Some Random Thoughts on Time Value of Money

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To me, a simple and generic way of linking time values of a single cashflow together is through the following formula:

\[ V_n = V_m (1 + r)^{(n-m)} \],

where \( V_n \) denotes the value of the cashflow at period \( n \), \( V_m \) denotes the value of the cashflow at period \( m \), \( r \) denotes the interest rate (return, discount rate) for each period. The textbook defines present value (today’s value) versus future value (value in the future?), but I feel those definitions are not that meaningful nor are those definitions exact. What’s really meaningful is to specify the value at what time period. We can always say \( V_0 \) is the present value if we regard today as time 0. Furthermore, with time 0 being today, \( V_j \) will be “future value” if time \( j > 0 \). But these are almost pure semantics. Let’s use examples to see how this general formula can be used to convert the value from one time to another.

**Examples:**

- If you receive $100 today (hence \( V_0 = 100 \)), how much will it be worth in 5 years at 5% annual compounding rate? — Answer: The rate for each period (in this case year) is 5%. The value you know is \( V_0 = 100 \) and you want to find is \( V_5 \). Apply the above equation, we have \( V_5 = V_0 (1 + r)^5 = 100(1 + .05)^5 = 127.63 \).

- If you receive $100 today (hence \( V_0 = 100 \)), how much will it be worth in 5 years at 5% semi-annual compounding rate? — Answer: The rate for each period (in this case half-year) is \( 5%/2 = 0.025 \). The value you know is \( V_0 = 100 \) and you want to find is \( V_{10} \) (value 10 periods later, which is five years later). Apply the above equation, we have \( V_{10} = V_0 (1 + r)^{10} = 100(1 + .025)^{10} = 128.01 \). The only difference between this and the previous question is how frequent one compounds the interest. You can see that the more frequent you compound, the more money you get, given the same quoted rate.

- If someone promises to give you $100 five years from now, what is its worth today (how much you are willing
to pay for it today) at 6% annual compounding rate? —Answer: The rate for each period is 6%. The value you
know is $V_5 = 100$ and the value you want to find is $V_0$. Hence, $V_0 = V_5 (1 + r)^{5-0} = 100 (1 + 0.06)^{-5} = 74.726$.

• If someone promises to give you $100 five years from now, what is its worth today (how much you are willing
to pay for it today) at 6% monthly compounding rate? —Answer: Each period is one month. The rate for each
period is $6%/12 = 0.005$. The value you know is $V_{5 \times 12} = V_{60} = 100$ and the value you want to find is $V_0$. Hence,
$V_0 = V_{60} (1 + r)^{0-60} = 100 (1 + 0.005)^{-60} = 74.137$. Note that $V_{60}$ in this problem refers to the same thing as the
$V_5$ in the previous problem, except 60 here means 60 months whereas in the previous proble we use year as the
unit and 5 denotes five years. For compounding to the future, the higher the frequency, the higher the future value.
For this question of discounting back to today, it is the opposite. The more frequent the discounting, the lower the
present value.

• The textbook formula for future value: $FV_n = PV (1 + r)^n$, where $PV$ is essentially $V_0$ in my notation.

• Sometimes, the textbook uses $n$ to denote the number of years, $m$ to denote the number of compounding per
year, and $i$ denote annual rate, then the future value formula can be written as $FV_n = PV \left(1 + \frac{i}{m}\right)^{nm}$. Under this
notation, my rate per period $r = i/m$ and the number of periods is simply $n \times m$ because there are $n$ years and $m$
compounding per year.

• Let’s do a calendar example. Suppose you received $100 in January 1, 2010 and saved the money in the bank for
a monthly compounding rate of 6%, you now want to know how much money you will have in the distant future
of January 1, 2020.— Answer: First, I know the period is per month and the monthly rate is $6%/12=0.005$. The
notation for period is a bit ugly. So let me reset January 1, 2010 to time 0, then January 2020 is 10 years later,
which is 120 months later. Hence, I can denote January 1, 2010 as time 0 and January 1, 2020 as time period 120.
$V_{120} = V_0 (1 + r)^{120} = 100 (1 + 0.005)^{120} = 181.94$. 