Stochastic Skew in Currency Options

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Overview

• There is a huge market for foreign exchange (FX), much larger than the equity market ... an understanding of FX dynamics is economically important.

• FX option prices can be used to understand risk-neutral FX dynamics, i.e. how the market prices various path bundles.

• Despite their greater economic relevance, FX options are not as widely studied as equity index options, probably due to the fact that the FX options market is now primarily OTC.

• We obtain OTC options data on 2 currency pairs (JPYUSD, GBPUSD).

• We use the options data to study the dynamics of the 2 currencies:
  – Document the behavior of the currency options.
  – Propose a new class of models to capture the behavior.
  – Estimate the new models and compare to older models such as Bates (1996).
  – Study the implications of the new model class for currency dynamics.
OTC FX Option Quoting Conventions

- Quotes are in terms of BS model implied volatilities rather than on option prices directly.

- Quotes are provided at a fixed BS delta rather than a fixed strike. In particular, the liquidity is mainly at 5 levels of delta:

  10 δ Put, 25 δ Put, 0 δ straddle, 25 δ call, 10 δ call.

- Trades include both the option position and the underlying, where the position in the latter is determined by the BS delta.
A Review of the Black-Scholes Formulae

- BS call and put pricing formulae:
  
  $c(K, \tau) = e^{-r_f \tau} S_t N(d_1) - e^{-r_d \tau} K N(d_2)$,
  
  $p(K, \tau) = -e^{-r_f \tau} S_t N(-d_1) + e^{-r_d \tau} K N(-d_2)$,

  with
  
  $d_{1,2} = \frac{\ln(F/K)}{\sigma \sqrt{\tau}} \pm \frac{1}{2} \sigma \sqrt{\tau}$,  
  $F = S e^{(r_d - r_f) \tau}$.

- BS Delta

  $\delta(c) = e^{-r_f \tau} N(d_1)$,  
  $\delta(p) = -e^{-r_f \tau} N(-d_1)$.

  $|\delta|$ is roughly the probability that the option will expire in-the-money.

- BS Implied Volatility (IV): the $\sigma$ input in the BS formula that matches the BS price to the market quote.
Data

- We have 8 years of data: January 1996 to January 2004 (419 weeks)

- On two currency pairs: JPYUSD and GBPUSD.

- At each date, we have 8 maturities: 1w, 1m, 2m, 3m, 6m, 9m, 12m, 18m.

- At each maturity, we have five option quotes at the five deltas.

All together, 40 quotes per day, 16,760 quotes for each currency.
The five option quotes at each maturity are in the following forms:

1. Delta-neutral straddle implied volatility (ATMV)
   - A straddle is a sum of a call and a put at the same strike.
   - Delta-neutral means $\delta(c) + \delta(p) = 0 \rightarrow N(d_1) = 0.5 \rightarrow d_1 = 0$.

2. ten-delta risk reversal (RR10)
   - $RR10 = IV(10c) - IV(10p)$.
   - Captures the slope of the smile (skewness of the return distribution).

3. ten-delta strangle margin (butterfly spread) (SM10)
   - A strangle is a sum of a call and a put at two different strikes.
   - $SM10 = (IV(10c) + IV(10p))/2 - ATMV$.
   - Captures the curvature of the smile (kurtosis of the distribution).

4. 25-delta RR and 25-delta SM
Convert Quotes to Option Prices

- Convert the quotes into implied volatilities at the five deltas:
  \[
  IV(0\delta_s) = ATMV;
  IV(25\delta_c) = ATMV + RR25/2 + SM25;
  IV(25\delta_p) = ATMV - RR25/2 + SM25;
  IV(10\delta_c) = ATMV + RR10/2 + SM10;
  IV(10\delta_p) = ATMV - RR10/2 + SM10.
  \]

- Download LIBOR and swap rates on USD, JPY, and GBP to generate the relevant yield curves \((r_d, r_f)\).

- Convert deltas into strike prices
  \[
  K = F \exp \left[ \mp IV(\delta, \tau) \sqrt{\tau} N^{-1}(\pm e^{r_f \tau} \delta) + \frac{1}{2} IV(\delta, \tau)^2 \tau \right].
  \]

- Convert implied volatilities into out-of-the-money option prices using the BS formulae.
Time Series of Implied Volatilities

Stochastic volatility — Note the impact of the 1998 hedge fund crisis on dollar-yen: During the crisis, hedge funds bought call options on yen to cover their yen debt.
The Average Implied Volatility Smile

- The mean implied volatility smile is relatively symmetric...
- The smile (kurtosis) persists with increasing maturity.
The strangle margins (kurtosis measure) are stable over time, ...

