Mechanics of Options Markets

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(Hull chapter: 8)
Outline

1. Definition
2. Payoffs
3. Mechanics
4. Other option-type products
5. Data source
6. Option behaviors
Definitions and terminologies

- An option gives the option holder the right/option, *but no obligation*, to buy or sell a security to the option writer/seller
  - for a pre-specified price (the *strike price*, $K$)
  - at (or up to) a given time in the future (the *expiry date* )

- An option has positive value.
  *Comparison: a forward contract has zero value at inception.*

- Option types
  - A *put option* gives the holder the right to sell a security. The payoff is $(S_T - K)^+$ when exercised at maturity.
  - A *call option* gives the holder the right to buy a security. The payoff is $(K - S_T)^+$ when exercised at maturity.
  - American options can be exercised at any time prior to expiry.
  - European options can only be exercised at the expiry.
More terminologies

- **Moneyness**: the strike relative to the spot/forward level
  - An option is said to be **in-the-money** if the option has positive value if exercised right now:
    - $S_t > K$ for call options and $S_t < K$ for put options.
    - Sometimes it is also defined in terms of the forward price at the same maturity (in the money forward): $F_t > K$ for call and $F_t < K$ for put.
    - The option has positive intrinsic value when in the money. The intrinsic value is $(S_t - K)^+$ for call, $(K - S_t)^+$ for put.
    - We can also define intrinsic value in terms of forward price.
  - An option is said to be **out-of-the-money** when it has zero intrinsic value.
    - $S_t < K$ for call options and $S_t > K$ for put options.
    - Out-of-the-money forward: $F_t < K$ for call and $F_t > K$ for put.
  - An option is said to be **at-the-money** spot (or forward) when the strike is equal to the spot (or forward).
More terminologies

- The value of an option is determined by
  - the current spot (or forward) price ($S_t$ or $F_t$),
  - the strike price $K$,
  - the time to maturity $\tau = T - t$,
  - the option type (Call or put, American or European), and
  - the dynamics of the underlying security (e.g., how volatile the security price is).

- Out-of-the-money options do not have intrinsic value, but they have time value.

- Time value is determined by time to maturity of the option and the dynamics of the underlying security.

- Generically, we can decompose the value of each option into two components:
  \[ \text{option value} = \text{intrinsic value} + \text{time value}. \]
Payoffs versus P&Ls

- For European options, the terminal **payoff** can be written as \((S_T - K)^+\) for calls and \((K - S_T)^+\) for puts at expiry date \(T\).

- Since options have positive value, one needs to pay an upfront price (**option price**) to possess an option.

- The P&L from the option investment is the difference between the terminal payoff and the initial price you pay to obtain the option.

- Do not confuse the two.

- The textbook likes to talk about P&Ls, but I like to talk about payoffs — Different perspectives:
  - P&Ls: If I buy/sell an option today, how much money can I make under different scenarios? What’s my return?
  - Payoffs: If I desire a certain payoff structure in the future, what types of options/positions I need to generate it?
An example: Call option on a stock index

Consider a European call option on a stock index. The current index level (spot $S_t$) is 100. The option has a strike ($K$) of $90$ and a time to maturity ($T - t$) of 1 year. The option has a current value ($c_t$) of $14$.

- Is this option in-the-money or out-of-the-money (wrt to spot)?

- What’s intrinsic value for this option? What’s its time value?

- If you hold this option, what’s your terminal payoff?
  - What’s your payoff and P&L if the index level reaches 100, 90, or 80 at the expiry date $T$?

- If you write this option and have sold it to the exchange, what does your terminal payoff look like?
  - What’s your payoff and P&L if the index level reaches 100, 90, or 80 at the expiry date $T$?
Payoffs and P&Ls from long/short a call option

\[(S_t = 100, K = 90, c_t = 14)\]

Long a call pays off, \((S_T - K)^+\), bets on index price going up. Shorting a call bets on index price going down.
Another example: Put option on an exchange rate

Consider a European put option on the dollar price of pound (GBPUSD). The current spot exchange rate ($S_t$) is $1.6285$ per pound. The option has a strike ($K$) of $1.61$ and a time to maturity ($T - t$) of 1 year. The 1-year forward price ($F_{t,T}$) is $1.61$. The dollar continuously compounding interest rate at 1-year maturity ($r_d$) is 5%. The option ($p_t$) is priced at $0.0489$.

