Chapter 7 - Part I
Capital Asset Pricing Model (CAPM)

Efficient Frontier with Riskfree Lending and Borrowing

- New efficient frontier with riskfree lending and borrowing
- Efficient frontier with risky assets only
- Optimal risky portfolio
- More risk-averse investor
- Less risk-averse investor

\[ E(r_p) \]
\[ \sigma_p \]
Capital Asset Pricing Model (CAPM)

- An equilibrium model underlying modern finance theory
- Answers the following question:
  - What determines the expected return of each security?
- Who can claim credit for CAPM?
  - Markowitz (Nobel Prize)
  - Sharpe (Nobel Prize), Lintner and Mossin

Assumptions of CAPM

- Individual investors are price-takers
  - Individual action does not affect stock prices
- Common single-period investment horizon
  - Investors maximize expected utility
- Homogeneous expectations
  - Investors agree on the return distributions
  - Investors may have different risk aversions
- Market is frictionless
  - No taxes or transaction costs
  - What you see is what you are going to get
Equilibrium Implications of CAPM

- All investors will hold the same portfolio of risky assets – market portfolio
  - Risk aversion only affects relative weights between riskfree security and the optimal risky portfolio
- Market portfolio contains all securities
  - The proportion of each security is the percentage of that security’s market cap in total market cap
  - The weights of risky securities in your risky portfolio are the same as their weights in the market portfolio
- Risk premium for an individual security depends on its contribution to the risk of market portfolio

Capital Market Line (CML)
Capital Market Line

- Slope and Market Risk Premium in CML

\[ M : \text{Market portfolio} \]
\[ r_f : \text{Risk free rate (the time value of money)} \]
\[ E[r_M] - r_f : \text{Market risk premium (compensation for risks)} \]
\[ \frac{E[r_M] - r_f}{\sigma_M} : \text{Market price of risk} \]

Expected Return of An Individual Security Based on CAPM

- The risk premium of an individual security depends on
  - The security’s contribution to the risk of the market portfolio, or
  - The security’s systematic risk measure
- The contribution to market (systematic) risk is measured by Beta

\[ \beta_i = \frac{\text{Cov} \left[ r_i, r_M \right]}{\sigma_M^2} \]
\[ E[r_i] - r_f = \beta_i \left( E[r_M] - r_f \right) \]
Facts about Beta

- We can think Beta as a normalized covariance
- Beta of the market portfolio is ____
- Beta of a riskfree asset is ______
- Can a security have a negative beta? ____
- Can a risky security have an expected return lower than riskfree rate?

If yes, then how?

Security Market Line (SML)

- Graphical Presentation of CAPM

Security Market Line (SML)

\[
E(r_i) = E(r_M) + \beta_i (E(r_M) - r_f)
\]

\[
\beta_M = 1.0
\]
CML vs. SML

- Both specify a risk-return relationship
  
  \[ CML: E[r_i] - r_f = \frac{E[r_M] - r_f}{\sigma_M} \]
  
  \[ SML: E[r_i] - r_f = \beta_i (E[r_M] - r_f) \]

- Measure of risk
  
  - CML: risk is measured by \( \sigma_i \)
  
  - SML: risk is measured by \( \beta_i \)

- Applicability:
  
  - CML is applicable only to the efficient portfolios
    
    - Combinations of risk-free asset and market portfolio
  
  - SML is applicable to individual security and portfolios (efficient or inefficient ones)

An Example of SML

- Expected return for Market portfolio is 11%, risk free rate is 3%, security X and Y have betas of 1.25 and 0.6 respectively.

- Q: What are the expected returns of X and Y?

- A: Market risk premium: \( E[r_M] - r_f = 11\% - 3\% = 8\% \)
  
  \[ E[r_x] = r_f + \beta_x (E[r_M] - r_f) = 3\% + 1.25 \times 8\% = 13\% \]
  
  \[ E[r_y] = r_f + \beta_y (E[r_M] - r_f) = 3\% + 0.6 \times 8\% = 7.8\% \]
Graph for the SML Example

Beta of a Portfolio

- Beta of a portfolio is the weighted average of the betas of its component securities
  \[ \beta_P = \sum_{i=1}^{N} W_i \beta_i \]

- Beta of the efficient portfolio on the CML is
  - \( 0 < \beta < 1 \) if portfolio is long in riskfree bond (lending)
  - \( \beta > 1 \) if portfolio is short in bond (borrowing)
An Example of Portfolio Beta

- Q1: What is the beta of a portfolio with
  - a. 70% in market portfolio and 30% in riskfree bond?
  - b. -50% in riskfree bond and 150% in market portfolio?

- Q2: Stock X has a beta of 1.2 while stock Y has a beta of -0.2. X and Y have expected returns of 18% and 4% respectively. What is the expected return of the market portfolio?

Market Vs. Non-Market Risk

- The total risk of security $i$, measured by its variance and denoted $\sigma_i^2$, consists of two parts

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\epsilon i}^2$$

  - First component on the RHS is related to movements of the market portfolio ($\beta_i^2 \sigma_M^2$)
  - Second component is unique to the $i^{th}$ security ($\sigma_{\epsilon i}^2$)

- $R^2 = \frac{\beta_i^2 \sigma_M^2}{\sigma_i^2}$ measures how much market risk explains the total risk of the security
An Example of Risk Decomposition

- If the std dev of the market portfolio is 20%, then how much does the market risk account for the total risk of the following portfolios:

- Portfolio A is on CML with a Beta of 1.5?

- Portfolio B has a standard deviation of 35% and a beta of 1.5?