Chapter 8

Risk and Return: Capital Asset Pricing Model

Liuren Wu
Overview

1. Portfolio Returns and Portfolio Risk
   Calculate the expected rate of return and volatility for a portfolio of investments and describe how diversification affects the returns to a portfolio of investments.

2. Systematic Risk and the Market Portfolio
   Understand the concept of systematic risk for an individual investment and calculate portfolio systematic risk (beta).

3. The CAPM
   Estimate an investor’s required rate of return using capital asset pricing model.
8.1 Portfolio Returns and Portfolio Risk

- By investing in many different stocks to form a portfolio, we can lower the risk without lowering the expected return.

- The effect of lowering risk via appropriate portfolio formulation is called *diversification*.

- By learning how to compute the expected return and risk on a portfolio, we illustrate the effect of diversification.
The Expected Return of a Portfolio

- To calculate a portfolio’s expected rate of return, we weight each individual investment’s expected rate of return using the fraction of money invested in each investment.

- **Example 8.1**: If you invest 25% of your money in the stock of Citi bank (C) with an expected rate of return of -32% and 75% of your money in the stock of Apple (AAPL) with an expected rate of return of 120%, what will be the expected rate of return on this portfolio?

  - Expected rate of return = 0.25(-32%) + 0.75(120%) = 82%

**Portfolio Expected Rate of Return**

\[
E(r_{portfolio}) = [W_1 \times E(r_1)] + [W_2 \times E(r_2)] + [W_3 \times E(r_3)] + \cdots + [W_n \times E(r_n)]
\]
Checkpoint 8.1

Calculating a Portfolio’s Expected Rate of Return

Penny Simpson has her first full-time job and is considering how to invest her savings. Her dad suggested she invest no more than 25% of her savings in the stock of her employer, Emerson Electric (EMR), so she is considering investing the remaining 75% in a combination of a risk-free investment in U.S. Treasury bills, currently paying 4%, and Starbucks (SBUX) common stock. Penny’s father has invested in the stock market for many years and suggested that Penny might expect to earn 9% on the Emerson shares and 12% from the Starbucks shares. Penny decides to put 25% in Emerson, 25% in Starbucks, and the remaining 50% in Treasury bills. Given Penny’s portfolio allocation, what rate of return should she expect to receive on her investment?
STEP 3: Solve

We can use Equation (8-1) to calculate the expected rate of return for the portfolio as follows:

\[
E(r_{\text{portfolio}}) = W_{\text{Treasury Bills}}E(r_{\text{Treasury Bills}}) + W_{EMR}E(r_{EMR}) + W_{SBUX}E(r_{SBUX})
\]

\[
= (1/2 \times .04) + (1/4 \times .08) + (1/4 \times .12) = .07 \text{ or } 7\%
\]

Alternatively, by filling out the table described above we get the same result.

<table>
<thead>
<tr>
<th>E(Return)</th>
<th>Weight</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury bills</td>
<td>4.0%</td>
<td>0.50</td>
</tr>
<tr>
<td>Emerson Electric (EMR)</td>
<td>8.0%</td>
<td>0.25</td>
</tr>
<tr>
<td>Starbucks (SBUX)</td>
<td>12.0%</td>
<td>0.25</td>
</tr>
<tr>
<td>Portfolio E(Return) =</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Checkpoint 8.1: Check Yourself

Evaluate the expected return for Penny’s portfolio where she places \(\frac{1}{4}\)th of her money in Treasury bills, half in Starbucks stock, and the remainder in Emerson Electric stock.

Answer: 9%.
Evaluating Portfolio Risk

- Unlike expected return, standard deviation is not generally equal to the a weighted average of the standard deviations of the returns of investments held in the portfolio. This is because of diversification effects.

- The diversification gains achieved by adding more investments will depend on the degree of correlation among the investments.

- The degree of correlation is measured by using the correlation coefficient ($r$).
Correlation and diversification

- The correlation coefficient can range from -1.0 (perfect negative correlation), meaning two variables move in perfectly opposite directions to +1.0 (perfect positive correlation), which means the two assets move exactly together.

- A correlation coefficient of 0 means that there is no relationship between the returns earned by the two assets.

- As long as the investment returns are not perfectly positively correlated, there will be diversification benefits.

- However, the diversification benefits will be greater when the correlations are low or negative.

