Introduction, Forwards and Futures

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(Hull chapters: 1,2,3,5)
Derivatives

- Derivatives are financial instruments whose returns are derived from those of another financial instrument.

- Cash markets or spot markets
  - The sale is made, the payment is remitted, and the good or security is delivered immediately or shortly thereafter.

- Derivative markets
  - Derivative markets are markets for contractual instruments whose performance depends on the performance of another instrument, the so-called underlying.
Derivatives Markets

- Over-the-counter market (OTC)
  - OTC securities are not listed or traded on an organized exchange.
  - An OTC contract is a private transaction between two parties (counterparty risk).
  - A typical deal in the OTC market is conducted through a telephone or other means of private communication.
  - The terms of an OTC contract are usually negotiated on the basis of an ISDA master agreement (International Swaps and Derivatives Association).
Exchange-traded instruments (Listed products)

- Exchange traded securities are generally standardized in terms of maturity, underlying notional, settlement procedures ...

- By the commitment of some market participants to act as market-maker, exchange traded securities are usually very liquid.
  - Market makers are particularly needed in illiquid markets.

- Many exchange traded derivatives require "margining" to limit counterparty risk.

- On some exchanges, the counterparty is the exchange itself yielding the advantage of anonymity.
Derivatives Products

- Forwards (OTC)
- Futures (exchange listed)
- Swaps (OTC)
- Options (both OTC and exchange listed)
Fundamental Ingredients of Derivative Trading/Pricing

- Arbitrage: Opportunity to lock into a risk-free profit.
  - Question: “What does risk-free mean?”

- Possibility to store an economic good: essential link between cash and derivative markets.

- Delivery and Settlement: important in the evaluation of relative price differences between cash and derivative markets.

- Price discovery: prices must be determined in an “objective” way. The instrument’s payoff structure must be irrevocably clear.
Arbitrage in a Micky Mouse Model

- The current prices of asset 1 and asset 2 are 95 and 43, respectively.
- Tomorrow, one of two states will come true
  - A good state where the prices go up or
  - A bad state where the prices go down

\[
\begin{align*}
\text{Asset 1} &= 95 \\
\text{Asset 2} &= 43 \\
\text{Asset 1} &= 100 \\
\text{Asset 2} &= 50 \\
\text{Asset 1} &= 80 \\
\text{Asset 2} &= 40
\end{align*}
\]

Do you see any possibility to make risk-free money out of this situation?
Arbitrage in a Micky Mouse Model

- Characteristics
  - More wealth is always preferred to less
  - Given two investment opportunities, we always prefer the one that performs at least as good as all the other opportunities in all states and better in at least one state
  - If two opportunities have equivalent outcomes, they must have equivalent prices
  - An investment opportunity that produces the same return in all states is risk-free and must yield the risk-free rate

- How do you like that?
Derivative Traders

- Hedgers
- Speculators
- Arbitrageurs

Some of the largest trading losses in derivatives have occurred because individuals who had a mandate to be hedgers or arbitrageurs switched to being speculators.
Forward contracts: Definition

- A forward contract is an OTC agreement between two parties to exchange an underlying asset
  - for an agreed upon price (the forward price)
  - at a given point in time in the future (the expiry date)

Example: On June 3, 2003, Party A signs a forward contract with Party B to sell 1 million British pound (GBP) at 1.61 USD per 1 GBP six month later.
  - Today (June 3, 2003), sign a contract, shake hands. No money changes hands.
  - December 6, 2003 (the expiry date), Party A pays 1 million GBP to Party B, and receives 1.61 million USD from Party B in return.
  - Currently (June 3), the spot price for the pound (the spot exchange rate) is 1.6285. Six month later (December 3), the exchange rate can be anything (unknown).
  - 1.61 is the forward price.
Foreign exchange quotes for GBPUSD June 3, 2003

<table>
<thead>
<tr>
<th>Maturity</th>
<th>bid</th>
<th>offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>spot</td>
<td>1.6281</td>
<td>1.6285</td>
</tr>
<tr>
<td>1-month forward</td>
<td>1.6248</td>
<td>1.6253</td>
</tr>
<tr>
<td>3-month forward</td>
<td>1.6187</td>
<td>1.6192</td>
</tr>
<tr>
<td>6-month forward</td>
<td>1.6094</td>
<td>1.6100</td>
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</tbody>
</table>

- The forward prices are different at different maturities.
  - **Maturity or time-to-maturity** refers to the length of time between now and expiry date (1m, 2m, 3m etc).
  - **Expiry (date)** refers to the date on which the contract expires.
  - **Notation:** Forward price $F(t, T)$: $t$: today, $T$: expiry, $\tau = T - t$: time to maturity.
  - The spot price $S(t) = F(t, t)$. [or $S_t, F_t(T)$]

- Forward contracts are the most popular in currency and interest rates.
Forward price revisited

- The forward price for a contract is the delivery price that would be applicable to the contract if were negotiated today. It is the delivery price that would make the contract worth exactly zero.