- From the above information, can you infer the continuously compounding interest rate at 1-year maturity on pound ($r_f$)?

- Is this option in-the-money or out-of-the-money wrt to spot? What’s the moneyness in terms of forward?

- In terms of forward, what’s intrinsic value for this option? What’s its time value?

- If you hold this option, what’s your terminal payoff, if the dollar price of pound reaches 1.41, 1.61, or 1.81 at the expiry date $T$?
Another example: Put option on an exchange rate

- Review the forward pricing formula: $F_{t,T} = S_t e^{(r_d - r_f)(T - t)}$.
  
  $r_f = r_d - \frac{1}{T-t} \ln(F_{t,T}/S_t) = .05 - \ln(1.61/1.6285)/1 = 6.14\%$.

  - Recall covered interest rate parity: Annualized forward return
    \[ \left( \frac{1}{T-t} \ln(F_{t,T}/S_t) \right) \]
    on exchange rates equals interest rate differential
    \[ (r_d - r_f) \]
    between the two currencies.

- Long a put option pays off, \((K - S_T)^+\), and bets on the underlying currency (pound) depreciates.

- Shorting a call option bets on pound appreciates.

- *How does it differ from betting using forwards?*
Payoffs and P&Ls from long/short a put option

\( (S_t = 1.6285, F_{t,T} = 1.61, K = 1.61, p_t = 0.0489) \)
What derivative positions generate the following payoff?
Assets underlying exchanged-traded options

- Stocks
- Stock indices
- Index return variance (new)
- Exchange rate
- Futures
Specification of exchange-traded options

- Expiration date ($T$)
- Strike price ($K$)
- European or American
- Call or Put (option class)

OTC options (such as OTC options on currencies) are quoted differently.
Options market making

- Most exchanges use market makers to facilitate options trading.
- A market maker is required to provide bid and ask quotes
  - with the bid-ask spread within a maximum limit,
  - with the size no less than a minimum requirement,
  - at no less than a certain percentage of time (lower limit)
  - on no less than a certain fraction of securities that they cover.
- The benefit of market making is the bid-ask spread; The risk is market movements.
  - The risk and cost of options market making is relatively large.
  - The bid-ask is wide (stock options). The tick size is 10 cents on options with prices higher than $3. It is 5 cents otherwise.
Options market making

Since there can be hundreds of options underlying one stock, when the stock price moves, quotes on the hundreds of options must be updated simultaneously.

- Quote message volume is dramatically larger than trade message volume.
- The risk exposure is large compared to the benefit.
  - When a customer who has private information on the underlying stock (say, going up), the customer can buy all the call options and sell all the put options underlying one stock.
  - The market maker’s risk exposure is the sum of all the quote sizes he honors on each contract.
  - Market makers hedge their risk exposures by buying/selling stocks according to their option inventories.
- Market makers nowadays all have automated systems to update their quotes, and calculate their optimal hedging ratios.
- Options market makers are no longer individual persons, but are well-capitalized firms.
Exchanges that trade stock options in the US

A rapidly changing landscape

- Old exchanges:
  - CBOE (Chicago Board of Options Exchange)
  - AMEX (American Stock Exchange)
  - PCX (Pacific Stock Exchange)
  - PHLX (Philadelphia Stock Exchange)

- New developments:
  - BOX (Boston Options Exchange, February 2004)

- Trend: The blink of an eye is no longer fast enough!
  (30 milliseconds)
Margins

- Margins are required when options are sold/written.

- When a naked option is written the margin is the greater of:
  - A total of 100% of the proceeds of the sale plus 20% of the underlying share price less the amount (if any) by which the option is out of the money
  - A total of 100% of the proceeds of the sale plus 10% of the underlying share price.

- For other trading strategies there are special rules
Dividends and stock splits

Suppose you own $N$ option contracts with a strike price of $K$:

- No adjustments are made to the option terms for cash dividends.
- When there is an $n$-for-$m$ stock split,
  - The strike price is reduced to $mK/n$
  - The number of options is increased to $nN/m$
- Stock dividends are handled in a manner similar to stock splits.

