Comprehensive Exam: Options Markets
Professor Liuren Wu
Date: September, 2007

Design a tractable model for the stock return that has the following key features:

1. At-the-money stock option implied volatility varies strongly over time.
2. The stock return is correlated with the S&P 500 index return
3. The at-the-money stock option implied volatility positively co-moves with the VIX, the volatility index on S&P 500 index.
4. When plotted against a standard measure of moneyness (say, \( \ln(K/S) / \sqrt{\tau} \), with \( K, S, \tau \) denoting the strike, the spot price, and the time-to-maturity, respectively), the implied volatilities show a U-shaped smile pattern at short maturities (say 30-day options), but a negatively skewed pattern at very long maturities (say 1-2 years).
5. The company currently has a credit rating of B.

For the exam, you are asked to do the following:

(a) Write down the return dynamics under the risk-neutral measure. There are potentially many models that can capture the above feature. Just use your imagination, and make sure that the model is simple and tractable enough for you to answer the subsequent questions. (30%)

(b) Discuss which part of your model can be used to capture which of the above features. (40%)

(c) Derive the generalized Fourier transform of the currency return under your model specification. Make a list of the free parameters in the model that you need to estimate, assuming that you are focusing on the risk-neutral dynamics and will estimate the model using stock options (and possibly also S&P 500 index options –You can decide whether you want to include this or not). (30%)