  - Example: Party A agrees to sell to Party B 1 million GBP at the price of 1.3USD per GBP six month later, but with an upfront payment of 0.3 million USD from B to A.

  - 1.3 is NOT the forward price. Why?

- The party that has agreed to buy has what is termed a long position. The party that has agreed to sell has what is termed a short position.

  - In the previous example, Party A entered a short position and Party B entered a long position on dollar.

  - But since it is on exchange rates, you can also say: Party A entered a long position and Party B entered a short position on pound.
Profit and Loss (P&L) in forward investments

- By signing a forward contract, one can lock in a price *ex ante* for buying or selling a security.

- Ex post, whether one gains or loses from signing the contract depends on the spot price at expiry.

- In the previous example, Party A agrees to sell 1 million pound at $1.61 per GBP at expiry. If the spot price is $1.31 at expiry, what’s the P&L for party A?

  - On Dec 3, Party A can buy 1 million pound from the market at the spot price of $1.31 and sell it to Party B per forward contract agreement at $1.61.

  - The net P&L at expiry is the difference between the strike price \( K = 1.61 \) and the spot price \( S_T = 1.31 \), multiplied by the notional (1 million). Hence, 0.3 million.

- If the spot rate is $1.71 on Dec 3, what will be the P&L for Party A? What’s the P&L for Party B?
Profit and Loss (P&L) in forward investments

\[(K = 1.61)\]

long forward: \((S_T - K)\)

short forward: \((K - S_T)\)

- Credit risk: There is a small possibility that either side can default on the contract. That’s why forward contracts are mainly between big institutions.

- How to calculate returns on forward investments?
Futures versus Forwards

Futures contracts are similar to forwards, but

- Buyer and seller negotiate *indirectly*, through the exchange.
- Default risk is borne by the exchange clearinghouse
- Positions can be easily reversed at any time before expiration
- Value is marked to market daily.
- Standardization: quality; quantity; Time.

  ▶ The short position has often different *delivery options*; good because it reduces the risk of squeezes, bad ... because the contract is more difficult to price (need to price the “cheapest-to-deliver”).

The different execution details also lead to pricing differences, e.g., effect of marking to market on interest calculation.
Futures versus Spot

- Easier to go short: with futures it is equally easy to go short or long. A short seller using the spot market must wait for an uptick before initiating a position.
- Lower transaction cost.
  - Fund managers who want to reduce or increase market exposure, usually do it by selling the equivalent amount of stock index futures rather than selling stocks.
  - Underwriters of corporate bond issues bear some risk because market interest rates can change the value of the bonds while they remain in inventory prior to final sale: Futures can be used to hedge market interest movements.
  - Fixed income portfolio managers use futures to make duration adjustments without actually buying and selling the bonds.
Futures on what?

- Just about anything. "If you can say it in polite company, there is probably a market for it," advertises the CME.

- For example, the CME trades futures on agricultural commodities, foreign currencies, interest rates, and stock market indices, including
  
  ▶ **Agricultural commodities**: Live Cattle, Feeder Cattle, Live Hogs, Pork Bellies, Broiler Chickens, Random-Length Lumber.
  
  ▶ **Foreign currencies**: Euro, British pound, Canadian dollar, Japanese yen, Swiss franc, Australian dollar, ...
  
  ▶ **Interest rates**: Eurodollar, Euromark, 90-Day Treasury bill, One-Year Treasury bill, One-Month LIBOR
  
  ▶ **Stock indices**: S&P 500 Index, S&P MidCap 400 Index, Nikkei 225 Index, Major Market Index, FT-SE 100 Share Index, Russell 2000 Index
How do we determine forward/futures prices?

Is there an arbitrage opportunity?

- The spot price of gold is $300.
- The 1-year forward price of gold is $340.
- The 1-year USD interest rate is 5% per annum, continuously compounding.

