The Volatility Surface: Errata

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• **Page xxiii** “what it is that analysis of stock prices can tell us about how options ought to be priced”.

• **Page 7** In the first equation, the coefficient of

\[ S \frac{\partial V}{\partial S} \]

should be \( +r \) not \( -r \). Moreover \( dZ_2 \) in the fourth line of the equation should have a prefactor \( \eta \). Thus the equation at the top of page 7 should now read:

\[
d\Pi_1 - r \Pi_1 \, dt \\
= \left\{ \frac{\partial V}{\partial t} + \frac{1}{2} v S^2 \frac{\partial^2 V}{\partial S^2} + \rho \eta v \beta S \frac{\partial^2 V}{\partial v \partial S} + \frac{1}{2} \eta^2 v \beta^2 \frac{\partial^2 V}{\partial v^2} + r S \frac{\partial V}{\partial S} - r V \right\} \, dt \\
+ \frac{\partial V}{\partial v} \, dv \\
= \beta \sqrt{v} \frac{\partial V}{\partial v} \left\{ \varphi(S,v,t) \, dt + \eta dZ_2 \right\}
\]

(Thanks to Jan Obľoj of Imperial College)

• **Page 10** In the first equation, there should be no \( S \) in front of \( \frac{\partial}{\partial S_T} \). The equation should read

\[
\frac{1}{2} \frac{\partial^2}{\partial S_T^2} \left( \sigma^2 S_T^2 \varphi \right) - \frac{\partial}{\partial S_T} \left( \mu S_T \varphi \right) = \frac{\partial \varphi}{\partial T}
\]
Also, the equation lower down on page 10:

\[
\frac{\partial C}{\partial T} = \frac{\sigma^2 K^2}{2} \varphi + \int_{K}^{\infty} dS_T \mu S_T \varphi \\
= \frac{\sigma^2 K^2}{2} \frac{\partial^2 C}{\partial K^2} + \mu (T) \left( -K \frac{\partial C}{\partial K} \right)
\]

should be corrected to:

\[
\frac{\partial C}{\partial T} = \frac{\sigma^2 K^2}{2} \varphi + \int_{K}^{\infty} dS_T \mu S_T \varphi \\
= \frac{\sigma^2 K^2}{2} \frac{\partial^2 C}{\partial K^2} + \mu (T) \left( C - K \frac{\partial C}{\partial K} \right)
\]

(Thanks to Jan Oblój of Imperial College)

- **Page 12** The last sentence should read “equation (1.8) becomes” rather than “equation (1.4) becomes”.
(Thanks to Dan Pirjol of Markit)

- **Page 33** In equation (3.14), \( \sqrt{v_t} \) should be inside the expectation. (3.14) would then read:

\[
du_t = -\lambda(u_t-\bar{v})dt + \frac{\rho \eta}{2} u_t dt + \rho \eta x_T d\hat{w}_t + \sqrt{1-\rho^2 \eta} E[\sqrt{v_t} dW_t | x_T]
\]

Also, the line just after equation (3.15) should read: “with \( \bar{v}' : = (v_0 - \bar{v}') e^{-\lambda'T} + \bar{v}' \)” (an extra subscript 0 on the \( v \)).

On the last line of page 33, we should replace the words “Brownian Bridge” with the words “Brownian Bridge-like”.
(Thanks to Jan Oblój)

- **Page 35** Formula (3.19) should read:

\[
\sigma_{BS}(K, T)^2 \approx \frac{\hat{w}_T}{T} + \frac{\rho \eta}{T} x_T \int_0^T dt \int_0^t e^{-\lambda(t-s)} ds \\
= \frac{\hat{w}_T}{T} + \rho \eta \frac{x_T}{\lambda T} \left\{ 1 - \frac{1 - e^{-\lambda T}}{\lambda T} \right\}
\]

– the current version has an extra factor \( \frac{1}{T} \) inside the first integral.
Also, further down on the same page there are three references to:

\[ \frac{\partial}{\partial x_t} \sigma_{BS}(K, T)^2 \]

These should of course read

\[ \frac{\partial}{\partial x_T} \sigma_{BS}(K, T)^2 \]

(Thanks to Paul Jones of Imperial College)

- **Page 44** Second paragraph, “from equation (3.11)” should read “from equation (3.15)”.
  (Thanks to Jan Oblój)

- **Page 49** “We note too that the Heston-Nandi model and its local volatility equivalent are single-factor”.

- **Page 57** Line 5 should read “The characteristic function for an exponential”.
  (Thanks to Jan Oblój)

- **Page 61** There is a subscript BS missing in equation (5.9) which should read:

  \[
  C_J(S, K, \Delta T) \approx (1 - \lambda \Delta T) C_{BS}(Se^{\mu \Delta T}, K, \Delta T) + \lambda \Delta T C_{BS}(JS, K, \Delta T) = C_{BS}(Se^{\mu \Delta T}, K, \Delta T) + O(\Delta T)
  \]
  (Thanks to Jan Oblój)

- **Page 68** Factor T missing in the denominator of the last equation on the page. It should read

  \[
  I(u, T) = \frac{1}{T} \int_0^T e^{\gamma v D(u, t)} dt = -\frac{1}{T} \frac{2\gamma v}{p_+p_-} \int_{-\gamma D(u, T)}^{-\gamma D(u, T)} e^{-z} dz (1 + z/p_+) (1 + z/p_-)
  \]
  (Thanks to Roger Lord)
• Page 89 6 lines from the bottom, “earlier in Chapter 7” should read “earlier in Chapter 3”.
(Thanks to Jan Obloj)

• Page 102 The expression for $v_{\text{loc}}$ should read:

$$v_{\text{loc}} = \frac{\partial w}{\partial T} \left( 1 - \frac{k w}{\partial k} + \frac{1}{4} \left( -\frac{1}{4} - \frac{1}{w} + \frac{k^2}{w^2} \right) \left( \frac{\partial w}{\partial k} \right)^2 + \frac{1}{2} \frac{\partial^2 w}{\partial k^2} \right)$$

(Thanks to John-Joe Hosking of Imperial College)

• Page 113 There should be an extra $\Delta K$ so that the formula reads

$$2N \left( d_2 \right)_{S=K} \Delta K$$

(Thanks to Craig Nelson)

• Page 124 should have $\text{MinCoupon} = -12\%$ not $\text{MinCoupon} = -1\%$.
(Thanks to Jining Han)

• Page 139 The definition of $d_2$ should have a minus sign:

$$z(k) = d_2 = -\frac{k}{\sigma \sqrt{T}} - \frac{\sigma \sqrt{T}}{2}$$

(Thanks to James LaDue of Imagine Software)

• Page 141 The upper limit of the $y$–integrations should be $\infty$ not $T$.
(Thanks to Fahmi Zaidi)

• Page 160 The VXB formula should read

$$E_{T_1} \left[ \sqrt{\langle x \rangle_{T_1,T_2}} \right] =: Y$$

(Thanks to Jining Han)

• Page 177 Add an entry: “Stochastic differential equation (SDE), 4,7,26,43,44,52,57,87,91.”