shift entails a decline in the mean from 14% to 11.2% and a decline in the standard deviation from 18.9% to 17.5%. Since both mean return \textit{and} standard deviation fall, it is not yet clear whether the move is beneficial.
The disadvantage of the shift is apparent from the fact that, if my client is willing to accept an expected return on his total portfolio of 11.2%, he can achieve that return with a lower standard deviation using my fund portfolio rather than the passive portfolio. To achieve a target mean of 11.2%, we first write the mean of the complete portfolio as a function of the proportions invested in my fund portfolio, \( y \):

\[
E(r_C) = 7 + y(17 - 7) = 7 + 10y
\]

Because our target is: \( E(r_C) = 11.2\% \), the proportion that must be invested in my fund is determined as follows:

\[
11.2 = 7 + 10y \Rightarrow y = \frac{11.2 - 7}{10} = 0.42
\]

The standard deviation of the portfolio would be:

\[
\sigma_C = y \times 27\% = 0.42 \times 27\% = 11.34\%
\]

Thus, by using my portfolio, the same 11.2% expected rate of return can be achieved with a standard deviation of only 11.34% as opposed to the standard deviation of 17.5% using the passive portfolio.

b. The fee would reduce the reward-to-variability ratio, i.e., the slope of the CAL. Clients will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let \( f \) denote the fee:

\[
\text{Slope of CAL with fee} = \frac{17 - 7 - f}{27} = \frac{10 - f}{27}
\]

\[
\text{Slope of CML (which requires no fee)} = \frac{13 - 7}{25} = 0.24
\]

Setting these slopes equal and solving for \( f \):

\[
\frac{10 - f}{27} = 0.24
\]

\[
10 - f = 27 \times 0.24 = 6.48
\]

\[
f = 10 - 6.48 = 3.52\% \text{ per year}
\]
Q9. Additional question:

a. First, the approximate formula:

\[ r \cong R - i \cong 18.43\% - 3.12\% = 15.29\% \]

Next, we compute real rates using the exact relationship:

\[ r = \frac{1 + R}{1 + i} - 1 = \frac{R - i}{1 + i} = \frac{15.29\%}{1.0312} = 14.83\% \]

b. Tax is collected on nominal returns. Your after-tax nominal return is

\[ R_{\text{after, tax}} = R \times (1 \text{- tax rate}) = 18.43\% \times (1 - 15\%) = 15.67\% \]

Hence you after-tax real return is

\[ r_{\text{after}} = \frac{1 + R_{\text{after, tax}}}{1 + i} - 1 = \frac{R_{\text{after, tax}} - i}{1 + i} = \frac{(15.67\% - 3.12\%)}{1.0312} = 12.17\% \]