Apply the principle of arbitrage:

- The key idea underlying a forward contract is to lock in a price for a security.
- Another way to lock in a price is to buy now and carry the security to the future.
- Since the two strategies have the same effect, they should generate the same P&L. Otherwise, short the expensive strategy and long the cheap strategy.
- The expensive/cheap concept is relative to the two contracts only. Maybe both prices are too high or too low, compared to the fundamental value ...
Pricing forward contracts via replication

- Since signing a forward contract is equivalent (in effect) to buying the security and carry it to maturity.
- The forward price should equal to the cost of buying the security and carrying it over to maturity:

  \[ F(t, T) = S(t) + \text{cost of carry} - \text{benefits of carry}. \]

Apply the principle of arbitrage: Buy low, sell high.

- The 1-year later (at expiry) cost of signing the forward contract now for gold is $340.
- The cost of buying the gold now at the spot ($300) and carrying it over to maturity (interest rate cost because we spend the money now instead of one year later) is:

  \[ S_t e^{r(T-t)} = 300e^{0.05 \times 1} = 315.38. \]

  (The future value of the money spent today)
- Arbitrage: Buy gold is cheaper than signing the contract, so buy gold today and short the forward contract.
Carrying costs

- **Interest rate cost**: If we buy today instead of at expiry, we endure interest rate cost — In principle, we can save the money in the bank today and earn interests if we can buy it later.
  - This amounts to calculating the future value of today’s cash at the current interest rate level.
  - If 5% is the annual compounding rate, the future value of the money spent today becomes, $S_t(1 + r)^1 = 300 \times (1 + .05) = 315$.

- **Storage cost**: We assume zero storage cost for gold, but it could be positive...
  - Think of the forward price of live hogs, chicken, ...
  - Think of the forward price of electricity, or weather ...
Carrying benefits

- **Interest rate benefit**: If you buy pound (GBP) using dollar today instead of later, it costs you interest on dollar, but you can save the pound in the bank and make interest on pound. In this case, what matters is the interest rate difference:

\[ F(t, T)[GBPUSD] = S_t e^{(r_{USD} - r_{GBP})(T-t)} \]

- In discrete (say annual) compounding, you have something like:

\[ F(t, T)[GBPUSD] = S_t (1 + r_{USD})^{(T-t)}/(1 + r_{GBP})^{(T-t)}. \]

- **Dividend benefit**: similar to interests on pound

  - Let \( q \) be the continuously compounded dividend yield on a stock, its forward price becomes, \( F(t, T) = S_t e^{(r-\frac{q}{2})(T-t)}. \)
  - Also think of piglets, eggs, ...
Another example of arbitrage

Is there an arbitrage opportunity?

- The spot price of gold is $300.
- The 1-year forward price of gold is $300.
- The 1-year USD interest rate is 5% per annum, continuously compounding.
Another example of arbitrage

Is there an arbitrage opportunity?

- The spot price of oil is $19
- The quoted 1-year futures price of oil is $25
- The 1-year USD interest rate is 5%, continuously compounding.
- The annualized storage cost of oil is 2%, continuously compounding.
Another example of arbitrage

Is there an arbitrage opportunity?

- The spot price of oil is $19
- The quoted 1-year futures price of oil is $16
- The 1-year USD interest rate is 5%, continuously compounding.
- The annualized storage cost of oil is 2%, continuously compounding.

Think of an investor who has oil at storage to begin with.
Another example of arbitrage?

*Is there an arbitrage opportunity?*

- The spot price of electricity is $100 (per some unit...)
- The quoted 3-month futures price on electricity is $110
- The 1-year USD interest rate is 5%, continuously compounding.
- Electricity cannot be effectively stored

*How about the case where the storage cost is enormously high?*
Covered interest rate parity

- The cleanest pricing relation is on currencies:
  \[ F(t, T) = S_t e^{(r_d - r_f)(T - t)}. \]

- Taking natural logs on both sides, we have the covered interest rate parity:
  \[ f_{t,T} - s_t = (r_d - r_f)(T - t). \]
  The log difference between forward and spot exchange rate equals the interest rate difference.

- Notation: \((f, s)\) are natural logs of \((F, S)\): \(s = \ln S, f = \ln F\).
Uncovered interest rate parity

- Since we use forward to lock in future exchange rate, we can think of forwards as the “expected value” of future exchange rate,

\[ F(t, T) = \mathbb{E}_t^Q [S_T] = S_t e^{(r_d - r_f)(T - t)}, \]

where \( \mathbb{E}[\cdot] \) denotes expectation and \( Q \) is a qualifier: The equation holds only if people do not care about risk; otherwise, there would be a risk premium term.

