Options Trading Strategies

Liuren Wu

Zicklin School of Business, Baruch College

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(Hull chapter: 10)
Types of strategies

- Take a position in the option and the underlying.
- Take a position in 2 or more options of the same type (a spread).
- Take a position in a mixture of calls & puts (a combination).
- Use European options (calls or puts or both) to replicate any arbitrary terminal payoff function $f(S_T)$.

Before you can do the replication, you need to be very familiar with the payoff structures of the building blocks (options, forwards, spots, bonds).
And you need to know how to combine them (either mathematically or graphically).
Put-call conversions

Plot the payoff function of the following combinations of calls/puts and forwards at the same strike $K$ and maturity $T$.

- Long a call, short a forward.
  - Compare the payoff to long a put.
- Short a call, long a forward.
  - Compare the payoff to short a put.
- Long a put, long a forward.
  - Compare the payoff to long a call.
- Short a put, short a forward.
  - Compare the payoff to short a call.
- Long a call, short a put.
  - Compare the payoff to long a forward.
- Short a call, long a put.
  - Compare the payoff to short a forward.
Put-call conversions

Payoff vs. Spot at expiry, $S_T$
The linkage between put, call, and forward

- The above conversions reveal the following parity condition in payoffs of put, call, and forward at the same strike and maturity:

  \[
  \text{Payoff from a call} - \text{Payoff from a forward} = \text{Payoff from a put} \\
  \text{Payoff from a put} + \text{Payoff from a forward} = \text{Payoff from a call} \\
  \text{Payoff from a call} - \text{Payoff from a put} = \text{Payoff from a forward}
  \]

- If the payoff is the same, the present value should be the same, too (\textit{put-call parity}):

  \[
  c_t - p_t = e^{-r(T-t)}(F_{t,T} - K).
  \]

- At a fixed strike ($K$) and maturity $T$, we only need to know the two prices of the following three: $(c_t, p_t, F_{t,T})$. One of the three contracts is redundant.
In the absence of forward, use spot and bond:

- Can you use a spot and bond to replicate a forward payoff?
- What’s the payoff function of a zero bond?
Can you generate the above payoff structure (solid blue line) using (in addition to cash/bond):

- two calls
- two puts
- a call, a put, and a stock/forward

Who wants this type of payoff structure?
Generating a bull spread

- **Two calls:** Long call at $K_1 = $90, short call at $K_2 = $110, short a bond with $10 par.

- **Two puts:** Long a put at $K_1 = $90, short put at $K_2 = $110, long a bond with $10 par.

- **A call, a put, and a stock/forward:** Long a put at $K_1 = $90, short a call at $K_2 = $110, long a forward at $K = 100$ (or long a stock, short a bond at $100$ par).
Pointers in replicating payoffs

- Each kinky point corresponds to a strike price of an option contract.
  - How many options do you need to replicate a quadratic payoff function \((\text{Payoff} = S_T^2)\) ?

- Given put-call party, you can use either a call or a put at each strike point, subject to adjustments using forward.

- Use bonds for parallel shifts (it is a matter of paying now or later).
Example: Bear spread

- How many (at minimum) options do you need to replicate the bear spread?
- Do the exercise, get familiar with the replication.
- Who wants a bear spread?
How many (at minimum) options do you need to replicate the straddle?

Do the exercise, get familiar with the replication.

Who wants long/short a straddle?
Example: Strangle

- How many (at minimum) options do you need to replicate the strangle?
- Do the exercise, get familiar with the replication.
- Who wants long/short a strangle?
Example: Butterfly spread

- How many (at minimum) options do you need to replicate the butterfly spread?
- Do the exercise, get familiar with the replication.
- Who wants long/short a butterfly spread?
Example: Risk Reversal

- How many (at minimum) options do you need to replicate the risk reversal?
- Do the exercise, get familiar with the replication.
- Who wants long/short a risk reversal?
Smooth out the kinks: Can you replicate this?

How many options do you need to replicate this quadratic payoff?

- You need a continuum of options to replicate this payoff.
- The weight on each strike $K$ is $2dK$.

Who wants long/short this payoff?

- The variance of the stock price is $\mathbb{E}[(S_T - F_{t,T})^2]$.
- Variance swap contracts on major stock indexes are actively traded.
Replicate any terminal payoff with options and forwards

\[ f(S_T) = f(F_t) + f'(F_t)(S_T - F_t) + \left\{ \begin{array}{l}
\int_{F_t}^{F_t} f''(K)(K - S_T)^+ dK \\
\int_{F_t}^{\infty} f''(K)(S_T - K)^+ dK
\end{array} \right\} \]

- Can you prove this formula: It looks easier than it really is.
- What does this formula tell you?
  - With bonds, forwards, and European options, we can replicate any terminal payoff structures.
  - More exotic options deal with path dependence, correlations, etc.
Replicating variance swap contracts with vanilla options

- Replicate the return variance swap using options and futures.

- Based on the replication idea, think of ways to summarizing the information in the options market.
  - Information about the directional movement of the underlying.
  - Information about return variance.
  - Information about large movements of a certain direction.
  - Information about large movements of either direction.


- Caveat: Far out-of-the-money options may not be actively traded. Quotes may not be reliable.
  Example: ATM volatility versus synthetic variance swap.
Variance swap rate as a portfolio of options

- Variance swap can be replicated by a static position in a portfolio of OTM options and dynamic trading in the underlying futures:

$$VS_{t,T} \equiv \mathbb{E}_t^Q [RV_{t,T}] \doteq \frac{1}{T-t} e^{r(T-t)} \int_0^\infty O_t(K, T) \frac{2dK}{K^2},$$

where $O_t$ — OTM option value.

- It can be written as a weighted average of implied variance:

$$VS_{t,T} \doteq \frac{1}{T-t} e^{r(T-t)} \left[ \int_0^{F_t} p_t(K, T) \frac{2dK}{K^2} + \int_{F_t}^\infty c_t(K, T) \frac{2dK}{K^2} \right]$$

$$= \ldots$$

$$= \int_{-\infty}^\infty n(d_2) \text{IV}(d_2)^2 \, d(d_2)$$