But the risk reversals (return skewness) vary greatly over time.

⇒ **Stochastic Skew**.
• Changes in risk reversals are positively correlated with contemporaneous currency returns ...

• But there are no obvious lead-lag effects.
How Does the Literature Capture Smiles?

Two ways to generate a smile or skew:

1. **Add jumps**: Merton (1976)’s jump-diffusion model

   \[
   dS_t/S_{t-} = (r_d - r_f)dt + \sigma dW_t + \int_{-\infty}^{\infty} (e^x - 1)[\mu(dx,dt) - \lambda n(\mu_j, \sigma_j)dxdt]
   \]

   • The arrival of jumps is controlled by a Poisson process with arrival rate \( \lambda \).
   • Conditional on one jump occurring, the percentage jump size \( x \) is normally distributed, with density \( n(\mu_j, \sigma_j^2) \).
   • Nonzero \( \mu_j \) generates asymmetry (skewness).

   \( \Rightarrow \) Stochastic skew would require \( \mu_j \) to be stochastic ...

   Not tractable!
How Does the Literature Capture Smiles?

Two ways to generate a smile or skew:

1. **Add jumps**

2. **Stochastic volatility**: Heston (1993)

\[
\begin{align*}
\frac{dS_t}{S_t} &= (r_d - r_f)dt + \sqrt{v_t}dW_t, \\
\frac{dv_t}{v_t} &= \kappa(\theta - v_t)dt + \sigma_v\sqrt{v_t}dZ_t, \quad \rho dt = E[dW_tdZ_t]
\end{align*}
\]

- Vol of vol ($\sigma_v$) generates smiles,
- Correlation ($\rho$) generates skewness.

$\Rightarrow$ Stochastic skew would require correlation $\rho$ to be stochastic ...

Not tractable!
How Do The Two Methods Differ?

• Jump diffusions (Merton) induce short term smiles and skews that dissipate quickly with increasing maturity due to the central limit theorem.

• Stochastic volatility (Heston) induces smiles and skews that increase as maturity increases over the horizon of interest.

• Bates (1996) combines Merton and Heston to generate stochastic volatility and smiles/skews at both short and long horizons ... but NOT stochastic skew.
Our Models Based on Time-Changed Lévy Processes

\[
\ln \frac{S_t}{S_0} = (r_d - r_f)t + \left( L^R_t - \xi^R T^R_t \right) + \left( L^L_t - \xi^L T^L_t \right),
\]

(1)

- \( L^R_t \) is a Lévy process that generates positive skewness (diffusion + positive jumps)

- \( L^L_t \) is a Lévy process that generates negative skewness (diffusion + negative jumps)

- \([T^R_t, T^L_t]\) randomize the clock underlying the two Lévy processes so that
  - \([T^R_t + T^L_t]\) determines total volatility: stochastic
  - \([T^R_t - T^L_t]\) determines skewness (risk reversal): ALSO stochastic

⇒ **Stochastic Skew Model (SSM)**
SSM In the Language of Merton and Heston

\[ dS_t/S_t^- = (r_d - r_f)dt \leftarrow \text{risk-neutral drift} \]
\[ + \sigma \sqrt{v_t^R} dW_t^R + \int_0^\infty (e^x - 1) [\mu^R(dx, dt) - k^R(x)dxv_t^R dt] \leftarrow \text{right skew} \]
\[ + \sigma \sqrt{v_t^L} dW_t^L + \int_{-\infty}^0 (e^x - 1) [\mu^L(dx, dt) - k^L(x)dxv_t^L dt] \leftarrow \text{left skew} \]
\[ dv_t^j = \kappa(1 - v_t)dt + \sigma_v \sqrt{v_t} dZ_t^j, \quad \rho^j dt = E[dW_t^j dZ_t^j], \quad j = R, L \leftarrow \text{activity rates} \]

- At short term, the Lévy density \( k^R(x) \) has support on \( x \in (0, \infty) \) \( \mapsto \) \text{Positive skew}. The Lévy density \( k^L(x) \) has support on \( x \in (-\infty, 0) \) \( \mapsto \) \text{Negative skew}.

- At long term, \( \rho^R > 0 \) \( \mapsto \) \text{Positive skew}. \( \rho^L < 0 \) \( \mapsto \) \text{Negative skew}.