- The returns on most stocks tend to be positively correlated.
For simplicity, let’s focus on a portfolio of 2 stocks:

\[ \sigma_{portfolio} = \sqrt{W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + 2W_1 W_2 \rho_{1,2} \sigma_1 \sigma_2} \]

**Important Definitions and Concepts:**

- \( \sigma_{portfolio} \) = the standard deviation in portfolio returns,
- \( W_i \) = the proportion of the portfolio that is invested in asset \( i \),
- \( \sigma_i \) = the standard deviation in the rate of return earned by asset \( i \), and
- \( \rho_{i,j} \) = the correlation coefficient between the rates of return earned by assets \( i \) and \( j \). The symbol \( \rho_{i,j} \) (pronounced “rho”) represents the correlation coefficient between the rates of return for asset 1 and asset 2.
Diversification effect

- Investigate the equation:

\[ \sigma_{\text{portfolio}} = \sqrt{W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\rho_{1,2}\sigma_1\sigma_2} \]

- When the correlation coefficient \( \rho = 1 \), the portfolio standard deviation becomes a simple weighted average:

\[ \sigma_{\text{portfolio}} = |W_1^1 + W_2^2|, \quad \text{when} \quad \rho = 1 \]

- If the stocks are perfectly moving together, they are essentially the same stock. There is no diversification.

- For most two different stocks, correlation is less than perfect (<1). Hence, the portfolio standard deviation is less than the weighted average. – This is the effect of diversification.
Determine the expected return and standard deviation of the following portfolio consisting of two stocks that have a correlation coefficient of .75.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Weight</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>.50</td>
<td>.14</td>
<td>.20</td>
</tr>
<tr>
<td>Coca-Cola</td>
<td>.50</td>
<td>.14</td>
<td>.20</td>
</tr>
</tbody>
</table>
Answer

- Expected Return   \[ = 0.5 \times 0.14 + 0.5 \times 0.14 = 0.14 \text{ or } 14\% \]

- Standard deviation

\[ = \sqrt{0.5^2 \times 0.2^2 + 0.5^2 \times 0.2^2 + 2 \times 0.5 \times 0.5 \times 0.75 \times 0.2 \times 0.2} \]
\[ = \sqrt{0.035} = 0.187 \text{ or } 18.7\% \]

- Lower than the weighted average of 20\%.
**Figure 8.1**

**Diversification and the Correlation Coefficient—Apple and Coca-Cola**

The effects of diversification on the risk of the portfolio is contingent on the degree of correlation between the assets included in the portfolio. If the correlation is +1 (meaning the two assets are perfectly correlated and move together in lockstep as was the case with sunglasses and sunscreen), then there is no benefit to diversification. However, if the correlation is −1 (meaning the two assets move in lockstep in opposite directions as was the case with sunglasses and umbrellas), it will be possible to construct a portfolio that completely eliminates risk.

Notice that if the correlation coefficient is equal to +1.0, the portfolio standard deviation is equal to a weighted average of the standard deviations of the individual stocks in the portfolio (20% in this case).

If the correlation coefficient is −1.0 the portfolio standard deviation drops to zero!
Portfolio return does not depend on correlation

Portfolio standard deviation decreases with declining correlation.

Legend:

<table>
<thead>
<tr>
<th>Correlation</th>
<th>E(Return)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>0.14</td>
<td>0%</td>
</tr>
<tr>
<td>-0.80</td>
<td>0.14</td>
<td>6%</td>
</tr>
<tr>
<td>-0.60</td>
<td>0.14</td>
<td>9%</td>
</tr>
<tr>
<td>-0.40</td>
<td>0.14</td>
<td>11%</td>
</tr>
<tr>
<td>-0.20</td>
<td>0.14</td>
<td>13%</td>
</tr>
<tr>
<td>0.0</td>
<td>0.14</td>
<td>14%</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>15%</td>
</tr>
<tr>
<td>0.40</td>
<td>0.14</td>
<td>17%</td>
</tr>
<tr>
<td>0.60</td>
<td>0.14</td>
<td>18%</td>
</tr>
<tr>
<td>0.80</td>
<td>0.14</td>
<td>19%</td>
</tr>
<tr>
<td>1.00</td>
<td>0.14</td>
<td>20%</td>
</tr>
</tbody>
</table>

All portfolios are comprised of equal investments in Apple and Coca-Cola shares.
Evaluating a Portfolio’s Risk and Return

Sarah plans to invest half of her 401k savings in a mutual fund mimicking S&P 500 and half in an international fund.

The expected return on the two funds are 12% and 14%, respectively. The standard deviations are 20% and 30%, respectively. The correlation between the two funds is 0.75.

What would be the expected return and standard deviation for Sarah’s portfolio?
Checkpoint 8.2: *Check Yourself*

- Verify the answer: 13%, 23.5%
- Evaluate the expected return and standard deviation of the portfolio, if the correlation is .20 instead of .75.
Answer

- The expected return remains the same at 13%.
- The standard deviation declines from 23.5% to 19.62% as the correlations declines from 0.75 to 0.20.
- The weight average of the standard deviation of the two funds is 25%, which would be the standard deviation of the portfolio if the two funds are perfectly correlated.
- Given less than perfect correlation, investing in the two funds leads to a reduction in standard deviation, as a result of diversification.
8.2 Systematic Risk and Market Portfolio

- It would be an onerous task to calculate the correlations when we have thousands of possible investments.

- Capital Asset Pricing Model or the CAPM provides a relatively simple measure of risk.

