Chapter 7 - Part II
Factor Model and Arbitrage Pricing Theory (APT)

CAPM vs. One-Factor Market Model

- Under CAPM, a security’s risk premium is determined by
  - its level of systematic risk (beta) and
  - the market risk premium

\[ E[r_i] - r_f = \beta_i (E[r_M] - r_f), \text{ where } \beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2} \]

- Empirically we can estimate beta using realized returns from historical data

\[ r_{it} - r_{ft} = a_i + b_i (r_{Mt} - r_{ft}) + \epsilon_{it} \]
Estimation of CAPM Beta

- How to find beta?
  I. Find individual stock return data: \( r_i \)
  II. Find the market return data: \( r_M \)
  III. Find the T-bill data: \( r_f \)
  IV. Calculate the excess returns of
    I. Individual stocks: \( R_i = r_i - r_f \)
    II. Market: \( R_M = r_M - r_f \)
  V. Run the regression
    \[ R_i = \alpha + \beta R_M + e_i \]

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GM Example (Monthly data)

<table>
<thead>
<tr>
<th>Month</th>
<th>( r_i ) (GM)</th>
<th>( r_M ) (Mkt)</th>
<th>( r_f ) (Tbill)</th>
<th>( r_i - r_f )</th>
<th>( r_M - r_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>6.06%</td>
<td>7.89%</td>
<td>0.65%</td>
<td>5.41%</td>
<td>7.24%</td>
</tr>
<tr>
<td>Feb</td>
<td>-2.86%</td>
<td>1.51%</td>
<td>0.58%</td>
<td>-3.44%</td>
<td>0.93%</td>
</tr>
<tr>
<td>Mar</td>
<td>-8.18%</td>
<td>0.23%</td>
<td>0.62%</td>
<td>-8.80%</td>
<td>-0.39%</td>
</tr>
<tr>
<td>Apr</td>
<td>-7.36%</td>
<td>-0.29%</td>
<td>0.72%</td>
<td>-8.08%</td>
<td>-1.01%</td>
</tr>
<tr>
<td>May</td>
<td>7.70%</td>
<td>5.58%</td>
<td>0.66%</td>
<td>7.10%</td>
<td>4.92%</td>
</tr>
<tr>
<td>Jun</td>
<td>0.52%</td>
<td>1.73%</td>
<td>0.55%</td>
<td>-0.03%</td>
<td>1.18%</td>
</tr>
<tr>
<td>Jul</td>
<td>-1.74%</td>
<td>-0.21%</td>
<td>0.62%</td>
<td>-2.36%</td>
<td>-0.83%</td>
</tr>
<tr>
<td>Aug</td>
<td>-3.00%</td>
<td>-0.36%</td>
<td>0.36%</td>
<td>-3.55%</td>
<td>-0.91%</td>
</tr>
<tr>
<td>Sep</td>
<td>-0.56%</td>
<td>-3.58%</td>
<td>0.60%</td>
<td>-4.16%</td>
<td>-1.16%</td>
</tr>
<tr>
<td>Oct</td>
<td>-0.37%</td>
<td>4.62%</td>
<td>0.85%</td>
<td>-1.02%</td>
<td>3.97%</td>
</tr>
<tr>
<td>Nov</td>
<td>6.93%</td>
<td>6.85%</td>
<td>0.61%</td>
<td>6.32%</td>
<td>6.24%</td>
</tr>
<tr>
<td>Dec</td>
<td>3.08%</td>
<td>4.55%</td>
<td>0.65%</td>
<td>2.43%</td>
<td>3.90%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.02%</td>
<td>2.36%</td>
<td>0.62%</td>
<td>-0.60%</td>
<td>1.76%</td>
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<tr>
<td>Std Dev</td>
<td>4.97%</td>
<td>3.33%</td>
<td>0.06%</td>
<td>4.97%</td>
<td>3.32%</td>
</tr>
</tbody>
</table>

Regression Statistics
- R Square: 0.57
- Adjusted R Square: 0.53
- Coefficient: 1.14
- Standard Error: 0.04
- Observations: 12.00
CAPM vs. One-Factor Market Model

- We call the above equation *market model*
  - A statistical relationship between the return of an asset and the return of a market index
  - Market return becomes a "factor" that can explain return of individual security
- So we can think CAPM, an *equilibrium* model, as the market model with restrictions
  - $a_i = 0$
  - $b_i = \beta_i$
- If $a_i > 0$ (positive Jensen’s Alpha)
  - Performance is better than what CAPM predicts

Arbitrage Pricing Theory (APT)

- In general, a factor model can have multiple economic or statistical variables as factors
- APT is an extension of factor models to equilibrium setup.
- APT is driven by the following thoughts:
  - Factors are risks
  - Sensitivity to factors (coefficient or factor loadings) measures how much factor risk a security is taking
  - If we pool many securities together then non-factor or firm-specific risk can be diversified away
    - Portfolio only has factor risks
Arbitrage Pricing Theory (APT)

- So in equilibrium, a security should be compensated based on its exposure to the factor risks
  - A security with zero exposures to all factor risks should receive only riskfree rate

\[ E[r_i] = r_f + b_i \lambda \]

- \( b_i \) - the exposure to factor risk for security i
- \( \lambda \) - factor risk premium; the risk premium for the factor portfolio

Wrap-up

- The CAPM relationship and SML
- Definition of beta
- How to estimate beta?
- What is arbitrage?
- What determines the expected return of a security based on APT?