Example: Consider a call option to buy 100 shares for $20 per share.

- How should the option contract terms be adjusted:
  - for a 2-for-1 stock split?
  - for a 5% stock dividend?
Other option-type products

- **Warrants**: options that are issued by a corporation
  - When call warrants are issued by a corporation on its own stock, exercise will lead to new stock being issued.

- **Executive stock options**: a form of remuneration issued by a company to its executives
  - usually at the money when issued.
  - When exercised, the company issues more stock and sells it to the option holder for the strike price.
  - They become vested after a period of time (1 to 4 years).
  - They cannot be sold.
Other option-type products

- **Convertible bonds**: regular bonds that can be exchanged for equity at certain times in the future according to a predetermined exchange ratio.
  - Very often callable, so that the issuer can force conversion at a time earlier than the holder might otherwise choose.

- **Stocks**: Can be regarded as call options on firm value.
  - The payoff is the difference between firm value and debt liability, \((Firm\, Value - Debt)^+\).
  - When firm value is less than debt value, the firm can apply for bankruptcy.
  - Limited liability guarantees that stock price is always positive.
  - When DCF method does not work well, one can value a stock like an option.
Data on stock options

- Single names options are American. Options on some indexes are European.

- Data source: (Baruch has two very rich data sets)
  - OptionMetrics: daily closing option prices, matched with spot prices, interest rates, dividend yields, implied volatilities. They also provide standardized implied volatility surface (quality is low). Available via WRDS for Baruch faculty.
  - OPRA: Real time messages on quotes and transaction from all US option exchanges. Available at the Subotnick Financial Service Center.
Data processing on stock options

- Early exercise premium
- Discrete dividend projection
- Interpolation/extrapolation to obtain implied volatility at fixed time-to-maturity and moneyness.
  - How to define moneyness?
  - How to interpolate/extrapolate (smoothness, fitting, no-arbitrage)?
  - How to match implieds of puts and calls at the same strike?
- Interesting time in-homogeneous behaviors:
  - Stock pinning at expiry
  - Behaviors around ex-dividend date, earnings announcement date, FOMC date.
  - Put-call parity violations.
Research on stock options

- My recent papers using stock options:
  - (OptionMetrics) Variance risk premia, RFS, forthcoming.
  - (OptionMetrics) Stock options and credit default swaps: a joint framework for valuation and estimation, wp.
  - (OptionMetrics) Static hedging of standard options, wp.

- Future works:
  - Microstructure research using OPRA.
  - Large scale model estimation that includes pricing of both single names and stock indexes.
Data on stock index options

- In addition to data at OptionMetrics (daily from CBOE) and OPRA (real-time message), banks often provide OTC quotes on index options at longer maturities.
  - Trading includes delta.
  - Quotes on implied volatility at fixed strike in percentages of the spot, and also at fixed time to maturity.

- Data description and research:

- Future research:
  - Design and estimate new models that better capture long-run behavior (up to 5 years) of index options.
Data on currency options

- The exchange market (PHLX) has died out. Currency options market has moved to over the counter.

- Trading and quoting convention:
  - Trading includes both the option and the Black-Scholes delta of the underlying.
  - Quotes are not in dollar prices, but in implied volatilities in the form of (1) delta-neutral straddle, (2) 5-,10-, 25-delta risk reversals, (3) 5-,10-,25-delta butterfly spreads.
  - Grid/matrix quote at fixed delta and fixed time to maturity.

- Data source: Bloomberg, BBA/Reuters.

- Recently, ISE started trading on currency (American) options.
Recent researches on currency options

- My recent papers using current options:
  - Stochastic skew in currency option, JFE, forthcoming.
  - Stochastic risk premiums, stochastic skewness in currency options, and stochastic discount factors in international economies, JFE, forthcoming.
  - Theory and evidence on the dynamic interactions between sovereign credit default swaps and currency options, JBF, 2007, 31(8), 2383–2403.