- CAPM assumes that investors choose to hold the optimally diversified portfolio that includes all risky investments. This optimally diversified portfolio that includes all of the economy’s assets is referred to as the market portfolio.

- According to the CAPM, the relevant risk of an investment relates to how the investment contributes to the risk of this market portfolio.
Risk classification

- To understand how an investment contributes to the risk of the portfolio, we categorize the risks of the individual investments into two categories:
  1. Systematic risk, and
  2. Unsystematic risk, or idiosyncratic risk

- The **systematic risk** component measures the contribution of the investment to the risk of the market. For example: War, hike in corporate tax rate.

- The **unsystematic risk** is the element of risk that does not contribute to the risk of the market. This component is diversified away when the investment is combined with other investments. For example: Product recall, labor strike, change of management.
Systematic versus Idiosyncratic Risk

- An investment’s systematic risk is far more important than its unsystematic risk.

- If the risk of an investment comes mainly from unsystematic risk, the investment will tend to have a low correlation with the returns of most of the other stocks in the portfolio, and will make a minor contribution to the portfolio’s overall risk.
Figure 8.2

Portfolio Risk and the Number of Investments in the Portfolio

Adding more investments to a portfolio that are not highly correlated with the other assets in the portfolio can dramatically reduce the portfolio’s risk. In fact, for randomly selected shares of common stock, the benefits of diversification can be virtually fully achieved with a portfolio of less than 50 stocks (assuming equal investment in each stock).
Diversification and Systematic Risk

- Figure 8-2 illustrates that as the number of securities in a portfolio increases, the contribution of the unsystematic or diversifiable risk to the standard deviation of the portfolio declines.

- Systematic or non-diversifiable risk is not reduced even as we increase the number of stocks in the portfolio.

- Systematic sources of risk (such as inflation, war, interest rates) are common to most investments resulting in a perfect positive correlation and no diversification benefit.

- Large portfolios will not be affected by unsystematic risk but will be influenced by systematic risk factors.
Systematic Risk and Beta

- Systematic risk is measured by **beta coefficient**, which estimates the extent to which a particular investment’s returns vary with the returns on the market portfolio.

- In practice, it is estimated as the slope of a straight line (see figure 8-3):

  \[ R_i = a + R_m + e \]

- Beta could be estimated using excel or financial calculator, or readily obtained from various sources on the internet (such as Yahoo Finance and Money Central.com)
Estimating Google’s (GOOG) Beta Coefficient

A firm’s beta coefficient is the slope of a straight line that fits the relationship between the firm’s stock returns and those of a broad market index. In the graph below, the market index used is the Standard and Poor’s (S&P) 500 Index.

Observation #5
S&P 500 return = -5.9%
Google return = -18.5%

Run = 12%
Rise = 9%

Google’s Slope
= 9%/12% = .75
Utilities companies can be considered less risky because of their lower betas.
The beta of a portfolio measures the systematic risk of the portfolio and is calculated by taking a simple weighted average of the betas for the individual investments contained in the portfolio.

Example 8.2 Consider a portfolio that is comprised of four investments with betas equal to 1.5, .75, 1.8 and .60. If you invest equal amount in each investment, what will be the beta for the portfolio?

Portfolio beta = 1.5*(1/4) + .75*(1/4) + 1.8*(1/4) + .6*(1/4) = 1.16
8.3 The CAPM

- CAPM also describes how the betas relate to the expected rates of return that investors require on their investments.

- The key insight of CAPM is that investors will require a higher rate of return on investments with higher betas. The relation is given by the following linear equation:

\[ E(r_{Asset\ j}) = r_f + \beta_{Asset\ j} \left[ E(r_{market}) - r_f \right] \]

- \( R_{market} \) is the expected return on the market portfolio

- \( R_f \) is the riskfree rate (return for zero-beta assets).
Example

\[ E(r_{Asset\ j}) = r_f + \beta_{Asset\ j} \left[ E(r_{market}) - r_f \right] \]

- **Example 8.2** What will be the expected rate of return on AAPL stock with a beta of 1.49 if the risk-free rate of interest is 2% and if the market risk premium, which is the difference between expected return on the market portfolio and the risk-free rate of return is estimated to be 8%?

- AAPL expected return = 2% + 1.49*8% = **13.92%**.
Checkpoint 8.3: *Check Yourself*

Estimate the expected rates of return for the three utility companies, found in Table 8-1, using the 4.5% risk-free rate and market risk premium of 6%. Use beta estimates from Yahoo:

\[
\text{AEP} = 0.74, \text{DUK} = 0.40, \text{CNP} = 0.82. \]
Solution

\[ E(r_{Asset,j}) = r_f + \beta_{Asset,j} \left[ E(r_{market}) - r_f \right] \]

- Beta (AEP) = 4.5% + 0.74(6%) = 8.94%
- Beta (DUK) = 4.5% + 0.40(6%) = 6.9%
- Beta (CNP) = 4.5% + 0.82(6%) = 9.42%
- The higher the beta, higher is the expected return.