- \textbf{Stochastic skew} is generated via the randomness in \([v_t^R, v_t^L]\), which randomizes the contribution from the two jumps and from the two correlations.
Our Jump Specification

• The arrival rates of upside and downside jumps (Lévy density) follow exponential dampened power law (DPL):

\[ k^R(x) = \begin{cases} 
\lambda e^{-\frac{|x|}{v_j}x^{-\alpha-1}}, & x > 0, \\
0, & x < 0.
\end{cases} \]

\[ k^L(x) = \begin{cases} 
0, & x > 0, \\
\lambda e^{-\frac{|x|}{v_j}x^{-\alpha-1}}, & x < 0.
\end{cases} \]  

(2)

• The specification originates in Carr, Géman, Madan, Yor (2002), and captures much of the stylized evidence on both equities and currencies (Wu, 2004).

• A general and intuitive specification with many interesting special cases:
  – \( \alpha = -1 \): Kou’s double exponential model (KJ), finite activity.
  – \( \alpha = 0 \): Madan’s VG model, infinite activity, finite variation.
  – \( \alpha = 1 \): Cauchy dampened by exponential functions (CJ), infinite variation.
Option Pricing Under SSM

- Our specifications are within the very general framework of time-changed Lévy processes (Carr&Wu, JFE 2004).

- Following the theorem in Carr&Wu, we can derive the generalized Fourier transform function (FT) of the currency return in closed forms.
  - Derive the characteristic exponent of the Lévy process
  - Convert the FT of the currency return into the Laplace transform of the random time change.
  - Derive the Laplace transform following the bond pricing literature.

- Given the FT, we price options using fast Fourier transform (FFT) (Carr&Madan, 1999; Carr&Wu, 2004).

- Analytical tractability and pricing speed are similar to that for the Heston model and the Bates model.
The Characteristic Exponents of Lévy Processes

- The Lévy-Khintchine Theorem describes all Lévy processes via their characteristic exponents:

\[ \psi_x(u) \equiv \ln E e^{iuX_1} = -iub + \frac{u^2\sigma^2}{2} - \int_{\mathbb{R}_0} \left( e^{iu} - 1 - iux 1_{|x|<1} \right) k(x) dx. \]

- The Lévy density \( k(x) \) specifies the arrival rate as a function of the jump size:

\[ k(x) \geq 0, x \neq 0, \quad \int_{\mathbb{R}_0} (x^2 \wedge 1) k(x) dx < \infty. \]

- To obtain tractable models, choose the Lévy density of the jump component so that the above integral can be done in closed form.
## Characteristic Exponents For Dampened Power Law

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha$</th>
<th>Right (Up) Component</th>
<th>Left (Down) Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>KJ</td>
<td>-1</td>
<td>$\psi^D - iu\lambda \left[\frac{1}{1-iuv_j} - \frac{1}{1-v_j}\right]$</td>
<td>$\psi^D + iu\lambda \left[\frac{1}{1+iuv_j} - \frac{1}{1+v_j}\right]$</td>
</tr>
<tr>
<td>VG</td>
<td>0</td>
<td>$\psi^D + \lambda \ln (1 - iuv_j) - iu\lambda \ln (1 - v_j)$</td>
<td>$\psi^D + \lambda \ln (1 + iuv_j) - iu\lambda \ln (1 + v_j)$</td>
</tr>
<tr>
<td>CJ</td>
<td>1</td>
<td>$\psi^D - \lambda (1/v_j - iu) \ln(1 - iuv_j)$ $+ iu\lambda (1/v_j - 1) \ln(1 - v_j)$</td>
<td>$\psi^D - \lambda (1/v_j + iu) \ln(1 + iuv_j)$ $+ iu\lambda (1/v_j + 1) \ln(1 + v_j)$</td>
</tr>
<tr>
<td>CG</td>
<td>$\alpha$</td>
<td>$\psi^D + \lambda \Gamma(-\alpha) \left[\left(\frac{1}{v_j}\right)^\alpha - \left(\frac{1}{v_j} - iu\right)^\alpha\right]$</td>
<td>$\psi^D + \lambda \Gamma(-\alpha) \left[\left(\frac{1}{v_j}\right)^\alpha - \left(\frac{1}{v_j} + iu\right)^\alpha\right]$</td>
</tr>
</tbody>
</table>

$\psi^D = \frac{1}{2} \sigma^2 (iu + u^2)$

---

$\lambda, \Gamma$ denote the damping parameter and Gamma function, respectively.
The new measure $\mathbb{M}$ is absolutely continuous with respect to the risk-neutral measure $\mathbb{Q}$ and is defined by a complex-valued exponential martingale,

$$\frac{d\mathbb{M}}{d\mathbb{Q}}_t \equiv \exp \left[ iu \left( L_{T_t}^R - \xi_T R_T^{R_T} \right) + iu \left( L_{T_t}^L - \xi_T L_T^{L_T} \right) + \psi_T R_T^R + \psi_T L_T^L \right].$$

Girsanov’s Theorem yields the (complex) dynamics of the relevant processes under the complex-valued measure $\mathbb{M}$. 