- New places to look:
  - It can be interesting in comparing the behavior of OTC and exchange (ISE) options.
  - Possibly more can be done on currency risk premium puzzle using currency options.
Interest rate options: caps/floors, swaptions

- OTC quotes at fixed time-to-maturity.
- Mainly at the money.
- Quotes at fixed strikes are also becoming available.
- Research: “Interest rate caps smile too!” Jarrow, Li, Zhang
- Future: More modeling efforts are needed to price interest rates and interest rate options consistently.
  - Term structure of interest rates, yield curve residuals, and the consistent pricing of interest rate derivatives, JFQA, forthcoming.
  - Time change Lévy may not be the way to go.
- Future: Joint analysis of currency, interest rate, and stock index options can be useful.
Notation

- $S$: stock price. $F_{t,T}$ — time-$t$ forward price with expiry $T$.
- $K$: strike price.
- Today: either time 0 or time $t$.
- $T$: expiry date (or maturity with $t = 0$).
- $\sigma$: Volatility (annualized standard deviation) of stock return.
- $r$: continuously compounded riskfree rate with maturity $T$ (same as option).
- $D$: present value of discrete dividends paid during option’s life.
- $q$: continuously compounded dividend yield during option’s life (for foreign interest rate for currency options).
Dependence of option values

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In examples given below, I use the following benchmark numbers:

$S_t = 100; K = 100; \sigma = 2\%; t = 0; T = 2/12; r = 5\%; q = 3\%.$

(unless otherwise specified)
Option variation with spot price

Dependence on spot follows from payoff function (when time to maturity is zero).
Dependence on strike follows from payoff function (when time to maturity is zero).
European option variation with time to maturity

Normally increase, but can decline.
American option value always increases with time, given the option to exercise early.
ATM option value is almost linear in volatility. Options market is mainly a market for volatility.
Increasing interest rate or reducing dividend yield increases the growth rate of the stock price.
American v. European options

- The above graphs are all generated for European options, based a simple option pricing model, which we’ll deal with later.

- An American option is worth at least as much as the corresponding European option: $C \geq c$ and $P \geq p$.
  - The difference is due to the extra option you get from an American option: You can exercise any time before the expiry date.
  - Hence, the difference is also referred to as the early exercise premium.

- Pricing American options is a bit harder — we’ll need a numerical algorithm to deal with the early exercise premium.
Obvious arbitrage opportunities on call options

1. Suppose that $c_t = 2, S_t = 20, T - t = 1, K = 17, r = 5\%, q = 0$. Is there an arbitrage opportunity?

2. Suppose that $c_t = 21, S_t = 20, T - t = 1, K = 1, r = 5\%, q = 0$. Is there an arbitrage opportunity?
Bounds on a call option

- Lower bound: \( c_t \geq S_t e^{-q(T-t)} - Ke^{-r(T-t)} = e^{-r(T-t)}(F_{t,T} - K) \).
  
  ▶ If violated, buy the call and short a forward at \( K \).
    
      ★ Today’s value from the transaction is: \(-c_t + e^{-r(T-t)}(F_{t,T} - K) > 0\).
      ★ At maturity, the payoff is \((S_T - K)^+ - (S_T - K) > 0\).

  ▶ Point to remember: An option is worth more than a forward.

- Upper bound: \( c_t \leq S_t e^{-q(T-t)} = e^{-r(T-t)}F_{t,T} \).
  
  ▶ If violated, write the call and long a forward at zero strike.
    
      ★ Today’s value from the transaction is: \(c_t - e^{-r(T-t)}F_{t,T} > 0\).
      ★ At maturity, the payoff is \(-(S_T - K)^+ + S_T > 0\).

- I am using forward instead of spot to avoid the complication of discrete dividends.
  
  ▶ How much is a call option with \( K = 0 \) worth?

- In the presence of discrete dividends, the bounds are: \([S_t - D - Ke^{-r(T-t)}, S_t - D]\).
Less obvious arbitrage opportunities on call options

- Suppose that $c_t = 3.2$, $S_t = 20$, $T - t = 1$, $K = 17$, $r = 5\%$, $q = 3\%$. Is there an arbitrage opportunity?
  
  $S_t e^{-q(T-t)} - Ke^{-r(T-t)} = 3.238.$

- Suppose that $c_t = 19.5$, $S_t = 20$, $T - t = 1$, $K = 1$, $r = 5\%$, $q = 3\%$. Is there an arbitrage opportunity?
  