- Replacing the forward price with the future exchange rate, we have the uncovered interest rate parity,

\[ s_T - s_t = f_t - s_t + error = (r_d - r_f)(T - t) + error, \]

The error is due to (i) the difference between expectation and realization (expectation error) and (ii) risk premium.

- Implication: High interest rate currencies tend to depreciate.
Violation of uncovered interest rate parity

- If you run the following regression,
  
  \[ s_T - s_t = a + b(r_d - r_f)(T - t) + \text{error}, \]
  
  or equivalently,
  
  \[ s_T - s_t = a + b(f_{t,T} - s_t)(T - t) + \text{error}, \]
  
  the slope estimate \( b \) is most likely negative!

- Implication: High interest rate currencies tend to appreciate, not depreciate!

- Carry trade: Invest in high interest rate currency, and you will likely earn more than the interest rate differential.

- Why?
Hedging using Futures

- A long futures hedge is appropriate when you know you will purchase an asset in the future and want to lock in the price.
- A short futures hedge is appropriate when you know you will sell an asset in the future and want to lock in the price.
- By hedging away risks that you do not want to take, you can take on more risks that you want to take while maintaining the aggregate risk levels.
  - Companies can focus on the main business they are in by hedging away risks arising from interest rates, exchange rates, and other market variables.
  - Insurance companies can afford to sell more insurance policies by buying re-insurance themselves.
  - Mortgage companies can sell more mortgages by packaging and selling some of the mortgages to the market.
Basis risk

- Basis is the difference between spot and futures ($S - F$).
- Basis risk arises because of the uncertainty about the basis when the hedge is closed out.
- Let $(S_1, S_2, F_1, F_2)$ denote the spot and futures price of a security at time 1 and 2.
  - Long hedge: Entering a long futures contract to hedge future purchase:
    \[
    \text{Future Cost} = S_2 - (F_2 - F_1) = F_1 + \text{Basis}.
    \]
  - Short hedge: Entering a short futures contract to hedge future sell:
    \[
    \text{Future Profit} = S_2 - (F_2 - F_1) = F_1 + \text{Basis}.
    \]
Optimal hedge ratio

For each share of the spot security, the optimal share on the futures (that minimizes future risk) is:

$$\frac{\rho \sigma_S}{\sigma_F}$$

$\sigma_S$ the standard deviation of $\Delta S$, $\sigma_F$ the standard deviation of $\Delta F$, $\rho$ the correlation between the two.

A simple way to obtain the optimal hedge ratio is to run the following least square regression:

$$\Delta S = a + b\Delta F + e$$

- $b$ is the optimal hedge ratio estimate for each share of the spot.
- The variance of the regression residual ($e$) captures the remaining risk of the hedged position ($\Delta S - b\Delta F$).
Many times, we estimate the correlation or we run the regressions on returns instead of on price changes for stability:

\[
\frac{\Delta S}{S} = \alpha + \beta \frac{\Delta F}{F} + e
\]

- Comparing $\beta$ from the return regression with the optimal hedging ratio in the price change regression, we need to adjust $\beta$ for the value (scale) difference to obtain the hedging ratio in shares: $b = \beta \frac{S}{F}$.

- Example: Hedge equity portfolios using index futures based on CAPM $\beta$. 

Example: hedging equity portfolio using index futures

What position in futures contracts on the S&P 500 is necessary to hedge the portfolio?

- Value of S&P 500 is 1,000
- Value of Portfolio is $5 million
- Beta of portfolio is 1.5.
  - For each percentage change in the portfolio return, the index return changes by 1.5 percentage point.
- Application: “Market (β)-neutral” stock portfolios.
Summary

- Understand the general idea of derivatives (products, markets).
- Understand the general idea of arbitrage
  Can execute one when see one.
- The characteristics of forwards/futures
  - Payoff under different scenarios, mathematical representation:
    \((S_T - K)\) for long, \((K - S_T)\) for short
  - Understand graphical representation.
  - Pricing: \(F(t, T) = S_t + \text{cost of carry}\). Know how to calculate carry cost/benefit under continuously/discrete compounding.
  - Combine cash and forward market for arbitrage trading
  - Hedging using futures (compute hedging ratios)