---

**CF of Return as LT of Clock**

\[
\phi_s(u) \equiv E^Q e^{iu \ln(S_T/S_0)} = e^{iu(r_d-r_f)t} E^Q \left[ e^{iu \left( L_{T_t}^R - \xi_T R_T^{R_T} \right) + iu \left( L_{T_t}^L - \xi_T L_T^{L_T} \right) } \right] = e^{iu(r_d-r_f)t} E^\mathbb{M} \left[ e^{-\psi_T T_t} \right] \equiv e^{iu(r_d-r_f)t} L^{\mathbb{M}}_T(\psi),
\]
The Laplace Transform of the Stochastic Clocks

- We chose our two new clocks to be continuous over time:
  \[ T^j_t = \int_0^t v^j_s \, ds, \quad j = R, L, \]
  where for tractability, the activity rates \( v^j \) follow square root processes.
- As a result, the Laplace transforms are exponential affine:
  \[ \mathcal{L}^M_T(\psi) = \exp \left( -b^R(t)v^R_0 - c^R(t) - b^L(t)v^L_0 - c^L(t) \right), \]
  where:
  \[ b^j(t) = \frac{2\psi^j(1 - e^{-\eta^j t})}{2\eta^j - (\eta^j - \kappa^j)(1 - e^{-\eta^j t})}, \]
  \[ c^j(t) = \frac{\kappa}{\sigma^2 v} \left[ 2 \ln \left( 1 - \frac{\eta^j - \kappa^j}{2\eta^j} \left( 1 - e^{-\eta^j t} \right) \right) + (\eta^j - \kappa^j)t \right], \]
  and:
  \[ \eta^j = \sqrt{\kappa^j}^2 + 2\sigma_v^2 \psi^j, \quad \kappa^j = \kappa - iu\rho^j \sigma \sigma_v, \quad j = R, L. \]
- Hence, the CFs of the currency return are known in closed form for our models.
We estimate six models: **HSTSV, MJDSV, KJSSM, VGSSM, CJSSM, CGSSM.**

Using quasi-maximum likelihood method with unscented Kalman filtering (UKF).

- State propagation equation: The time series dynamics for the 2 activity rates:
  \[ dv_t = \kappa^P (\theta^P - v_t) dt + \sigma_v \sqrt{v_t} dZ, \quad (2 \times 1) \]

- Measurement equations: \( y_t = O(v_t; \Theta) + e_t, \quad (40 \times 1) \)
  * \( y \) — Out-of-money option prices scaled by the BS vega of the option.
  * Turn option price into the volatility space.

- UKF generates efficient forecasts and updates on conditional mean and co-variance of states and measurements sequentially (from \( t = 1 \) to \( t = N \)).

- Maximize the following likelihood function to obtain parameter estimates:
  \[ L(\Theta) = \sum_{t=1}^{N} l_{t+1}(\Theta) = -\frac{1}{2} \sum_{t=1}^{N} \left[ \log |A_t| + \left( (y_{t+1} - \bar{y}_{t+1})^\top \left( A_{t+1} \right)^{-1} (y_{t+1} - \bar{y}_{t+1}) \right) \right]. \]
<table>
<thead>
<tr>
<th></th>
<th>HSTSV</th>
<th>MJDSV</th>
<th>KJSSM</th>
<th>VGSSM</th>
<th>CJSSM</th>
<th>CGSSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>JPY: rmse</td>
<td>1.014</td>
<td>0.984</td>
<td>0.822</td>
<td>0.822</td>
<td>0.822</td>
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<td>GBP: rmse</td>
<td>0.445</td>
<td>0.424</td>
<td>0.376</td>
<td>0.376</td>
<td>0.376</td>
<td>0.378</td>
</tr>
</tbody>
</table>

- SSM models with different jumps perform similarly.
- All SSM models perform much better than MJDSV, which is better than HSTSV.
Likelihood Ratio Tests

Under the null $H_0 : E[l_i - l_j] = 0$, the statistic $(M)$ is asymptotically $N(0, 1)$.