  $S_t e^{-q(T-t)} = 19.41.$

- Suppose that $c_t = 19.5$, $S_t = 20$, $T - t = 1$, $K = 1$, $r = 5\%$, $D = 1$. Is there an arbitrage opportunity?
  
  $S_t - D = 19.$
Bounds on a put option

- **Lower bound:** 
  \[ p_t \geq Ke^{-r(T-t)} - S_t e^{-q(T-t)} = e^{-r(T-t)}(K - F_{t,T}). \]
  - If violated, buy the put and long a forward at \( K \).
    - Today’s value from the transaction is: \(-p_t - e^{-r(T-t)}(F_{t,T} - K) > 0\).
    - At maturity, the payoff is \((K - S_T)^+ - (K - S_T) > 0\).
  - Point to remember: *An option to sell is worth more than a forward to sell.*

- **Upper bound:** 
  \[ p_t \leq Ke^{-r(T-t)}. \]
  - If violated, write the put and save \( Ke^{-r(T-t)} \) in the bank. Spend the rest at water park.
    - Today’s value: water slides.
    - At maturity, the return from the bank is \( K \). The worst possible obligation from writing the put is \( K \) when the company goes bankruptcy \((S_T = 0)\). So you make even in the worst case, make money otherwise.

- In the presence of discrete dividends, the lower bound is: \( Ke^{-r(T-t)} - S_t + D \).
Arbitrage opportunities on put options

- Suppose that $p_t = 2$, $S_t = 20$, $T - t = 1$, $K = 23$, $r = 5\%$, $q = 3\%$. Is there an arbitrage opportunity?
  - $Ke^{-r(T-t)} - S_t e^{-q(T-t)} = 2.47$.

- Suppose that $p_t = 2$, $S_t = 20$, $T - t = 1$, $K = 23$, $r = 5\%$, $D = 1$. Is there an arbitrage opportunity?
  - $Ke^{-r(T-t)} - S_t + D = 2.88$. 

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Options Markets Mechanics
Option Pricing, Fall, 2007
Put call parity

Recall the put-call parity condition:
The difference between a call and a put equals the forward.

\[ c_t - p_t = e^{-r(T-t)}(F_t, T - K_t) \]
\[ = S_t e^{-q(T-t)} - K_t e^{-r(T-t)} \]
\[ = S_t - D - K_t e^{-r(T-t)} \]

Suppose that \( c_t = 8, S_t = 100, T - t = 1, K = 100, r = 0, q = 0. \)
Is there an arbitrage opportunity if

- \( p_t = 7. \)
- \( p_t = 8. \)
- \( p_t = 9. \)
Early exercise of American options

- Usually there is some chance that an American option will be exercised early.
  - Exercise when the intrinsic value \((S_t - K)^+\) for call) is higher than the option value \((C_t)\).

- As a result, the American option is more expensive (valuable) than the European option.

- An exception is an American call on a non-dividend paying stock, which should never be exercised early.
  - \(C_t \geq c_t \geq S_t - Ke^{-r(T-t)} \geq S_t - K = \text{intrinsic value}\)
  - Option value is always higher than intrinsic value.
Cheat sheet

- **Forward pricing:**
  
  \[ F_t = e^{(r-q)(T-t)} S_t = e^{r(T-t)}(S_t - D_t), \]

  \( D_t \) the present value of discrete dividends paid (carrying benefits) during the remaining life of the contract \((t, T)\).

- **European options bounds:**

  \[ c_t \in [e^{-r(T-t)}(F_{t,T} - K), \quad e^{-r(T-t)}F_{t,T}] \]
  \[ p_t \in [e^{-r(T-t)}(K - F_{t,T}), \quad e^{-r(T-t)}K] \]

- **Put-call parity for European options:**

  \[ c_t - p_t = e^{-r(T-t)}(F_{t,T} - K_t) = e^{-q(T-t)} S_t - e^{-r(T-t)}K. \]

- **Put-call inequality for American options:**

  \[ S_t - D - K \leq C_t - P_t \leq S_t - Ke^{-r(T-t)} \]
  \[ S_te^{-q(T-t)} - K \leq C_t - P_t \leq S_t - Ke^{-r(T-t)} \]