<table>
<thead>
<tr>
<th>Curr</th>
<th>$M$</th>
<th>HSTSV</th>
<th>MJDSV</th>
<th>KJSSM</th>
<th>VGSSM</th>
<th>CJSSM</th>
<th>CGSSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>JPY</td>
<td>HSTSV</td>
<td>0.00</td>
<td>-2.55</td>
<td>-4.92</td>
<td>-4.88</td>
<td>-4.75</td>
<td>-4.67</td>
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<tr>
<td>JPY</td>
<td>MJDSV</td>
<td>2.55</td>
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<td>-5.07</td>
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<tr>
<td>JPY</td>
<td>KJSSM</td>
<td>4.92</td>
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<td>JPY</td>
<td>VGSSM</td>
<td>4.88</td>
<td>5.33</td>
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<td>0.00</td>
<td>-0.72</td>
<td>-1.21</td>
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<td>0.00</td>
<td>-1.59</td>
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<td>4.67</td>
<td>5.07</td>
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<tr>
<td>GBP</td>
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<tr>
<td>GBP</td>
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<td>3.85</td>
<td>0.00</td>
<td>-0.04</td>
<td>0.34</td>
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<td>GBP</td>
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<td>0.00</td>
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<td>GBP</td>
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<td>-0.56</td>
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<td>GBP</td>
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<td>4.71</td>
<td>4.19</td>
<td>0.37</td>
<td>0.39</td>
<td>0.51</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The outperformance of SSM models over MJDSV/HSTSV is statistically significant.
## Out of Sample Performance Comparison

<table>
<thead>
<tr>
<th></th>
<th>JPYUSD</th>
<th>GBPUSD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HSTSV</strong></td>
<td>0.00</td>
<td>-3.34</td>
</tr>
<tr>
<td><strong>MJDSV</strong></td>
<td>2.14</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>KJSSM</strong></td>
<td>4.44</td>
<td>4.42</td>
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<td><strong>VGSSM</strong></td>
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<td><strong>CGSSM</strong></td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>rmse</th>
<th>$\mathcal{L}/N$</th>
<th>$\mathcal{M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HSTSV</strong></td>
<td>1.04</td>
<td>-23.69</td>
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</tr>
<tr>
<td><strong>MJDSV</strong></td>
<td>1.02</td>
<td>-23.03</td>
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<tr>
<td><strong>KJSSM</strong></td>
<td>0.85</td>
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<tr>
<td><strong>VGSSM</strong></td>
<td>0.85</td>
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<td><strong>CJSSM</strong></td>
<td>0.85</td>
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<td><strong>CGSSM</strong></td>
<td>0.85</td>
<td>-16.47</td>
<td>-4.61</td>
</tr>
</tbody>
</table>

**In-Sample Performance: 1996-2001**

Similar to the whole-sample performance
Out of Sample Performance Comparison

<table>
<thead>
<tr>
<th></th>
<th>JPYUSD</th>
<th>GBPUSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>rmse</td>
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<tr>
<td>$\mathcal{L}/N$</td>
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<tr>
<td>$\mathcal{M}$</td>
<td>-18.47</td>
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<tr>
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<tr>
<td>CGSSM</td>
<td>6.12</td>
<td>3.48</td>
</tr>
</tbody>
</table>

Out of-Sample Performance: 2001-2002

Similar to the in-sample performance: SSMs >> MJDSV >> HSTSV

Infinite variation jumps are more suitable to capture large smiles/skews.
Theory (Bates) and Evidence on Stochastic Volatility

Top panels: Implied vol (data). Bottom panels: Activity rates (from Bates model)

Demand for calls on yen drives up the activity rate during the 98 hedge fund crisis.
Theory (SSM) and Evidence on Stochastic Volatility

Top panels: Implied vol (data). Bottom panels: Activity rates (from KJSSM)

The demand for yen calls only drives up the activity rate of upward yen moves (solid line), but not the volatility of downward yen moves (dashed line).
Theory and Evidence on Stochastic Skew

Three-month ten-delta risk reversals: data (dashed lines), model (solid lines).

Bates:

SSM:
Conclusions

• Using currency option quotes, we find that under a risk-neutral measure, currency returns display not only stochastic volatility, but also **stochastic skew**.

• State-of-the-art option pricing models (e.g. Bates 1996) capture stochastic volatility and **static** skew, but not stochastic skew.

• Using the general framework of time-changed Lévy processes, we propose a class of models (SSM) that capture both stochastic volatility and **stochastic skewness**.

• The models we propose are also highly tractable for pricing and estimation. The pricing speed is comparable to the speed for the Bates model.

• Future research: The economic underpinnings of the stochastic skew.